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A BLOCK RECURSIVE ALGORITHM FOR THE LINEAR COMPLEMENTARITY PROBLEM WITH AN M-MATRIX

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Abstract. *We have proposed an $O(n^3)$ recursive algorithm for the linear complementarity problem LCP(A, q), where A is an M-matrix. In this paper, a block form of the algorithm is presented. Many numerical examples show that the block version takes fewer number of the arithmetic operations than the non-block version.*
Keywords: Linear complementarity problem, M-matrix, Block, Recursive algorithm

1. **Introduction.** It is well known that the following Linear Complementarity Problem [1] often appears in fields of the mathematical programming.

LCP(A, q): Let $A \in R^{n\times n}$ and $q \in R^n$, finding one or all real vectors z with satisfying

$$Az + q \geq 0, \qquad z \geq 0, \qquad z^T(Az + q) = 0. \tag{1}$$

A matrix $A \in R^{n\times n}$ is called a P-matrix if all of its principal minors are positive. It is well known that for any real vectors $q \in R^n$, LCP(A, q) has an unique solution if and only if A is a P-matrix [2]. Testing whether an n-by-n real matrix is a P-matrix, seems inevitably of exponential time complexity. As it is shown in Coxson [3], this problem is co-NP-complete. The time complexity for testing P-matrix problem has been reduced [4] from $O(n^3 2^n)$ to $O(2^n)$ by applying recursively a criterion for P- matrices based on Schur complementation. For solving LCP(A, q), some traditional methods, including the principal method and the complementary algorithm have been shown [2]. Recently, some authors discussed the verification methods [5] for LCP(A, q), and the multisplitting methods [1,6] for large sparse LCP(A, q).

If A is a special matrix, it is an interesting problem to show the computational time complexity for direct algorithms of the LCP(A, q). Y. Fathi presented an $O(2^n)$ computational time complexity for LCP(A, q) associated with positive definite matrix A by the two well known complementary pivot methods [7]. But [10] shown that the positive semidefinite LCP can be solved in a (weak) polynomial time with $O(n^3L)$ arithmetic operations. We know that M-matrices $A = (a_{ij})_{n\times n}$ gives an important class of special matrices with satisfying

$$a_{ii} > 0, \qquad a_{ij} \leq 0, \quad i \neq j, \qquad A^{-1} \geq 0,$$

We have proposed an $O(n^3)$ recursive algorithm for solving LCP(A, q) with A being an $n \times n$ M-matrix [8]. In this paper, we will consider its block version.