PEAK TO AVERAGE POWER RATIO REDUCTION OF MULTICARRIER TRANSMISSION SYSTEMS USING ELECTROMAGNETISM-LIKE METHOD

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ABSTRACT. In this paper, a reduced-complexity partial transmit sequences (PTS) scheme is proposed to resolve the intrinsic high peak-to-average power ratio (PAPR) problem of multi-carrier transmission systems signal with low computational complexity. The conventional PTS technique is highly successful in PAPR reduction for multi-carrier transmission systems signals, but the considerable computational complexity for the required search through a high-dimensional vector space is a potential problem for practical implementation. To reduce the search complexity while still improving the PAPR statistics, stochastic optimization techniques such as the simulated annealing (SA) algorithm, genetic algorithm (GA) and particle swarm optimization (PSO) have recently been proposed to search for a phase factor that reduces both the PAPR statistics and the computational load. In this paper, a novel stochastic optimization approach, that is, the electromagnetism-like (EM) algorithm is applied in reducing the PAPR of a multi-carrier transmission systems signal. From the results, it can be seen that a proposed scheme based-iterative PTS can be easily implemented with low combining complexity, still maintaining a better BER performance, compared to the conventional PTS approach. Simulations have been conducted and the results show that the BER performance of the investigated EM-PTS is increased by minimizing the signal’s nonlinear distortion caused by HPA.

Keywords: Multi-carrier transmission systems, Peak-to-average power ratio, Partial transmitting sequence, Electromagnetism-like method

1. Introduction. Multi-carrier code division multiple-access (MC-CDMA) communication systems have emerged recently as a promising candidate for next generation broadband mobile networks [1-4]. However, there are still some challenging issues remained unresolved in the design of the MC-CDMA. One of the major problems is high peak to average power ratio (PAPR) of transmitted MC-CDMA signals. Therefore, the MC-CDMA receiver’s detection efficiency is very sensitive to the nonlinear devices used in its signal processing loop, such as high power amplification, digital-to-analog converter, which may severely impair system performance due to induced spectral re-growth and detection efficiency degradation. Thus, PAPR reduction technique plays an important role in multi-carrier transmission systems. Various methods for reducing PAPR have been proposed for MC-CDMA system, such as deliberate clipping [5], partial transmit sequences [6-9], block coding [10], code selecting [11], selective mapping (SLM) [12,13] and
subcarrier scrambling [14], etc. In deliberate clipping, the simplest method, signals are deliberately clipped before amplification. Although some techniques of PAPR reduction have been summarized in [15], it is still indeed needed to give a comprehensive review including some motivations of PAPR reductions, such as power saving, and to be compared with some typical methods of PAPR reduction through theoretical analysis and simulation. In code selecting, the number of codes with low PAPR available is limited and the resulting PAPR may not be really low. SLM multiples a frequency domain signal by prescribed randomly generated vectors and selects among them the vector that gives the time-domain symbol with lowest PAPR. PTS is based on the same principle as SLM. However, it provides PAPR reduction flexibility. In this scheme, sub-carriers are partitioned into multiple disjoint sub-blocks. Then, the phase of each sub-block is modified by phase rotation factors to make PAPR to be as low as possible. PTS significantly improves PAPR performance, but unfortunately, search the optimal phase weighting factors search from all combinations of allowed phase weighting factors. It turns out that search complexity increases exponentially with the number of sub-blocks. To reduce the searching complexity and avoid/reduce the usage of side information, many extensions of PTS have been developed recently [7-10,16,17]. Famous stochastic techniques for PAPR reduce include the simulation annealing algorithm [17], Genetic algorithm (GA) [18] and particle swarm optimization [19]. Based on above points, we state our interest to employ a novel PTS technique based on the EM algorithm to reduce the PAPR of OFDM signals through this paper. The simulations demonstrate that the proposed EM can not only achieve significant PAPR reduction but also enjoy complexity advantages compared with the other well-known stochastic approaches.

