A NOVEL TECHNIQUE FOR DESIGNING DECENTRALIZED STABILIZERS FOR ROBUST CONTROL IN POWER SYSTEMS USING AN $H_\infty$ CRITERION

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Abstract. This paper addresses the problem of excitation control for damping the electromechanical oscillation modes (EOM) in power systems with decentralized stabilizers (PSS) of reduced order to ensure robust stability and good performance of a power system under disturbances and uncertainties. Good stabilizer coordination is achieved by simultaneously designing all PSS. A frequency domain $H_\infty$ mixed sensitivity optimization approach is proposed. The proposed technique is not affected by the pole-zero cancellation problem associated with the conventional mixed sensitivity techniques.

Keywords: Power systems stabilizers, $H_\infty$ control, Decentralized control

1. Introduction. Large scale, interconnected power systems are ubiquitous in present days. As a consequence, quality indexes based on reliability [1], frequency and magnitude of voltage [2] among others are strictly required. Low frequency electromechanical oscillations have been a problem of great concern and continuous interest for interconnected power systems. These oscillations are mainly associated with local and interarea EOM. It is known that the operating conditions of the system changes continuously. Thus, for secure operation of power systems, it is required that the EOM must be sufficiently dampened at all common operating conditions. This means that the PSS must be robust for these changes [3, 4]. Conventional lead-lag PSS have been successfully applied to dampened EOM over decades. Its use is consolidated in power industry all over the world. However, its application in multimachine power systems have been carried out by independent designs or by a sequential design, which, due to interactions, may not provide the optimal settings. Moreover, the conventional PSS design does not take into account the disturbances, measurement noises and uncertainties due to nonlinearities, neglected dynamics in the model, changes in operating conditions, etc [4]. Recently, some researchers have been concerned in providing sufficient damping for the EOM in spite of uncertainties. The techniques most used for robust design of controllers in power systems are: linear matrix inequality [4, 5, 6] and mixed sensitivity optimization [7, 8, 9]. All these techniques have a common limitation of resulting into large centralized controllers when directly designed in a multimachine power system model. In order to minimize this problem, the model order is reduced and then controller order is also reduced [8].
The mixed sensitivity approach deserves a special consideration due to its attractive frequency response shape where robust stability and robust performance improvements can be easily visualized. However, its conventional approach has two additional drawbacks: the pole-zero cancellation and the needing of selection of three weighting matrices \[10\]. These drawbacks make it difficult to apply this approach in large systems. The basic concern of this paper consists of exploring improvements for the control structure already existing in the power industry, leaving for the future the replacement of current PSS for controllers by new structures. In this paper, a new procedure is proposed for PSS tuning, aiming $H_{\infty}$ robust control of a power system, with decentralized stabilizers with reduced order. The use of decentralized stabilizers is a practical requirement in order to avoid the need for remote feedback over long distances, as expected for decentralized power systems. The frequency domain propositions of $H_{\infty}$ robust control together with an optimization procedure are applied in order to determine the PSS gains and time constants. The PSS systems connected to the set of selected generators are tuned simultaneously in order to take into account the interactions among generators and controllers. The optimization procedure applied to tune PSS consists on minimizing a weighted transfer function matrix, yielding a mixed-sensitivity objective function \[8\] suitable for the application under study. The cancellation problem related to EOM complex zeros is avoided by constraining the weights, poles and zeros of PSS to be in the real domain.

2. Signal Analysis for Decentralized Control. Only a limited number of PSS require the damping of the critical EOM. Thus, the most suitable generators for application of PSS should be previously selected. The PSS function is the EOM damping, aiming to reduce the undesired effects of disturbances and noise measurements and also to achieve a good response despite of model uncertainties. Tracking is not of concern in PSS design.

Frequency domain techniques applied to damp EOM are closely related with the physical phenomenon and with the field practice in power systems. Besides, frequency domain multivariable techniques may be successfully applied for interactions analysis, input-output selection and for designing robust controllers.

A general power system with $n$ generators is described by
\[ y(j\omega) = G(j\omega)u(j\omega) \] (1)
where $G(j\omega)$ is the transfer function matrix, $y(j\omega)$ and $u(j\omega)$ are respectively the output and the input vector.

