

A NUMERICAL SCHEME FOR APPROXIMATE OPTIMAL CONTROL OF NONLINEAR HYBRID SYSTEMS

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ABSTRACT. *This paper presents a measure-theoretical approach for finding approximate solution of complex optimal control problems. The general continuous optimal control problem for a class of hybrid dynamical systems is considered, and it is detailed that how to replace the problem by one in measure space and solve the equivalent finite-dimensional linear problem in a straightforward manner. Computational experiments show that the proposed algorithm could substantially reduce the solving time, compared with the other techniques.*

Keywords: Nonlinear hybrid system, Optimal control, Measure theory, Linear programming.

1. **Introduction.** Hybrid systems are characterized by the interaction of continuous models, logic rules, automata and other discrete components, each one governed by its own characteristic dynamical law. They can switch between many operating modes where each mode, typically, may be triggered internally (variables crossing specific thresholds, or the elapse of certain time periods), or externally (controlled input events).

Recently, there has been an even increasing interest in the study of such systems, which is mainly due to the problems' significance in theories and applications. They can be found in chemical processes, automotive systems, and embedded control systems, to name a few. Several researchers have investigated the problem of determining optimal control laws for hybrid systems and many results have appeared in the literature [1, 8, 16, 17, 25].

For continuous-time hybrid systems, they mostly either studied necessary optimality conditions for a trajectory [19, 21, 24], or used the maximum principle or convex dynamic programming to approximate hybrid optimal control laws [4, 14, 18, 20].

The hybrid optimal control problem becomes less complex when the dynamics is expressed in discrete-time, and some authors have determined optimal control sequences of discrete time hybrid systems through transforming the problem into a mixed-integer linear programming (MILP) problem [2, 25]. Examples of applications where hybrid modeling and optimal control based on MILP were employed are reported in [9, 11, 12].

The advantage of an MILP-approach is that it provides a general framework for modelling a large variety of problems, however, the major difficulty lies in the computational expense that may be involved in solving the resulting MILP. While great progress has