In this paper, we present a new technique for computing the phase weighting factors that achieves a better performance than the exhaustive search approach does. In proposed techniques, an Electromagnetism-like (EM) method [20-22] is performed to find the phase weighting factors. It shows that a low-complexity technique has been implemented in the PTS approach, which seeks the best trade-off between performance and complexity. Simulation results show that the performance of this new technique is similar to that of the optimum case; however, our technique only needs significantly lower complexity. Moreover, it will bring significant reduction in PAPR and will bring in slight change to the BER performance between with and without PTS which makes it feasible to be used and can be considered as potential future research work [26-29].

The rest of this paper is organized as follow. In Section 2, a typical MC-CDMA system is given and the PAPR problem is formulated and PTS is explained. Then, EM method is proposed to search the optimal combination of phase weighting factors for PTS in Section 3. In Section 4, the performance of MC-CDMA signals are studied and evaluated by using the proposed scheme to reduce the PAPR through computer simulation and Section 5 is the conclusion.

2. System Model. In this section, we describe the multi-carrier transmission systems model of MC-CDMA systems and peak to-average power ratio (PAPR) definition.

2.1. System model of MC-CDMA systems. MC-CDMA system has been proposed for a variety of topologies. The configuration used in this paper is similar to the design in [15]. Let \( N \) be the number of sub-carriers, \( L \) be the spreading factor of frequency domain and \( M \) be the number of parallel input data symbol per MC-CDMA symbol. The modulated signals of each user are fed into serial to parallel converter. The parallel signals are copied into \( L \) parallel sub-carriers. First, as the number of sub-carriers is \( N \), the same as the length of spreading code, a data symbol \( d_k \) is copied to \( N \) parallel taps. Each
copy is multiplied by a single chip of the spreading sequence, \( c_{k,n}, k = 0, 1, \ldots, K \) and \( n = 0, \ldots, N - 1 \), which is a chip of the \( k \)th user’s spreading code at the \( n \)th sub-carrier. The \( k \)th user’s frequency-domain spread spectrum \( X_k \), is given by

\[
X_k = d_k c_k
\]  

where \( X_k = [X_{k,0}, X_{k,1}, \ldots, X_{k,N-1}]^T \) and \( c_k = [c_{k,0}, c_{k,1}, \ldots, c_{k,N-1}]^T \). \( X_{k,n} \) and \( c_{k,n} \) denote the \( k \)-th user’s spread data and chip of the spreading code, respectively, at the \( n \)-th sub-carrier. Each user’s channel is modeled as an independent flat fading channel \( H_k = \text{diag} \{H_{k,0}, H_{k,1}, \ldots, H_{k,N-1}\} \), where \( H_{k,n} \) is a frequency domain channel response at the \( n \)-th sub-carrier for the \( k \)-th user. The received signal also experiences additive white Gaussian noise of zero mean and variance [9].

These signals are converted into time domain using inverse fast Fourier transform of size \( N = V \times L \). MC-CDMA signals \( x(t) \) can be written as

\[
x(t) = \sum_{n=0}^{N-1} \sum_{i=0}^{N_d-1} \sum_{k=0}^{K-1} d_{k,n}(t - iT_s) c_{k,n} e^{j2\pi nt/T_s}
\]

where \( N_d \) is the number of symbols in a frame and \( T_s \) is inserted between symbols to eliminate the ISI caused by multipath fading.

2.2. **Peak to-average power ratio (PAPR) definition.** Similar to orthogonal frequency division multiplexing (OFDM) system, the PAPR, can be defined as the maximum power of the transmitted signal divided by the average, is expressed as follow:

\[
PAPR(x) \triangleq \max_{0 \leq t \leq N-1} \frac{|x(t)|^2}{E[|x(t)|^2]} 
\]

where \( x(t) \) is an MC-CDMA signal, \( |x(t)|^2 \) represents the envelope power and \( E[\cdot] \) denotes the expected value [15]. PAPR is evaluated per symbol.

