SURFACE TEXTURE CHARACTERIZATION OF FIBERS USING FRACTIONAL BROWNIAN MOTION MODEL

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Abstract. The texture of the surface of individual fiber is an important characteristic. The fibers can be classified and recognized using the surface texture by a specific parameter. To describe the texture, the fractal parameter (or Hurst coefficient) is a proper value. In this study, we use the Fractional Brownian Motion (FBM) to model the texture of the fiber surface. And we apply a Fourier-domain Maximum Likelihood Estimator (FDMLE) to calculate the fractal parameter of FBM. According to the experimental results, we can objectively classify different types of fibers.

Keywords: Texture characterization, Fiber classification, Fractional Brownian motion, Fourier-domain maximum likelihood estimator

1. Introduction. In recent years, the developments in sciences and technology, such as Internet, digital image processing and micro-manufacture, etc, is drastically changing the life of the mankind. Science and technology have been used in radar systems, robots, networks and communications [1-4]. Using science and technology to investigate crime scenes is the major aspect of fighting crime in democracies. They are necessary not only to fight crime efficiently, but also to protect human rights and maintain the social order. Evidence collected from the crime scene includes fingerprints, DNA, blood and fibers. The evidence can be used to identify the criminal or to reduce the number of suspects. Many classification schemes were proposed for fingerprints, DNA and blood, but nothing for fabrics. However, fiber identification has not only been an important object in the forensic science, but also in many applications such as wool manufacture, textile processing, archeology and zoology [5]. In the textile industry, accurate classification of animal fibers proved very difficult. In the past, physical and chemical properties of fibers have been used to solve the classification problem. Recently, some techniques have been proposed to identify animal fibers by the cuticle on the surface of the fiber, such as image processing methods and artificial intelligence techniques [6-11]. As seen from the proposed literature, classification of fibers is actually important and the texture of fiber surface is a characteristic useful in classifying fibers.
The properties of texture have been analyzed more than 30 years ago. Chetverikov discussed the texture analysis methods and proposed some schemes of texture inspections [12]. The texture can be segmented in the spatial domain using the sub-band domain co-occurrence matrix and the neighboring gray level dependent matrix. Some researchers also extract the texture characterizations from the frequency domain. Wavelet transform is used to inspect the defects of texture with segmentation algorithm [13]. Several blocks are segmented by using wavelet technology, and the features of texture can be obtained by computing the co-occurrence matrix. In this way, the small block selection may lose some features. However, this scheme can not exactly recognize the location of the defect. To improve efficiency of the texture characterization, the advanced wavelet transforms were developed. Many researchers use them to compute the detail values for determining the texture features. To obtain the homogeneous feature, the roughness index and the homogeneous index are built [14]. The technologies of texture characterization can be summarized as following.

1. The stochastic model is used to analyze the statistical properties of textures [15,16].
2. The matrix methods are used to extract the texture features for classification [17,18].
3. The transform-based methods are applied to characterizing textures and locating defects [19,20].

However, those previous works concentrated upon the feature extraction of textures, not the description of textures. The natural texture is very difficult to describe by conventional Euclidean geometry (such as lines, circles, triangles, rectangles, . . . , etc). For modeling the natural texture, the fractal geometry was proposed [21]. The kernels of the fractal theory are the statistical self-similarity concept and the fractal dimension is not necessary an integer. Fractal geometry had been successfully used in many applications [22-29].

In the use of the fractal models, the most important procedure is to measure the fractal parameter (or the Hurst coefficient). The fractal parameter is related to the fractal dimension and characterizes the surface roughness of natural textures. A fractal analysis method for estimating the fractal parameter is called box-counting [30], but some literatures mention that box-counting does not perform very well [31,32].

Among the fractal models, a generalized form of ordinary Brownian motion, which is called Fractional Brownian Motion (FBM), is proposed [33,34]. FBM is one of the most useful fractal models for the random fractals found in nature. And FBM is widely used to analyze data in many applications, such as random fractals in nature [27], target detection of radar techniques [35] and bone radiographic images [24]. It has also been shown that FBM is suitable for digital images. Since the probability density function of FBM is well known, a Maximum Likelihood Estimator (MLE) has been used to estimate the fractal parameter on a self-similar texture image [32]. Later a Fourier-domain Maximum Likelihood Estimator (FDMLE) is developed to save extensive computations based on MLE [36].

