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THE ALGORITHM FOR COMPUTING EXACT MINKOWSKI SUM OF 3D CONVEX POLYHEDRAL

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Abstract. *The paper separated from the previous algorithm based on the traditional Gaussian map and proposed a new algorithm of computing exact Minkowski sum of convex polyhedral. Map the convex polyhedron into the bottom of tetrahedron according to the definition of Regular Tetrahedron Map and Point Projection, so we can reduce the problem in 3D to 2D, and compute Minkowski sum of two convex polyhedral amounts to computing the overlay of only one pair of planar subdivision. Comparing with the previous methods, this algorithm establishes mapping from 3D to 2D directly, and gives the function of specific type. Through the programming, we can get the procedure of mapping. The experiment shows that it improves the efficiency greatly.*
Keywords: Minkowski sum, Convex polyhedron, Regular tetrahedron map, Point projection, Overlay algorithm

1. **Introduction.** The definition of Minkowski sum was proposed by the German mathematician, Hermann. In three-dimensional space, let P and Q be two closed convex polyhedron, the Minkowski sum of P and Q is defined as the convex polyhedron M, $M = P \oplus Q = \{p + q | p \in P, q \in Q\}$ [1], here p and q are the points of polyhedron P and Q respectively, and $p + q$ is the sum of position vectors p and q. It can be also viewed as a polyhedron P translated by a vector t denoted by P^t.

Minkowski sum algorithm has the great significance in theory and application as an embranchment of Computational Geometry, which plays an important role in robot, computer graphics, dynamic simulation, CAD/CAM, and so on [1]. Especially in robotics, it is an important tool for computing collision-free path. Lozano-Perez firstly used Minkowski sum to compute configuration space obstacle and applied to robot path planning in 1983 [2]. The free configuration space is a set of all possible positions where the robot avoids contacting with the obstacles. It can be expressed as the complement of the Minkowski sum of the robot of obstacles. Suppose that polyhedron P is the obstacle, polyhedron Q is a robot, so computing the configuration space obstacle in 3D can be reduced to computing the Minkowski sum of P and $-Q$ got by , that is $P \oplus -Q$, and $-Q$ is got by the polyhedron Q rotating 180°.

Now, how to calculate the path of obstacle avoidance quickly and accurately has been an important research subject at home and abroad. So far, many methods for computing Minkowski sum of two convex polyhedral have been proposed, the goals are typically to compute the boundary of Minkowski sum and provide some representation of it. In