2.3. **Partial transmit sequences (PTS) method.** PTS method is one kind of multi-signal representation technique. A block diagram of MC-CDMA system using PTS is shown in Figure 1 as that in [7-9]. In PTS technique, the input data block is partitioned into the \( V \) disjoint clusters that approach, the input data block is partitioned into disjoint sub-blocks. Each sub-block is multiplied by a phase weighting factor, which is obtained by the optimization algorithm to minimize the PAPR value. We define the data block as a vector \( X = [X_1, X_2, \ldots, X_N]^T \), where \( N \) denotes the number of sub-carriers in the MC-CDM frame. Then, \( X \) is partitioned into \( V \) disjoint sub-blocks represented by the vector \( X_i, i = 1, 2, \ldots, V \) such that

\[
X = \sum_{i=1}^{V} X_i
\]

Here, it is assumed that the clusters \( X_i \) consist of a set of sub-blocks and are of equal size. Then, a weighted sum combination of the \( V \) sub-blocks which is written as

\[
X' = \sum_{i=1}^{V} W_i X_i, \quad W_i = e^{j\phi_i}
\]

where \( W_i, i = 1, 2, \ldots, V, \) is the phase weighting factor which has phases consisting of \( \varphi = [0, 2\pi) \). The phase weight factor can be chosen freely within \([0, 2\pi)\); however, phase weighting factor can be generally chosen from a certain discrete group such as \{±1\} for
reducing the calculation complexity. After transforming to the time domain, the new time
domain vector becomes

$$
x = \text{IFFT} \{ \sum_{i=1}^{V} W_i X_i \} = \sum_{i=1}^{V} W_i \text{IFFT} \{ X_i \} \tag{6}
$$

The optimal phase weighting factor $W_i$ that minimize the PAPR can be obtained from a
comprehensive simulation of all possible combination, $2^{V-1}$. The objective of the optimum
PTS (OPTS) method is to choose a vector $W = \{ W_1, W_2, \cdots, W_i \}$ to reduce the PAPR
of $X'$, and the optimum phase weight factor for an MC-CDMA symbol is given by

$$
\hat{W} = \arg \min_{W} \left\{ \max \left\{ \sum_{i=1}^{V} W_i x_i \right\} \right\} \tag{7}
$$

For OPTS technique, minimized PAPR can be found after searching $2^{V-1}$ computation if
the number of sub-block is $V$.

**Figure 1.** A block diagram of the EM based PTS technique

2.4. **Sub-block partition schemes (SPS).** The sub-block partition [23] for PTS tech-
nique is a method of division on sub-carriers into multiple disjoint sub-blocks. Figure 2
shows a generation of the sub-blocks partitioning method. In general, it can be classi-
ified into three categories: adjacent method, pseudo-random method and interleaving method.
For the adjacent method, $N/V$ successive sub-carriers are assigned into the same sub-
block sequentially. In pseudo-random method, $N/V$ sub-carriers are randomly assigned
into each sub-block. In interleaving method, every sub-carrier space $L$ part is allocated
in the same sub-block. In the viewpoint of PAPR reduction [23,24], pseudo-random sub-
block partitioning has a better performance than others.

2.5. **Nonlinear amplifier model.** To simulate a non-linear power amplifier, the follow-
ing Rapp’s model [11,25] is employed for amplitude conversion.

$$
g(A) = \frac{A}{\left( 1 + (|A|/A_{sat})^{2p} \right)^{\frac{1}{2p}}} \tag{8}
$$
where \( A \) is the amplitude of an input signal, \( A_{sat} \) is the saturation amplitude of an amplifier, \( g(A) \) is the amplitude of an output signal and \( p \) is a constant representing the characteristic of a non-linear amplifier. In this paper, \( p = 3 \) is assumed, which is a general value for solid-state power amplifiers (SSPA) [11]. The operating point of an amplifier is determined by input back off (IBO) given by

\[
IBO = \frac{P_{sat}}{E\{|x(t)|^2\}},
\]

where \( P_{sat} \) is the input power corresponding to the saturation point of an amplifier and \( E\{|x(t)|^2\} \) is the average input power.