FBM model and FDMLE technique have been applied to inspect the surface defects of textile fabrics (defect segmentation) [37]. However, FBM and FDMLE methods do not deal the problem of the surface texture characterization of fiber.

The surface texture of an individual fiber can be classified and recognized by using a specific parameter. The fractal parameter (Hurst coefficient) is a proper value to describe the texture of a fiber surface. In this paper, we use FBM to model the texture of the fiber surface. And we apply a FDMLE technique to calculate the fractal parameter of FBM. The fractal parameter can be used to characterize the surface texture of the fiber.

2. Fractional Brownian Motion Model. Fractional Brownian Motion (FBM), $B_H(t)$, was first proposed by Mandelbrot [33]. In the FBM model, the most important parameter
is the fractal parameter, $H$, which is also called Hurst coefficient and it ranges from zero to one. FBM is a non-stationary zero-mean Gaussian random function, and it is a single variable Brownian motion. FBM is an ordinary Brownian motion when $H = 0.5$. The path of $B_H(t)$ is smoother when $H$ is larger than 0.5 and it is rougher when $H$ is smaller than 0.5. Three paths of $B_H(t)$ with different $H$ are shown in Figures 1(a), 1(b) and 1(c), respectively.

FBM is a non-stationary process and it is not convenient to analyze. The increment of FBM is called Fractional Gaussian Noise (FGN), and it is a stationary process, therefore, is more suitable for mathematical analysis than FBM.

The sampled data (the image of texture) is a discrete FBM model and it can be denoted as $B_H[n]$ ($n$ means the $n$th data), and the discrete FGN (DFGN) [32] can be written as

$$x_H[n] = B_H[n] - B_H[n-1].$$  \hspace{1cm} (1)

And the autocorrelation of DFGN can be derived as

$$r_{xH}[n] = \frac{\sigma^2}{2} (|n + 1|^{2H} + |n - 1|^{2H} - 2n^{2H}),$$ \hspace{1cm} (2)

where $n$ means the $n$th data and $\sigma^2$ is the variance of $x_H$.

3. **Fourier-domain Maximum Likelihood Estimator.** Since the probability density function of FBM (and FGN) is well known, the Hurst coefficient ($H$) can be obtained by a Maximum Likelihood Estimator (MLE) [32]. However, the computation time of MLE is huge and it is not suitable for practical applications. In this section, we briefly summarize a faster method that is called the Fourier-domain Maximum Likelihood Estimator (FDMLE) [36,37] for computing the Hurst coefficient.

According to Equation (1), the discrete-time Fourier transform of DFGN is denoted as $X_H$, and each element in $X_H$ is written as following:

$$X_H(u) = \sum_{n=0}^{N-1} x_H[n] e^{-j2\pi un/N},$$ \hspace{1cm} (3)

where $N$ is the number of the data and $u = 0, 1, \ldots, N - 1$. The auto-covariance of $X_H$ can be written as

$$E[X_H(u)X_H(v)^T] = \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} r_{xH}[m - n] e^{-j2\pi(um/N - vn/N)},$$ \hspace{1cm} (4)

where $N$ is the number of the data and $u,v = 0, 1, \ldots, N - 1$. The value of $E[X_H(u)X_H(v)^T]$. 

![Figure 1](image-url). The example signals of FBM with different value of $H$: (a) $H = 0.3$, $B_H(t)$ is rougher; (b) $H = 0.5$, $B_H(t)$ is an ordinary Brownian motion; (c) $H = 0.7$, $B_H(t)$ is smoother.
is real because $r_{x_H}[n] = r_{x_H}[-n]$. Then we can define a matrix, $\Lambda_H$, and each element in $\Lambda_H$ is written as

$$a_{pq} = E[X_H(p)X_H(q)^T],$$

(5)

where $p, q = 0, 1, \ldots, N - 1$, and $\Lambda_H$ can be approximated a diagonal matrix [36]. Because of linearity, we can assume that the discrete-time Fourier transform of DFGN still has a Gaussian distribution, and the probability density function, $pro()$, of DFGN can be written as

$$pro(X_H; H) = \frac{1}{(2\pi)^{N/2} |\Lambda_H|^{1/2}} \exp \left\{ -\frac{1}{2} X_H \Lambda_H^{-1} X_H^T \right\},$$

(6)

where $\Lambda_H$ is the auto-covariance matrix given in Equation (5). After log-likelihood calculation of Equation (6), we can obtain

$$\log pro(X_H; H) = -\frac{N}{2} \log 2\pi - \frac{N}{2} \log \left( \frac{X_H \Lambda_H^{-1} X_H^T}{N} \right) - \frac{N}{2} - \frac{1}{2} \log |\Lambda_H^*|,$$

(7)

where $\Lambda_H^* = \frac{1}{\sigma^2} \Lambda_H$. We obtain $\Lambda_H^*$, which has to be inverted, and its determinant has to be computed. We can approximate $\Lambda_H^*$ as a non-zero diagonal matrix and obtain its inverse and determinant directly.

