*International Journal of Innovative Computing, Information and Control*
Volume **2**, Number **4**, August **2006**
 ISSN 1349-4198
pp. 863–874

# OPTIMAL FILTERING FOR NONLINEAR POLYNOMIAL SYSTEMS OVER LINEAR OBSERVATIONS WITH DELAY

Michael Basin, Joel Perez
Department of Physical and Mathematical Sciences
Autonomous University of Nuevo Leon
Cd. Universitaria, San Nicolas de los Garza
Nuevo Leon, 66450, Mexico
mbasin@fcfm.uanl.mx; joelperezp@yahoo.com

Rodolfo Martinez-Zuñiga
Department of Electrical and Mechanical Engineering
Autonomous University of Coahuila
Calle Barranquilla, S/N, Col. Guadalupe
Monclova, Coahuila, 25750, Mexico
rmz10474@mail.uadec.mx

Received September 2005; revised January 2006

Abstract. *In this paper, the optimal filtering problem for nonlinear systems over linear observations with time delay is investigated proceeding from the general expression for the stochastic Ito differential of the optimal estimate and the error variance. As a result, the Ito differentials for the optimal estimate and error variance corresponding to the stated filtering problem are first derived. The procedure for obtaining a closed system of the filtering equations for a polynomial state over linear observations with delay is then established, which yields the explicit closed form of the filtering equations in the particular case of a bilinear system state. In the example, performance of the designed optimal filter is verified for a quadratic state over linear observations with delay against the optimal bilinear filter over linear observations without delay and the conventional extended Kalman-Bucy filter.*
**Keywords:** Filtering, Stochastic system, Nonlinear system, Time-delay system

1. **Introduction.** Although the general optimal solution of the filtering problem for nonlinear state and observation equations confused with white Gaussian noises is given by the Kushner equation for the conditional density of an unobserved state with respect to observations [1], there are a very few known examples of nonlinear systems where the Kushner equation can be reduced to a finite-dimensional closed system of filtering equations for a certain number of lower conditional moments. The most famous result, the Kalman-Bucy filter [2], is related to the case of linear state and observation equations, where only two moments, the estimate itself and its variance, form a closed system of filtering equations. However, the optimal nonlinear finite-dimensional filter can be obtained in some other cases, if, for example, the state vector can take only a finite number of admissible states [3] or if the observation equation is linear and the drift term in the state equation satisfies the Riccati equation $df/dx + f^2 = x^2$ (see [4]). The complete classification of the "general situation" cases (this means that there are no special assumptions on the structure