![Figure 2. Generation of sub-blocks partitioning method](image)

### 3. Electromagnetism-like Method-based PTS.

#### 3.1. The EM optimization algorithm.

Electromagnetism-like mechanism (EM) is a heuristic method proposed by Birbil and Fang [20,21] for global optimization. Electromagnetism-like mechanism uses the attraction repulsion mechanism of the electromagnetism theory to determine the optimal solution. Like the original EM, the proposed method includes four main stages: (1) initialization, (2) local search, (3) calculation of the total force vector of each particle, and (4) movement based on the total force. These four stages are mainly based on the procedures of EM. The flow-chart of applying electromagnetism-like mechanism on feature selection shown in Figure 1. These procedures are interpreted as follows. The first procedure, initialization, is used to sample \( M \) points (particles) randomly form the feasible region. The next procedure, local search, is a neighborhood search procedure which can be applied to one or many points for local refinement to get better solutions at each iteration. The total force exerted on each point by all other points is calculated in the calculation of the total force procedure. The remaining procedure of the EM algorithm is the movement of the particles procedure, which is used for moving the sample points along the direction of the total force.
Algorithm 1 Electromagnetism-like Algorithm

1: Initialize()
2: while termination criteria are not satisfied do
3: Local search()
4: Calculation of the total force()
5: Movement of the particles()
6: end which

3.2. The EM algorithm based PTS technique. Electromagnetism-like mechanism is a population based meta-heuristic that has been proposed to solve continuous problems effectively. In general, EM simulates the attraction-repulsion mechanism of electromagnetism theory that is based on Coulomb’s law. Each particle represents a solution, and the charge of each particle relates to its objective function value. At iteration $\kappa$, a population with $M$ points is generated. Each solution point is considered as a point in a multidimensional solution space with a certain charge. This charge is related to the objective function values associated with all the solution points. In the following, we employ the EM method to search the optimal phase factor for the PTS technique in order to reduce the PAPR. The procedure of the proposed EM-based PTS can be described as follows:

Step 1. Setting of EM-like algorithm parameters and Initialization of particles: At first, the dimension of the solution will be determined according to the fitness function. Secondly, we determine the upper and lower bounds in each dimension. Furthermore, the population size $P_{\text{Max}}$ should be determined in applying EM-like algorithm to the optimization of phase weight factor pattern. Like most stochastic algorithms, the EM method starts with generating $M$ random sample points (or particles) \( \{ \{ \theta_{m,v}^K \}_{V=1}^V \}_{m=1}^M \) from the feasible region, where $V$ is the dimension of the problem (i.e., the number of sub-blocks) and $\theta_{m,v}^K$ denotes the $v$th coordinate of the particle $m$ of the population at iteration $k$. Analogous to electromagnetism, each point $\theta_{m,v}^K$ is regarded as a virtually charged particle that is released in the space. It should be noted that in a multidimensional solution space where each point represents a solution, a charge is associated with each point. As such, each coordinate of a point, denoted as $\theta_{m,v}^K$, is computed by

$$\theta_{m,v}^K = l_v + \lambda (u_v - l_v)$$

where $u_v$ is the upper bound of the $v$-th dimension; $l_v$ is the lower bound of the $v$-th dimension; and $\lambda$ is a uniform random number generator within $[0, 1]$.

Step 2. Determination of fitness function and bounds: In this study, we use the position and weight as parameters to compute the fitness function of each particle, i.e., given a point (i.e., phase factor vector $W$) $\Theta_m^k$, the fitness function, defined as the amount of PAPR reduction, can be expressed as

$$f(\Theta_m^k) = \frac{1}{10 \log_{10} \left[ \max_j \left| x'(\Theta_m^k) \right|^2 \right] E \left[ \left| x'(\Theta_m^k) \right|^2 \right]}.$$  

When the $M$ points are all identified, the point with the best objective function value is stored into $\Theta_{\text{best}}^k = \{ \theta_{\text{best},v}^k \}_{V=1}^V$. As we are interested in the values of phase factors in the range of 0 to $2\pi$, the upper bound and lower bound are set to 0 and $2\pi$, respectively.