We use the gray level variation of image to compute values of the left side of Equation (7), $\log pro(X_H; H)$, with $H$ values varying from 0.01 to 0.99, then we can obtain the estimated $H$ which is with the maximum value of $\log pro(X_H; H)$.

In the practical application, we select a size of a window in the image of the fiber surface. We assume that the selected size is $M \times M$, in the other words, there are $M$ vertical lines and $M$ horizontal lines in the window. Each line is used to estimate a fractal parameter, $H$. The $H$ value of the selected window is represented by the mean of $H$ values of all lines on the window. We take Figure 2 for an example: Figure 2(a) is an Acrylic fiber image. We select a window on a fiber surface. The selected window is shown in Figure 2(b) and the size of selected window is $25 \times 25$. In the other words, there are 25 vertical lines and 25 horizontal lines in the window. Vertical lines are denoted as v1, v2, ..., v25 and horizontal lines are denoted as h1, h2, ..., h25. We extract two vertical lines (the 1st and 17th) and two horizontal lines (the 2nd and 19th) to show the gray level variation (as shown in Figures 2(c) - 2(f)). The estimated $H$ of each line and the mean $H$ of the selected window are shown in Table 1. The mean $H$ of the selected window is used to characterize the surface texture of the fiber.

Table 1. The estimated $H$ of each line and the mean value $H$ of the selected window (see Figure 2(b))

<table>
<thead>
<tr>
<th></th>
<th>v1</th>
<th>v2</th>
<th>v3</th>
<th>v4</th>
<th>v5</th>
<th>v6</th>
<th>v7</th>
<th>v8</th>
<th>v9</th>
<th>v10</th>
<th>v11</th>
<th>v12</th>
<th>v13</th>
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<tbody>
<tr>
<td>estimated $H$</td>
<td>0.87</td>
<td>0.85</td>
<td>0.86</td>
<td>0.85</td>
<td>0.87</td>
<td>0.88</td>
<td>0.91</td>
<td>0.87</td>
<td>0.89</td>
<td>0.88</td>
<td>0.87</td>
<td>0.88</td>
<td>0.89</td>
</tr>
<tr>
<td>the horizontal line</td>
<td>h1</td>
<td>h2</td>
<td>h3</td>
<td>h4</td>
<td>h5</td>
<td>h6</td>
<td>h7</td>
<td>h8</td>
<td>h9</td>
<td>h10</td>
<td>h11</td>
<td>h12</td>
<td>h13</td>
</tr>
<tr>
<td>estimated $H$</td>
<td>0.68</td>
<td>0.83</td>
<td>0.86</td>
<td>0.81</td>
<td>0.87</td>
<td>0.87</td>
<td>0.77</td>
<td>0.74</td>
<td>0.67</td>
<td>0.63</td>
<td>0.65</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>the horizontal line</td>
<td>h14</td>
<td>h15</td>
<td>h16</td>
<td>h17</td>
<td>h18</td>
<td>h19</td>
<td>h20</td>
<td>h21</td>
<td>h22</td>
<td>h23</td>
<td>h24</td>
<td>h25</td>
<td></td>
</tr>
<tr>
<td>estimated $H$</td>
<td>0.86</td>
<td>0.89</td>
<td>0.75</td>
<td>0.77</td>
<td>0.86</td>
<td>0.82</td>
<td>0.64</td>
<td>0.82</td>
<td>0.72</td>
<td>0.65</td>
<td>0.72</td>
<td>0.86</td>
<td></td>
</tr>
</tbody>
</table>

mean value $H$ of the selected window: 0.8314
4. Experiments.

