CONTROLLABILITY, OBSERVABILITY AND STABILIZABILITY OF
A CLASS OF MATRIX LINEAR SYSTEMS

BIN ZHOU, GUANG-REN DUAN AND ZHEN ZHONG
Center for Control Theory and Guidance Technology
Harbin Institute of Technology
Harbin, 150001, Heilongjiang, P. R. China
binzhou@hit.edu.cn, binzhou@msn.com (B. Zhou)

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Abstract. In this paper we consider the controllability, observability and stabilizability
properties of a class of matrix linear systems. For controllability and observability, nec-
essary and sufficient conditions are derived. Based on these conditions, stabilizability of
such class of matrix linear systems by state feedback is also considered.

Keywords: Matrix linear systems, Controllability, Observability, Stabilizability by state
feedback

1. Introduction. The description of plant dynamics by vector equations is a very com-
mon one and has been well studied in the past few decades (e.g., [3], [6], [10] and [16]).
However, there are problems in which the variables are most naturally described by means
of matrices, like the inertia, stiffness, and damping matrices of a given structural system.
This is the case, for instance, of the direction cosine matrix in rigid body kinematics
problems (see, for example, [7], pp.512), and of the estimation error covariance matrix in
a Kalman-Bucy filter. Consider the problem of analysis and design for a matrix plant,
where the state is presented by matrix. In principle, the tools for tackling these analysis
and design problems are already available. After all, one can decompose the matrix plant
into a set of vector equations and proceed with the application of the conventional tools
for conventional vector linear systems (see, for example [8] and [17]). One drawback of
that approach, however, is the loss of physical insight in the vectorized analysis frame-
work. For a high-dimensional model, an excessive number of equations result and it may
be almost impossible to determine any structure and properties of the solution.

For the reasons mentioned above, some works have been done directly on the original
matrix linear systems. In [12]-[13], a closed form solution to a class of special matrix linear
systems was obtained via explicit solutions to its corresponding scalar differential equa-
tion. In a recent paper [4], Kalman filter for a stochastic linear time-varying discrete-time
plant with a state matrix observed by matrix measurements is considered. The authors
have proposed a general state matrix Kalman filter (MKF) for this plant. The MKF has
the statistical properties of the ordinary Kalman filter while retaining the advantages of
a compact matrix notation by expressing the estimated matrix in terms of the original
plant parameters. In [1], the elementary methods and operator identities are used to
solve linear matrix differential equations in the simple form $\dot{X}(t) = AX(t) + F(t)$, where
$X$ is a matrix, and explicit formulas are obtained. In [11], a class of matrix differential
equations, such as error covariance propagation is also considered by using the original
system parameters. Very recently, the authors present a closed-form solutions to a class of
matrix linear systems by using the so-called double matrix exponential functions without
transforming the matrix linear system into its equivalent vectorized form ([2]). These