Step 3. Local search and update: Local search should be able to find better solution in theory. Local search is used to gather the neighborhood information for a sampled point, which can be applied to one point or to all point in the population for local refinement at each iteration. Theoretically, the local search is expected to find a better solution.
especially when it is applied to all particles. However, the local search is usually time-consuming. Therefore, in this study, the EM algorithm is implemented with local search on the current better particle. The procedure of the local search can be described as follows:

Step 3.1) Calculate maximum feasible step length $s_{\text{max}}$ based on the parameter $\delta \in [0, 1]$, where the maximum feasible step length can be computed using the following equation:

$$S_{\text{max}} = \delta \left( \max_{1 \leq v \leq V} (u_v - l_v) \right).$$

(12)

Step 3.2) Generate a candidate of point $\tilde{\Theta} = \left\{ \tilde{\theta}_v \right\}_{v=1}^V$: A new particle $\tilde{\Theta}$ is generated from the current best point $\tilde{\Theta}_{\text{best}}^k$. As $\tilde{\Theta}$ is a small random change coming from $\tilde{\Theta}_{\text{best}}^k$, here, we randomly change two coordinates to generate $\tilde{\Theta}$, where the modified coordinate of the current best point, denoted as $\tilde{\theta}_v$, is computed using the following equation:

$$\tilde{\theta}_v = \theta_{\text{best},v}^k + \lambda \cdot s_{\text{max}}.$$ 

(13)

Step 3.3) Decide whether to update the current best point $\tilde{\Theta}_{\text{best}}^k$: If the new point $\tilde{\Theta}$ observes a better point, the sample point $\tilde{\Theta}_{\text{best}}^k$ is replaced by this new point $\tilde{\Theta}$. Step 3.4) Repeat Step 3.1 to Step 3.3 until the maximum number of local search iteration is met.

Step 4. Calculation of charge: The particle moves according to Coulomb’s force produced among the particles, as we assign a charge-like value to each particle. The charge of each particle is determined by its fitness function value, which can be evaluated as: the artificial charge $q_m^k$ at point $\Theta_m^k$ is determined by the fitness function value, and is calculated using the following equation:

$$q_m^k = \exp \left\{ -V \sum_{m=1}^M \frac{f(\Theta_m^k) - f(\Theta_{\text{best}}^k)}{f(\Theta_{\text{best}}^k) - f(\Theta_{\text{best}}^k)} \right\}.$$ 

(14)

By observing (14), we can find that 1) a large $f(\Theta_m^k)$ results in a small $q_m^k$, and vice versa; and 2) the artificial charges are all positive. Now, the problem on hand is how to determine the force of attraction or repulsion between each pair of particles $\Theta_m^k$ and $\Theta_r^k$. Suppose that $f(\Theta_m^k) < f(\Theta_r^k)$, which implies that $q_m^k > q_r^k$; in this case, the one that has better fitness function value is preferred, that is $\Theta_m^k$ is the preferred point and particle $\Theta_r^k$ should be “attracted” to particle $\Theta_m^k$. That means the particle attracts other particles with better with better fitness function values and repels other particles with fitness cost function values.