The EOM controllability may be defined as the ability of the system to damp the mode in order to achieve acceptable performance with bounded inputs and outputs. Likewise, the observability of an EOM may be defined as the contribution of the mode on system response. For analysis of controllability and observability of MIMO power systems, the singular values of the matrix $G(j\omega)$ are required.

The maximum singular value of $G$ is denoted by $\bar{\sigma}(G)$, the minimum singular value is denoted by $\underline{\sigma}(G)$ and the condition number is given by $\gamma(G) = \bar{\sigma}(G)/\underline{\sigma}(G)$.

The singular values $\bar{\sigma}$ and $\underline{\sigma}$ in the frequency of an EOM are, respectively, measurements of observability and controllability of the mode. EOM with small damping and strongly observable in the output signals display large peaks on the graph of $\bar{\sigma}(j\omega)$. Furthermore, a large $\underline{\sigma}$ in the frequency of an EOM means that the mode is strongly controllable. A dip on the graph of $\underline{\sigma}(j\omega)$ represents a complex system zero with significant effect on response and possibly on the control.

The effect of $\underline{\sigma}$ on system performance with respect to set-point changes, $R$, disturbances, $d$, and measurement noises, $\eta$, is introduced, considering the power system $G(s)$, with feedback controllers $H(s)$, as usually found in practice, as shown in Figure 1.
The following relations are obtained from Figure 1, omitting $s = j\omega$

$$y = TR + Sd - TH\eta \quad (2)$$

$$u = (I + HG)^{-1}R - (I + HG)^{-1}H(\eta + d) \quad (3)$$

where $S = (I + GH)^{-1}$ is the sensitivity transfer function matrix and $T = SG$ is the closed-loop transfer function matrix.

Consider a change on setpoints, assuming $d = 0$ and $\eta = 0$. Using singular value properties \cite{11} from (2), it is obtained that $\|y\|/\|R\| = \bar{\sigma}(T) \leq \bar{\sigma}(G)/\sigma(GH + I)$, resulting $\bar{\sigma}(T) \leq \bar{\sigma}(G)/(\sigma(G)\sigma(H) - 1)$.

Similarly, by considering only the effect of disturbances on $y$, it is verified that $\|y\|/\|d\| \leq 1/(\sigma(G)\sigma(H) - 1)$. Then, $\sigma(G)$ should be large enough in order to reduce $\bar{\sigma}(T)$ which is usually large at the frequencies of the EOM, and also to reduce the effects of disturbances and measurement noises. Obviously, the controller should be designed to contribute on both cases.

It is also important to impose bounds in order to avoid input saturation, which constrains the control action. Considering only the effect of setpoints in (3), results that $\|u\| \leq \|R\|/(\sigma(G)\sigma(H) - 1)$. Again, $\sigma(G)$ must be large.

Therefore, the most suitable generators set which should be selected for applications of PSS is the set with the largest $\sigma(G)$ in the frequency range of the EOM.

3. Robust $H_{\infty}$ Damping Control. The effects of unstructured uncertainties are now taken into account. Two common types of uncertainties are presented in Figure 2 \cite{11}, where $\Delta A$ and $\Delta M$ are respectively the additive and the multiplicative uncertainty matrices and, $W_A$, $W_M$ and $W_1$ are weighting function diagonal matrices.

The goal is to design stabilizers in order to damp the EOM, yielding robust stability and satisfactory system performance, not only for the nominal plant $G(s)$, but for the set of all plants.
This set is defined by the expression \( G'(s) = (I + W_M(s)\Delta_M W_1(s))G(s) + W_A(s)\Delta_A W_1(s) \).

Introducing the uncertainties into the model of Figure 1, considering one uncertainty at a time, and arranging the blocks, results the model \( M - \Delta \) of Figure 3, where \( M(s) = W_M TH W_1 \) for \( \Delta = \Delta_M \) and \( M(s) = W_A HS W_1 \) for \( \Delta = \Delta_A \).

