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ON THE SIGN VARIATIONS OF A CLASS OF STURM CHAINS INITIATED BY FORWARD AND BACKWARD POLYNOMIALS

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Abstract. *We study in this paper the sign variations of Sturm sequences initiated by forward and backward polynomials respectively, where a backward polynomial is obtained by reversing the coefficient sequence order of the original, called forward polynomial, and the Sturm sequences are generated by the Euclidean-like algorithm. We show under non-singularity assumption that the numbers of sign variations in the leading and constant terms of the Sturm sequence initiated by forward polynomials are the same as that of sign variations in the constant and leading terms ,respectively, of the Sturm sequence initiated by backward polynomials. A simpler version of the result as the initial polynomials are of degree 3 and 2 is alternatively derived via matrix determinant calculation. Other numerical examples are also provided to illustrate our work.*
Keywords: Sturm chain, Cauchy index, Routh criterion, Argument principle, Zero distribution of continuous-time system

1. **Introduction.** The Sturm chain, or Sturm sequence is a powerful symbolic tool in the zero distribution analysis of polynomials. Its usefulness is best known in the application of Sturm's theorem, which allows one to determine the number of a polynomial's distinct real roots in a specified interval. A more important application of the chain is to the analysis of zero distribution with respect to the imaginary axis. The most successful example should be from Routh (1875), who proposed an algorithm to identify the number of zeros of a real polynomial in the right half plane [2]. He suggested to construct the so-called Routh table by generating a special Sturm chain. By Sturm theorem, Cauchy index and the Argument principle, it can be shown that the number of sign variations in the first column of the table is equal to the number of zeros in the right half plane. If the polynomial represents the characteristic function of a linear continuous-time system, the criterion for the system to be stable is that the induced Routh table has no sign variation in the first column. Among various approaches to solve similar problems [1, 5, 8, 13, 16], including the classical works by Hermite (1854), Lyapunov (1892), Hurwitz (1896), and Lienard-Chipart (1914), the Sturm chain and its concept play the essential role in dealing with the problem of zero distribution of a polynomial. In spite of its main function in serving as a premise, to validate the applicability of the Sturm theorem, hardly, however, is the chain studied concerning other properties, for example, the dependence of the sign changes on initial polynomials. This deficiency motivates our work in this paper and hopefully our pioneering exploration inspires more research activity along this line.

It is not difficult to see that the number of sign changes of the chain is subject to two factors. That is: 1. the initial polynomials from which the sequence starts; and 2. the value at which the sequence is evaluated. In this work, efforts are thus made to explore