International Journal of Innovative Computing, Information and Control
Volume **1**, Number **1**, March **2005**

ICIC International ©2005 ISSN 1349-4198
pp. 1–16

A NEW PROOF OF MCCANN'S THEOREM AND THE GENERALIZATION OF LYAPUNOV'S EQUATION TO NONLINEAR SYSTEMS

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Received October 2004; revised January 2005

Abstract. *This paper gives a simple proof of McCann's theorem for smooth dynamical systems based on the solution of partial differential equations. This theorem basically states that globally asymptotically stable dynamical systems are equivalent to linear systems. The new approach is then applied to the generalization of Lyapunov's equation for the stability of linear dynamical systems to globally asymptotically stable nonlinear systems by defining a Lyapunov equation for the transformed nonlinear system. This is a similar result as the generalization obtained by means of a Carleman linearization of an analytic globally asymptotically stable nonlinear system.*
Keywords: Global stability, Lyapunov equation, Lyapunov function, Lyapunov methods, Nonlinear systems

1. Introduction.

The use of Lyapunov's equation

$$A^T P + PA = -Q \tag{1}$$

where Q is any positive definite symmetric matrix, is fundamental in the classical theory of linear system stability. If the equation (1) has a solution for P which is positive definite and symmetric, then the linear system

$$\dot{x} = Ax$$

is asymptotically stable and a Lyapunov function is given by

$$V = \langle x, Px \rangle.$$

Moreover, the converse result is true.

It has been shown before [8] that it is possible to generalize a Lyapunov equation for nonlinear analytic systems by using a Carleman linearization of the nonlinear system. Practically, this Carleman linearization gives an approximation to the solutions of the nonlinear system and has been used before in different control problems, such as in the design of observers with linear error dynamics [4] and in the estimation of domains of attraction of nonlinear feedback systems [6].

From [8], a nonlinear analytic system of the form