Assume that the nominal system \( M(s) \) is stable and that the uncertainties \( \Delta \) are stable. Thus, the \( M - \Delta \) systems are stable for all uncertainties \( \Delta \), satisfying \( \bar{\sigma}(T) \leq 1 \), if and only if [11]:

\[
\bar{\sigma}(M(j\omega)) \leq 1 \quad (4)
\]

For simplicity sake, it is assumed round weights, that is, \( W_A(s) = \omega_a(s)I \), \( W_M(s) = \omega_m(s)I \) and \( W_1(s) = \omega_1(s)I \). Then, from (4), results:

\[
\omega_1(s)\omega_m(s)\bar{\sigma}(H)\bar{\sigma}(T) \leq 1 \quad (5)
\]

for multiplicative uncertainties, and

\[
\omega_1(s)\omega_a(s)\bar{\sigma}(H)\bar{\sigma}(S) \leq 1 \quad (6)
\]

for additive uncertainties.

The weight \( \omega_1(s) \) helps in scaling \( T \) and \( S \), since, with feedback control, as in Figure 1, \( T + S \neq I \), it becomes difficult to satisfy both inequalities of (5) and (6) simultaneously.

Considering, from (2), that \( S \) should be small for disturbance rejection and \( T \) should be small enough for noise modulation, then, for robust stability and robust performance, \( \bar{\sigma}(T) \) and \( \bar{\sigma}(S) \) should be minimized. To satisfy these requirements, the following mixed sensitivity optimization problem is proposed:

\[
\min \left[ \sup \bar{\sigma} \left( \begin{bmatrix} \omega_1 \omega_m TH & \omega_1 \omega_a HS \end{bmatrix} \right) \right] \quad (7)
\]

3.1. Justification for using identical controller structures. For analysis of this case, the uncertainties of Figure 2 are lumped into a single multiplicative output uncertainty matrix given by \( \Delta_E = \omega_1 \omega_m \Delta_M + \omega_1 \omega_a \Delta_A G^{-1} \). The true power system (with uncertainties) is given by \( G'(s) = [I + \Delta_E]G \) and the true sensitivity matrix of the system affected by uncertainties is given by \( S' = [I + G'H]^{-1} = [I + (I + \Delta_E)GH]^{-1} (I + GH)S \).

Assuming that in the frequency range of main concern (frequency range of EOM), occurs that \( \bar{\sigma}(GH) \gg 1 \), results:

\[
\bar{\sigma}(S') \leq \frac{\gamma(H)\gamma(G)}{\bar{\sigma}(I + \Delta_E)} \bar{\sigma}(S) \quad (8)
\]

From (8) it can be seen that the sensitivity of the system may be significantly deteriorated by uncertainties, mostly in the frequencies of the critical EOM, where \( \gamma(G) \gg 1 \). Also, the controller may constrain the deterioration on sensitivity if \( \gamma(H) = 1 \), which means identical decentralized controllers on all units.

Assuming that \( \omega_a(s) = \omega_m(s) = \omega_0(s) \), the optimization problem of (7), satisfying (5) and (6), consists on adjusting the parameters of \( H \) in order to minimize \( \bar{\sigma} \left( \begin{bmatrix} T & S \end{bmatrix} \right) \),
resulting:

$$\bar{\sigma} \left( \begin{bmatrix} T \\ S \end{bmatrix} \right) < \frac{1}{\omega_1(s)\omega_0(s)\sigma(H)} \forall \omega$$

(9)

That allows to leave $T$ and $S$ separated from the weights, allowing the application of the proposed technique to larger systems, since the use of MATLAB is limited by the dimensions of the matrix when obtaining singular values. Besides, $\omega_0(s)$ and $H(s)$ should have only first order terms in order to avoid the pole-zero cancellations of the EOM and complex zeros.

If the last inequality is met (9), the robustness of stability with good performance is attained. If the inequality is not met, the order of the controller should be increased. It is obvious that, with the minimization of $\bar{\sigma} \left( \begin{bmatrix} T \\ S \end{bmatrix} \right)$, the peaks of $T$ and $S$ are flattened, meaning that the EOM are dampened.

The objective function to be minimized in (9) is not an explicit expression. Then, an optimization technique to determine the minimum of a function without calculation of derivatives seems attractive. The direct pattern search optimization technique of Hooke and Jeeves [12] is used due to its good adaptation to this problem. The limitation of the technique is that it converges more slowly than the derivative methods.

