STABILITY OF ZEROS OF DISCRETE-TIME MULTIVARIABLE SYSTEMS WITH GSHF

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ABSTRACT. This paper is concerned with the stability of zeros of the discrete-time multivariable system composed of a generalized sample hold function (GSHF), a continuous-time plant with the degrees of infinite elementary divisors being two or three, i.e., \( \mu_1 = \cdots = \mu_{m-k} = 2 \) and \( \mu_{m-k+1} = \cdots = \mu_m = 3 \) \((k = 1, \ldots, m, 0 \leq k \leq m-1)\), and a sampler in cascade. The properties of the limiting zeros are studied and conditions for ensuring the stable limiting zeros of the discrete-time systems for sufficiently small sampling periods are derived. It is a natural extension of Liang et al.’s result for single-input-single-output systems to multivariable systems.

Keywords: Zeros, Stability, Generalized sampled hold function, MIMO, Discrete-time system

1. Introduction. It is well known that unstable zeros limit the performance that can be achieved. Some techniques for control system design are hard to be applied when a plant has unstable zeros [1, 2, 3]. Therefore, the properties of discrete-time zeros have received considerable attention from researchers [4, 5, 6, 7, 8, 9]. Some results therein showed the corresponding discrete-time systems arising from ZOH may have unstable discretization zeros when continuous-time systems with the relative degree \( p \geq 2 \). On the other hand, it was shown that the discretisation zeros can be placed inside the unit circle by the parameters of generalized sample hold function (GSHF) even if relative degree \( p \geq 2 \) [16, 17]. Moreover, the properties of zeros of the discrete-time systems with a piecewise constant GSHF, including the stability conditions, were presented. However, the properties of zeros of the discrete-time systems with GSHF are discussed mainly on SISO systems [17].

The properties of the zeros for multivariable systems are characterized by the degrees of the infinite elementary divisors \( \mu_1, \ldots, \mu_m \) \((\mu_1 \leq \cdots \leq \mu_m)\) [10] of a system matrix. It was shown [11] that ÅSTRÖM et al.’s result [4] holds in the case of square multivariable systems when the difference between the largest and the smallest degrees of the infinite elementary divisors of the underlying continuous-time system is less than two, i.e., \( \mu_m - \mu_1 < 2 \). Weller [12] also demonstrated ÅSTRÖM et al.’s result [4] can be extended directly to decouplable MIMO systems. Hayakawa et al. [11] showed that if all the degrees of the infinite elementary divisors of a continuous-time system matrix are two, i.e., \( \mu_1 = \cdots = \mu_m = 2 \), and all zeros of the continuous-time system are stable, then