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RISK-SENSITIVE APPROACH TO OPTIMAL FILTERING AND CONTROL FOR LINEAR STOCHASTIC SYSTEMS

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Abstract. *The optimal exponential quadratic control problem and exponential mean-square filtering problems are considered for stochastic Gaussian systems with polynomial first degree drift terms and intensity parameters multiplying diffusion terms in the state and observations equations. To the best of the author's knowledge, this is the first paper that designs the optimal closed-form regulator and filter with respect to the exponential quadratic and mean-square criteria. The closed-form optimal control and filtering algorithms are obtained using quadratic value functions as solutions to the corresponding Hamilton-Jacobi-Bellman equations. The performance of the obtained risk-sensitive regulator and filter for stochastic first degree polynomial systems is verified in a numerical example against the conventional linear-quadratic regulator and Kalman-Bucy filter, through comparing the exponential quadratic and exponential mean-square criteria values. The simulation results reveal strong advantages in favor of the designed risk-sensitive algorithms in regard to the final criteria values.*
Keywords: Risk-sensitive filtering and control, Stochastic systems

1. **Introduction.** After the optimal linear stochastic control problem was solved (see [1], [2]), the optimal control theory for nonlinear stochastic systems is based on dynamic programming (Hamilton-Jacobi-Bellman) equation [2] and the maximum principle of Pontryagin [3]. A long tradition of the optimal control design was developed for nonlinear systems with respect to a quadratic Bolza-Meyer criterion (see, for example, [4]-[10]). The optimal control problems with respect to nontraditional criteria were also considered: the stochastic linear exponential quadratic regulator (LEQR) problem was introduced in [11], where it is formulated for both, discrete and continuous, systems. Besides, it was demonstrated that the solutions to LEQR problems are equivalent to the solutions of cooperative and noncooperative linear-quadratic zero-sum differential games. Analysis of nonlinear stochastic LEQR problems and differential games was conducted in [12]. Further connection between the LEQR problem and H_∞ control via a minimum entropy principle was given in [13]. Whittle ([14], [15]) considered problems on a finite-time horizon, using "small-noise" asymptotics. When the process being controlled is governed by stochastic differential equation, the Whittle's formula for the optimal large-derivations rate was obtained using partial differential equation viscosity solution method in [12], [16] and it is proposed in [17], [18]. Runolfsson [19], [20] used Ponsker-Varadham-type large-derivations ideas to obtain a corresponding stochastic differential game for which the game payoff is an ergodic (expected average cost per unit time) criterion. The associated state-space formulation was found via Fourier analysis (see Francis [21], Dyn and McKean [22]). This state-space formulation can naturally be viewed in terms of two-player, zero-sum games