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CONTROLLABILITY, OBSERVABILITY AND STABILIZABILITY OF A CLASS OF MATRIX LINEAR SYSTEMS

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Abstract. *In this paper we consider the controllability, observability and stabilizability properties of a class of matrix linear systems. For controllability and observability, necessary and sufficient conditions are derived. Based on these conditions, stabilizability of such class of matrix linear systems by state feedback is also considered.*
Keywords: Matrix linear systems, Controllability, Observability, Stabilizability by state feedback

1. **Introduction.** The description of plant dynamics by vector equations is a very common one and has been well studied in the past few decades (e.g., [3], [6], [10] and [16]). However, there are problems in which the variables are most naturally described by means of matrices, like the inertia, stiffness, and damping matrices of a given structural system. This is the case, for instance, of the direction cosine matrix in rigid body kinematics problems (see, for example, [7], pp.512), and of the estimation error covariance matrix in a Kalman-Bucy filter. Consider the problem of analysis and design for a matrix plant, where the state is presented by matrix. In principle, the tools for tackling these analysis and design problems are already available. After all, one can decompose the matrix plant into a set of vector equations and proceed with the application of the conventional tools for conventional vector linear systems (see, for example [8] and [17]). One drawback of that approach, however, is the loss of physical insight in the vectorized analysis framework. For a high-dimensional model, an excessive number of equations result and it may be almost impossible to determine any structure and properties of the solution.

For the reasons mentioned above, some works have been done directly on the original matrix linear systems. In [12]-[13], a closed form solution to a class of special matrix linear systems was obtained via explicit solutions to its corresponding scalar differential equation. In a recent paper [4], Kalman filter for a stochastic linear time-varying discrete-time plant with a state matrix observed by matrix measurements is considered. The authors have proposed a general state matrix Kalman filter (MKF) for this plant. The MKF has the statistical properties of the ordinary Kalman filter while retaining the advantages of a compact matrix notation by expressing the estimated matrix in terms of the original plant parameters. In [1], the elementary methods and operator identities are used to solve linear matrix differential equations in the simple form $\dot{X}(t) = AX(t) + F(t)$, where X is a matrix, and explicit formulas are obtained. In [11], a class of matrix differential equations, such as error covariance propagation is also considered by using the original system parameters. Very recently, the authors present a closed-form solutions to a class of matrix linear systems by using the so-called double matrix exponential functions without transforming the matrix linear system into its equivalent vectorized form ([2]). These