Step 5. Calculation of resultant force: The resultant force among particles determines the effect for optimization process. The resultant force of each particle can be evaluated by Coulomb’s law and superposition principle as: determining the charge of each point on $\{\Theta_m^k\}_{m=1}^M$ and defining the rule of attraction-repulsion mechanism of artificial charge the force vector, $F_{m,r}^k$, between two points $\Theta_m^k$ and $\Theta_r^k$, is computed as

$$F_{m,r}^k = \begin{cases} 
(q_r^k - q_m^k) \frac{q_r^k \cdot q_m^k}{||q_r^k - q_m^k||^2}, & \text{if } f(\Theta_r^k) < f(\Theta_m^k) \text{(attraction)} \\
(q_m^k - q_r^k) \frac{q_m^k \cdot q_r^k}{||q_m^k - q_r^k||^2}, & \text{if } f(\Theta_r^k) \geq f(\Theta_m^k) \text{(repulsion)}
\end{cases}$$

(15)
The total force $\varphi^k_m$ exerted on each point $\Theta^k_m$ by the other $(M - 1)$ points is then calculated by

$$\varphi^k_m = \sum_{r \neq m}^M F^k_{r,m}, \quad m = 1, 2, \cdots, M. \quad (16)$$

**Step 6. Movement of the particles:** After calculating the total force $\varphi^k_m$, the point $m$ is updated in the $v$-th coordinate of the force by the force by a random step length as given as

$$\theta^k_{m,v}^{+1} = \begin{cases} 
\theta^k_{m,v} + \lambda \frac{\varphi^k_{m,v}}{\|\varphi^k_{m,v}\|} (u_v - \theta^k_{m,v}), & \text{if } \varphi^k_{m,v} > 0 \\
\theta^k_{m,v} + \lambda \frac{\varphi^k_{m,v}}{\|\varphi^k_{m,v}\|} (\theta^k_{m,v} - l_v), & \text{if } \varphi^k_{m,v} \leq 0
\end{cases} \quad m = 1, 2, \cdots, M; \quad m \neq \text{best}. \quad (17)$$

**Step 7. Criterion:** Running the EM-like procedures until the predetermined iteration or allowable optimal value is met. In other words, the procedures will be terminated as the criterion is reached.

### 3.3. Complexity comparison for finding suboptimal solutions.

As GA, PSO and the EM are all population-based search methods, we may therefore fix the number of samples, to find the suboptimal solutions with low complexity. In this case, the complexity for GA [18], PSO [19] and EM method can further be expressed in terms of the number of samples, where each sample is calculated using the $N$-point IFFT. Accordingly, the number of samples for GA, PSO and the EM are $P_{\text{Max},\text{pop}}$, $P_{\text{Max},\text{pop}} + \text{pop}$ and $(\text{pop} + P_L) \cdot P_{\text{Max}}$, respectively, where $P_{\text{Max}}$ is the maximum number of iteration, $\text{pop}$ is the number of sample point (particle) and $P_L$ is the maximum number of local search iterations. It should be that the complexity for each sample to find a suboptimal solution is $O(N \log N)$ multiplications.

### 4. Results and Discussions.

Simulation experiments are conducted in this section to verify the PAPR performance of the proposed EM method presented in Section 3 for multi-carrier transmission systems signal. In the simulations, the numbers of subcarriers are set to be $N = 64$, $N = 128$ and $N = 256$ subcarriers, respectively, which are divided into $V = 8$ subblocks and data symbols are modulated using the QPSK constellation with four times oversampling. The criteria for performance measurement considered here are the complementary cumulative distribution function $\text{CCDF} = P_r[\text{PAPR} > \text{PAPR}_0]$ of the PAPR and the average PAPR performance, where the CCDF is the probability that the PAPR of a symbol exceeds the threshold level $P$ PAPR$_0$. In order to generate the CCDF of the PAPR, 10,000 OFDM blocks are generated randomly. For comparison, we also tested some existing stochastic optimization-based approaches for PAPR reduction, including the SA algorithm [17], the GA method [18] and PSO [19]. These parameters are the number of clusters and the number of allowed phase weighing factors $W$ for transmit sequences. The cumulative distribution function (CDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The CDF of the amplitude of a signal sample is given by $\text{CDF} = 1 - \exp(\text{PAPR}_0)$. In the performance comparison, the parameter of CCDF [15] is defined as

$$\text{CCDF} = P_r(\text{PAPR} > \text{PAPR}_0) = 1 - P_r(\text{PAPR} \leq \text{PAPR}_0) = 1 - (1 - \exp(-\text{PAPR}_0))^N \quad (18)$$

This expression assumes that the $N$ time domain signal samples are mutually independent and uncorrelated. In the EM method, the population size is assumed as $\text{pop} = 20$;
the maximum number of iterations is \( P_{\text{Max}} \), the maximum number of local search iterations is \( P_L = 10 \); and the corresponding maximum number of iterations are \( P_{\text{Max}} = 20, 40, 60, 80 \) and 100, respectively.

