International Journal of Innovative Computing, Information and Control
Volume **5**, Number **9**, September **2009**

ICIC International ©2009 ISSN 1349-4198
pp. 2791–2808

ON THE SUB-OPTIMAL FEEDBACK CONTROL LAW SYNTHESIS OF UNDERACTUATED SYSTEMS

J. Patricio Ordaz-Oliver[1], Omar J. Santos-Sánchez[2] and Virgilio López-Morales[2]

[1]Mechatronics Faculty
Universidad Politécnica de Pachuca
Ex. Hacienda Sta. Bárbara, Municipio de Zempoala, Hgo., México
patoordazoliver@yahoo.com.mx

[2]Centro de Investigación en Tecnologías de Información y Sistemas
Universidad Autónoma del Estado de Hidalgo
Carr. Pachuca-Tulancingo Km. 4.5, 42184, Pachuca, Hgo. México
omarj@uaeh.reduaeh.mx; virgilio@uaeh.edu.mx

Received June 2008; revised October 2008

Abstract. *In this paper, a complete solution on the suboptimal feedback control law synthesis for underactuated systems, based on the general optimization framework of the Dynamic Programming Theory is introduced. Control method proposed keeps the general structure of a suboptimal control approach, while the functional defining performance index is based on the underactuated system energy. Main application target of this contribution is on the electromechanical underactuated system stabilization about an unstable equilibrium point. This kind of system, has been stabilized by using two different control laws: the first one is used to swing up and the second one to balance around the unstable equilibrium point. Our main contribution is a single control synthesis methodology in order to swing up and asymptotically stabilize the underactuated system. In order to illustrate the application of the proposed technique, it is applied on two well-known systems: the Pendubot and the Rotatory Pendulum.*
Keywords: Non holonomic systems, Passivity, Dynamic programming theory, Euler-Lagrange model

1. **Introduction.** Since the late 1970s, trends in controlling degraded systems (underactuated), yielding a non-holonomic system, has grown interest in many fields.

An underactuated system can be found in different dynamical systems for instance: Flexible systems, degraded controlled systems, manipulator, spacecraft, degraded underwater vehicles, or systems designed as underactuated systems due to restrictions of cost weight, and complexity, as well as, some reliability advantages.

Underactuated system control schemes have different targets as regulation, trajectories tracking, obstacles avoidance, stabilization in an equilibrium point designed as a security zone among others. One approach addressing these issues is the optimal control on which present analysis is carried out.

In this paper, a synthesis of a suboptimal control for underactuated systems is proposed, which is based on the complete system energy analysis, the underactuated passivity properties, and the Lyapunov stabilization theory. The main contribution lies on the integration of dynamic programming theory, by introducing a functional defining performance index based on the complete system energy, which is used in the whole closed loop system control. I.e., it is used only one stabilizing control law in the control complete system workspace.