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NUMERICAL SOLUTIONS OF TIME-VARYING GENERALIZED DELAY SYSTEMS VIA GENERAL LEGENDRE WAVELETS

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Abstract. *General Legendre wavelets are presented. Using this method the delay function is expanded by the general Legendre wavelets. The property of the general operational matrix of integration is given and the direct algorithm for a product of a matrix function and a vector function is proposed. Furthermore the time-varying generalized delay systems are solved by the general Legendre wavelets. It is shown that the results can be applied to optimal control problem.*
Keywords: Generalized delay systems, General Legendre wavelets, Optimal control

1. **Introduction.** Many orthogonal functions or polynomials, such as Walsh [1], block-pulse [2], Chebyshev [3], Laguerre [4], Legendre [5], and Fourier [6], were developed for solving various problems of dynamic systems. If approximating a function which has inherent discontinuities, these methods may model the discontinuities unnaturally. Motivated by the hybrid functions [7, 8, 9] and Legendre wavelets [10], we provide the general Legendre wavelets to solve the time-varying generalized delay systems. The advantages of this method are that we can divide the integration internal arbitrarily and choose series numbers willfully. So the results will model the discontinuities properly. We derive the direct algorithm for a product of a matrix function and a vector function, the formula of delay functions and the general operational matrix by the general Legendre wavelets expression. By the general Legendre wavelets the time-varying generalized delay systems are reduced to the algebraic equations. Applying the results to the optimal control problem we obtain the approximate solutions of the optimal control and state as well as the optimal value of the objective functional. A numerical example is illustrated.

2. **Preliminaries.**

2.1. **Definitions.** General Legendre wavelets $\psi_{nm}(t),\ n = 1, 2, \ldots, N,\ m = 0, 1, \ldots, M-1$ on the interval $[t_0, T)$ are defined as

$$\psi_{nm}(t) = \begin{cases} [(2m+1)d_n^{-1}]^{\frac{1}{2}} L_m(d_n^{-1}(2t - t_{n-1} - t_n)),\ t_{n-1} \le t < t_n, \\ 0,\ \text{otherwise}, \end{cases}$$