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SOLVING A CLASS OF SADDLE POINT PROBLEMS BY NEURAL NETWORKS

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Abstract. *In this paper, we propose a differential inclusion for solving a wider class of saddle point problems (not necessarily smooth). The nonsmoothness and the mixed linear equality constraints are the two significant characters of the problem considered in this paper. Under a suitable assumption on the feasible region, we prove the global existence and uniqueness of the solution to the differential inclusion. Moreover, we get some convergence results about the solution to the differential inclusion and the exactness of the proposed differential inclusion. Furthermore, one illustrative example further demonstrates the effectiveness and characteristics of the proposed equation modeled by a differential inclusion.*

Keywords: Convergence in finite time, Differential inclusion, Nonsmooth saddle point problems, Generalized gradient

1. Introduction.

Consider the following saddle point problem:

$$\begin{aligned} & V(x,y) = V_1(x) - V_2(y), \\ \text{subject to} \quad & G(x,y) \le 0,\ B_1x + B_2y = b. \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $V_1(x) : \mathbb{R}^n \to \mathbb{R}$ and $V_2(y) : \mathbb{R}^m \to \mathbb{R}$ are convex functions (not necessarily smooth), $G = (G_1, G_2, \cdots, G_r)^T : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^r$ is an r-dimensional vector-valued function of $n \times m$ variables and $G_i(i = 1, 2, \cdots, r) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ are convex functions(not necessarily smooth), $B_1 \in \mathbb{R}^{s\times n}$, $B_2 \in \mathbb{R}^{s\times m}$ and $b \in \mathbb{R}^s$.

Throughout this paper, we denote the region where the constraints are satisfied (feasible region) is given as $\mathcal{S} = \{(x, y) \in \mathbb{R}^{n+m} : \ G(x, y) \le 0 \ and \ B_1x + B_2y - b = 0\}$. Moreover, we denote $\mathcal{S}_1 = \{(x, y) \in \mathbb{R}^{n+m} : G(x, y) \le 0\}$, $\mathcal{S}_2 = \{(x, y)^T \in \mathbb{R}^{n+m} : B_1x + B_2y - b = 0\}$.

A point $(x^*, y^*) \in \mathcal{S}$ is said to be a saddle point of $V(x, y)$ if

$$V(x^*, y) \le V(x, y^*),\ \forall (x, y) \in \mathcal{S}. \tag{2}$$

It is well-known that the constrained saddle point problem which is a kind of constrained optimization problem provides a useful reformulation of optimality conditions and also arises in a variety of engineering and economic contexts including game theory, military scheduling, automatic control, and so on. In many engineering and scientific

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