4.1. Computer generated fractal images. In order to demonstrate the performance of the Fractional Brownian motion model with Fourier-domain maximum likelihood estimator (FDMLE), we apply the Cholesky decomposition method to generate two-dimensional synthetic fractal images [38] and use those images to test the FDMLE method. Nine $32 \times 32$ images with different $H$ values ($0.1 \sim 0.9$) are obtained (see Figure 3). We choose 64 lines (1D data, 32 rows, 32 columns) for each image, and apply the FDMLE method (Section 2.3) to compute their mean $H$ values. The results are shown in Table 2. We can see that the FDMLE method provides a good estimation of those synthetic images.
4.2. **Classification of synthetic fibers.** Two different Acrylic fiber images are taken with a microscope (with 250×): Sample A1 is the Acrylic with two-component, 3 denier per filament and semi-dull luster (Figure 4(a)); Sample A2 is the Acrylic with modified wet spun, 3 denier per filament and semi-dull luster (Figure 4(b)). We select seven windows on the surface of each fiber. The selected windows of Figures 4(a) and 4(b) are shown in Figures 4(c) and 4(d), respectively. We apply the FDMLE technique to estimate the \( H \) value of each window, the results are shown in Table 3. The \( H \) range of Sample A1 and Sample A2 are from 0.82 to 0.86 and 0.70 to 0.75, respectively. From the experiment results, the differently processed Acrylic fibers can be classified by the \( H \) value.

**Table 3.** The results of Acrylic fibers (Figure 4)

<table>
<thead>
<tr>
<th>Sample</th>
<th>selected window</th>
<th>estimated ( H )</th>
<th>range of ( H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1W1</td>
<td>0.8200002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1W2</td>
<td>0.8310000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1W3</td>
<td>0.8604000</td>
<td>0.8200002</td>
<td></td>
</tr>
<tr>
<td>A1W4</td>
<td>0.8447999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1W5</td>
<td>0.8502000</td>
<td>0.8604000</td>
<td></td>
</tr>
<tr>
<td>A1W6</td>
<td>0.8482001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1W7</td>
<td>0.8487999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2W1</td>
<td>0.7011254</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2W2</td>
<td>0.7402000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2W3</td>
<td>0.7480000</td>
<td>0.7011254</td>
<td></td>
</tr>
<tr>
<td>A2W4</td>
<td>0.7270001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2W5</td>
<td>0.7204000</td>
<td>0.7513999</td>
<td></td>
</tr>
<tr>
<td>A2W6</td>
<td>0.7513999</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A2W7</td>
<td>0.7316000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.3. **Classification of animal fibers.** There are two animal fibers: Sample W1 is mohair (Figure 5(a)) and Sample W2 is merino (Figure 5(b)). The selected windows of Figures 5(a) and 5(b) are shown in Figures 5(c) and 5(d), respectively. The $H$ value of each window and the $H$ range of each fiber are shown in Table 4. From the experimental results, we can see that the $H$ value is different with different fiber of wool, in other words, we can classify different types of animal fibers using the FDMLE technique.

**Table 4.** The results of animal fibers (Figure 5)

<table>
<thead>
<tr>
<th>Sample</th>
<th>Selected window</th>
<th>estimated $H$</th>
<th>range of $H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample W1</td>
<td>W1W1</td>
<td>0.3985000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W1W2</td>
<td>0.3696186</td>
<td>0.3696186</td>
</tr>
<tr>
<td></td>
<td>W1W3</td>
<td>0.3756666</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W1W4</td>
<td>0.3754167</td>
<td>0.3985000</td>
</tr>
<tr>
<td></td>
<td>W1W5</td>
<td>0.3837629</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W1W6</td>
<td>0.3915001</td>
<td></td>
</tr>
<tr>
<td>Sample W2</td>
<td>W2W1</td>
<td>0.6399423</td>
<td>0.6147701</td>
</tr>
<tr>
<td></td>
<td>W2W2</td>
<td>0.6259195</td>
<td></td>
</tr>
<tr>
<td></td>
<td>W2W3</td>
<td>0.6147701</td>
<td>0.6399423</td>
</tr>
</tbody>
</table>

5. **Conclusions.** In this paper, we first summarized the definitions of FBM (modeling the surface) and the FDMLE technique (estimating the $H$ value). Then we have applied the
FDMLE technique to calculate the $H$ value (fractal parameter) of the fiber surface. Based on experimental results, now we can objectively classify different types of fibers. This method can be applied not only in animal fibers, but also to synthetic fibers. The fiber identification can be used not only in the forensic science, but also in wool manufacture, textile processing, archeology and zoology.

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