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A LINEAR-TIME ALGORITHM FOR THE RESTRICTED PAIRED-DOMINATION PROBLEM IN COGRAPHS*

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Abstract. *Let $G = (V, E)$ be a graph without isolated vertices. A matching in G is a set of independent edges in G. A perfect matching M in G is a matching such that every vertex of G is incident to an edge of M. A set $S \subseteq V$ is a paired-dominating set of G if every vertex not in S is adjacent to a vertex in S, and if the subgraph induced by S contains a perfect matching. The paired-domination problem is to find a paired-dominating set of G with minimum cardinality. This paper introduces a generalization of the paired-domination problem, namely, the restricted paired-domination problem, where some vertices are restricted so as to be in paired-dominating sets. Further, possible applications are also presented. We then present a linear-time constructive algorithm to solve the restricted paired-domination problem in cographs.*
Keywords: Graph algorithms, Linear-time algorithms, Paired-domination, Restricted paired-domination, Cographs

1. Introduction.

The problem of placing monitoring devices in a system such that every site in the system (including the monitoring devices themselves) is adjacent to a monitor and every monitor is paired with a backup monitor, can be modeled by paired-domination in graphs. There are applications in which it is also important that monitors are placed at critical sites (restricted set) in the system for instant monitoring; such an application can be modeled by a combination of paired-domination and a restricted set. In this paper, we consider this combination, called restricted paired-domination.

A set S of vertices of a graph $G = (V, E)$ is a *dominating set* of G if every vertex in $V - S$ is adjacent to a vertex in S. The *domination problem* is to find a dominating set of G with minimum cardinality. Variations of the domination problem seek to find a minimum dominating set with some additional properties, e.g., to be independent or to induce a connected graph. These problems arise in a number of distributed network applications, where the problem is to locate the smallest number of centers in networks such that every vertex is nearby at least one center. Domination and its variations in graphs have been thoroughly studied, and the literature on this subject has been surveyed and detailed in two books [13, 14].

A *matching* in a graph G is a set of independent edges in G. A *perfect matching* M in G is a matching in G such that every vertex of G is incident to an edge of M. A *paired-dominating set* of a graph G is a dominating set S of G such that the subgraph $G[S]$ induced by S contains a perfect matching M. Two vertices joined by an edge of M are said to be *paired*, and they are also called *partners* in S. Every graph without isolated

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