4. Application to a Multimachine Power System. A multimachine power system is shown for illustration of proposed technique application [13]. Its transmission configuration is shown in Figure 4. The generator #10 is a large equivalent generator, representing the rest of the system. The data set used for tests, which may be obtained from the authors, was omitted. It is assumed that speed signals are used as outputs and input voltages of the exciters are the control inputs.

In Figure 5, the singular values $\bar{\sigma}(T)$ and $\sigma(T)$ of the complete system are displayed. The peaks on the graph of $\bar{\sigma}(T)$ reveal that the system has four EOM insufficiently damped. These EOM are critical and they were named: Mode 1 (0.46 Hz), Mode 2 (0.86 Hz), Mode 3 (0.89 Hz) and Mode 4 (0.92 Hz). The two former ones are interarea modes and the latter ones are local modes.

It is known that modern exciter systems with fast responses and high gains are more appropriate for application of PSS and that small generators may not provide sufficient...
Table 1. Values of inertia constants and exciter gains of the generators

<table>
<thead>
<tr>
<th>Generator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$</td>
<td>11.0</td>
<td>11.5</td>
<td>10.0</td>
<td>6.4</td>
<td>9.9</td>
<td>9.9</td>
<td>9.0</td>
<td>9.0</td>
<td>15.0</td>
</tr>
<tr>
<td>$K_A$</td>
<td>50.0</td>
<td>50.0</td>
<td>20.0</td>
<td>15.0</td>
<td>100.0</td>
<td>100.0</td>
<td>10.0</td>
<td>10.0</td>
<td>50.0</td>
</tr>
</tbody>
</table>

damping for EOM [13, 14]. Then, only the largest generators with high gains, $K_A$, will be considered for application of PSS.

Table 1 shows the values of inertia constants, $H$, and exciter gains, $K_A$, of the generators. From Table 1, the generators #3, 4, 7 and 8 do not satisfy the requirements, and are consequently discarded, because these are generators with low gains $K_A$. Then, generators #1, 2, 5, 6 and 9 are selected for further analysis. Usually, a number of PSS equal to the number of critical EOM are usually enough to damp these modes. Thus, it is assumed that PSS should be applied, as an attempt, on four generators.

Thus, there are $\binom{5}{4} = 5$ sets of generators for final analysis. The graphic analysis of these sets shows that the set of generators #1, 2, 5 and 6 presents the largest $\sigma(T)$ in the range of the EOM. The graphs of $\bar{\sigma}(T)$ and $\sigma(T)$ of this set are presented in Figure 6. Since this set presents the largest controllability with good observability of all critical EOM, the generators #1, 2, 5 and 6 are selected for application of PSS. The graph of Figure 6 shows that the selected set in this paper provides better controllability of the critical EOM than the complete set of 9 generators in Figure 5, because they present higher values of $\bar{\sigma}(T)$. This is due to the interactions among generators, which may constrain the control of EOM, mainly Mode 4. It means that PSS applied on the complete system may not provide results as good as the ones shown for the selected set.

As can see the graph of $\sigma(T)$ in Figure 6, there is a zero complex (dip on the graph). However, the graph analysis leads to the conclusion that the zero does not affect the controllability of any EOM, because it hasn’t the same frequency of the modes.

![Figure 5. Graphs of $\bar{\sigma}(T)$ and $\sigma(T)$ of the complete system](image-url)
In [13], by using eigenanalysis techniques, a different set of generators was selected in
order to damp the same EOM. In the modal technique of that paper, the one at a time
selection was applied. Although acceptable results were obtained, the set obtained by
simultaneous selection in this paper yields better controllability and observability of the
EOM, which are required for robust control.

For PSS design, it is considered $\omega_1(s) = 0.13\frac{s^{0.95s+1}}{0.05s+1}$ and $\omega_0(s) = \frac{0.25s+0.15}{0.5s+1}$. The mixed
weight $\omega_1(s)\omega_0(s)$ is a unique upper bound of uncertainty associated with all control
channels.

