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OPTIMAL H_2 MODEL REDUCTION FOR MECHANICAL SYSTEMS

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Abstract. *The H_2 optimal model reduction problem is addressed to obtain reduced order models for mechanical systems described by matrix second order linear systems such that the H_2 norm of the error system is minimized. The structure of the original second order model is preserved in the model reduction procedure and the reduced order models are guaranteed to be stable. The expression of the error and its gradient are explicitly given in terms of the solutions of certain Lyapunov equations. An effective algorithm is derived to obtain the reduced order model. Two numerical examples are provided to illustrate the effectiveness of the proposed method.*
Keywords: Mechanical system, Model reduction, Matrix second order linear system, H_2 norm.

1. **Introduction.** To describe the behaviour of complex physical systems accurately, high or even infinite order mathematical models are often required to model them, such as, economic and ecological models, flexible structure, flutter suppression in aircraft, satellite pointing and slewing, spaceborne aperture shape control systems, finite element models, and power and communication networks [16]. However, high order models are more difficult to handle in control design problems from both engineering and computational perspectives. So, the approximation of a high order system by a low order model is one of the important problems in the system theory since the use of reduced order model can often obtain a better insight into the system behaviour, and make it easier to carry out analyses, simulations and control designs. A general requirement in any model reduction procedures is that the approximate systems must preserve certain properties of the original systems, such as stability. Moreover, model reduction should have a small approximation error in some well-defined sense. Consequently, several methods have been suggested for determining the low order reduced model in the literature, as can be seen in [3, 4, 5, 11, 12, 15, 17, 18, 20, 21, 24, 26]. The optimal model reduction under the H_2 specification is to minimize the root mean square value of the model reduction error over the entire frequency spectrum, which has received most of the attention in the past few years. The characterization of the optimal solutions by state-space methods are given in [9, 19]. Convergent algorithms are used to solve the frequency weighted L_2 model reduction for a given stable linear system [23]. A gradient flow formulation of the approximated H_2 model reduction problem is given in [22], while in a subsequent development [8], a near H_2 optimal solution is obtained by a well-posed smooth constrained minimization