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THE D_p-METRIC SPACE OF SET-VALUED RANDOM VARIABLES AND ITS APPLICATION TO COVARIANCES

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Abstract. *In this paper, we introduce the D_p-metric of set-valued random variables. We prove that the D_p-metric space of set-valued random variables is complete, and represent the equivalent propositions of convergence for set-valued random variables with respect to D_p-metric. Finally, we give several definitions of the variance, covariance and correlation coefficient of set-valued random variables by the D_p-metric, and discuss some useful results.*

Keywords: Set-valued random variable, D_p-metric, Convergence in D_p-metric, Covariance

1. **Introduction.** A set-valued random variable is a random variable whose values are the subsets of d-dimensional Euclidean space $\mathbb{R}^d$. In the 60's of the 20th century, a large number of economists represented the concept of a set-valued mapping and did a lot of work in the economic equilibrium. Since Aumann [**?**] introduced the concept of the integral of a set-valued mapping in 1965, many researchers naturally considered the set-valued random variables as measurable set-valued mappings. Debreu[**?**] gave another type of definition of the integral of a set-valued mapping. And after that, the authors discussed its equivalence to the Aumann integral under some conditions, and gave the Random-Nikodym theorem of set-valued mappings in 1970's. This not only sets a theoretic foundation for the research of economic problems, but also presents a new research field as a new branch of modern probability——the theory of set-valued random variables. Through the development of more than 30 years, there have been many important contributions in this branch, for example, Artstein, Vitale etc (cf. [**?**],[**?**],[**?**], [**?**],[**?**]) proved laws of large numbers and central limit theorems for set-valued random variables, Hiai etc (cf.[**?**],[**?**]–[**?**]) obtained some results on convergence theorems of set-valued martingales.

The Hausdorff metric d_H is often used when investigating the convergence of the sequence of set-valued random variables. But it is too strong sometimes. For example, on discussing set-valued martingales, even $\{X_n, \mathcal{F}_n : n \in N\}$ is a martingale generated by an integrable set-valued random variable X, i.e. $X_n = E[X|\mathcal{F}_n]$, it can't be concluded that $\{X_n\}$ converges to X in Hausdorff (cf. Li and Ogura[**?**]). So it is necessary to discuss a little weaker but useful convergence. In this paper we shall discuss the D_p-metric of set-valued random variables and give its applications.