The design methodology consists of applying identical PSS, with the conventional struc-
ture of reduced order, $h(s) = K_s \frac{T_{w}s}{1+T_{w}s}\left(\frac{1+T_1s}{1+T_2s}\right)^2$, on the selected generators and tuning
$K_s$, $T_1$ and $T_2$ in order to minimize $\bar{\sigma}\left(\begin{array}{c} T \\ S \end{array}\right)$, as shown in (9) Assuming that $T_w = 20s$
and using optimization technique, results that $K_s = 0.7$, $T_1 = 0.05s$ and $T_2 = 0.009s$.

In Figure 7, it is shown the graph of $\bar{\sigma}\left(\begin{array}{c} T \\ S \end{array}\right)$ for the set of generators #1, 2, 5 and 6
with PSS and, also, the same graph with the PSS gain of generator #6 decreased by 50%,
representing an uncertainty in the generator, when compared with the graph of $\frac{1}{\omega(\omega T_0)}$.
The Equation (9) is satisfied, thus the robustness of stability with good performance is
achieved. The minimization of the peaks resulted in flat graphs, as required for robustness.

In Table 2, the damping ratios of critical EOM with and without the four PSS applied
are compared. The damping ratio provides a mathematical means of expressing the
damping level of a system [14]. A higher ratio leads to a higher damping of the oscillations.
It is observed that all critical EOM were damped with the PSS. It is interesting to see in
Table 2 that for obtaining a robust control, only small increases in damping of the EOM
were required. The small displacements of the eigenvalues were due to the low gains $K_s$.
With the low gains of the PSS, good dampings of the exciter modes are preserved. This
is an important result, since exciter modes usually limit the robustness of PSS.

In Figure 8, it is shown the generator #6 time responses of the system with and without
PSS, considering the input uncertainty on machine #6. In both cases, it was assumed

![Figure 6. Singular values $\bar{\sigma}$ and $\bar{\sigma}$ of the set of generators 1, 2, 5 and 6](image-url)
an impulsive disturbance on the mechanical torque of the generator. It is shown that the system returns to the conditions before the disturbance. A good performance is achieved under disturbance and uncertainty.

5. Conclusions. A new design technique was proposed aiming to satisfy robust control requirements with good performance, by using decentralized reduced order stabilizers

Figure 7. Graphs of $\bar{\sigma} \left( \begin{bmatrix} T \\ S \end{bmatrix} \right)$ for generators #1, 2, 5 and 6 with and without 50% uncertainty in the input channel of generator #6 and $|\frac{1}{\omega_1 \omega_0 h}|$

Figure 8. Time responses of $\omega_6$ for the system with and without PSS under an impulsive disturbance on machine #6
A NOVEL TECHNIQUE FOR DESIGNING DECENTRALIZED STABILIZERS

Table 2. Damping ratios of critical EOM

<table>
<thead>
<tr>
<th>Modes</th>
<th>without PSS</th>
<th>with 4 PSS</th>
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<tbody>
<tr>
<td>1</td>
<td>0.0395</td>
<td>0.0767</td>
</tr>
<tr>
<td>2</td>
<td>0.0405</td>
<td>0.0758</td>
</tr>
<tr>
<td>3</td>
<td>0.0394</td>
<td>0.0808</td>
</tr>
<tr>
<td>4</td>
<td>0.0296</td>
<td>0.0779</td>
</tr>
</tbody>
</table>

in power systems. A simple mixed sensitivity approach with simple weights is applied in order to reach this goal. Previous reduction of the system model and reduction of the controller are not required. PSS with the conventional structure is applied and its parameters are tuned by an optimization technique. In order to achieve robust control, the best pole placement for PSS application is previously selected by using a multivariable frequency domain technique. All pole placements are simultaneously selected, providing a complete visualization of observability and controllability of all critical oscillation modes. This paper contributes to show that the conventional lead compensation type of PSS needs to be tuned in order to provide $H_\infty$ robust control in power systems, if the best placement for application of PSS is previously selected. This paper also contributes in proposing efficient techniques which can be applied in the design of reduced order decentralized controllers in general MIMO systems. The limitation of this analysis is the representation of unstructured uncertainties with a single bound for all generators. It represents a conservative measure for all procedure. However, the frequency domain technique provides an unique way of providing a decentralized reduced order with known structure as robust controller.

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