**Figure 3.** Comparison of the PAPR CCDF of the different numbers of the maximum number of iterations of the EM method for \( N = 64 \)

**Figure 4.** Comparison of the PAPR CCDF of the different numbers of the maximum number of iterations of the EM method for \( N = 128 \)

Figures 3 and 4 show the variation in CCDF with the proposed EM method for different numbers of the maximum number of iterations, \( P_{\text{Max}} \), with \( N = 64 \) and \( N = 128 \), respectively. In addition, we selected the exhaustive search algorithm mentioned in [3] to compare the performance of PAPR reduction with that of the EM searching method. In the ESA, the selection of the phase factors was limited to a set of finite number of elements \( W \). The ESA was then employed to find the best phase factor. Here, four allowed phase factors \(+1, -1, +j\) and \(-j\) \((W = 4)\) were used for the OPTS, and the
PAPR reduction performance was obtained by a Monte Carlo search with a full enumeration of $W^V(4^8 = 65,536)$ phase factors. As shown in Figures 3 and 4, as the maximum number of iterations was increased, and the CCDF of the PAPR has been improved. When $P_r [PAPR > PAPR_0] = 10^{-3}$, we can see that the performance of the proposed EM method provides an approximate PAPR reduction as with that of the conventional PTS.

![Figure 5](image-url)

**Figure 5.** CCDFs comparison of the EM, PSO and GA based-PTS scheme with different combinations of sub-blocks when $N = 256$, $V = 2$ and $W = 2$

![Figure 6](image-url)

**Figure 6.** CCDF of the sub-block phase weighting using three partitioning techniques

Figure 5 illustrates the CCDFs of the PAPR of QPSK-modulated OFDM signals in OPTS, GA-, PSO- and EM-based PTS when $W = 2$, respectively, for $N = 256$, $V = 2$ and $M = 16$ ($V$ is the number of subblocks). Clearly, the PAPR reduction performance of EM-based PTS is worse than that of OPTS, but the performance for PAPR reduction of the proposed EM method is almost the same as that of OPTS. This means that the EM method outperforms GA- and PSO-based PTS schemes. It can be regard as that when
the number of sub-blocks increases; the performance of peak power depression improves. For instance, given CCDF = 0.1%, the PAPR of the normal MC-CDMA is about 10.3 dB, and those of OPTS, EM-, PSO- and GA-based PTS are 7, 8.2, 9.2 and 9.8 dB for $V = 2$, respectively; those of OPTS, EM-, PSO- and GA-based PTS are 7, 7.2, 7.4 and 7.8 dB for $V = 16$, respectively. It indicates relatively increasing the number of sub-block $V$ will improve the system performance evidently it can be regard as that the EM-based PTS curves are very close to the OPTS. In addition, we can see that the performance of the proposed EM method not only provides an approximate PAPR reduction as with that of the OPTS but also has a much lower computational complexity than OPTS.

Figure 6 compares the performance of restraint PAPR with three categories of sub-block partitions, when $V = 8$, $W = 8$. It can be observed that probability of very large peak power has been increased significantly if PTS techniques are not used. That is, probability of $\text{PAPR} \geq 7.2$ dB is $10^{-3}$ at the pseudo-random SPS while it almost reaches 10.3 dB at the system without using PTS. The probability of $\text{PAPR} \geq 7$ dB is about $10^{-1}$ at the interleaved SPS and 0.5 at the adjacent SPS, respectively. And it is about $10^{-2}$ at pseudo-random SPS. It is clear that the pseudo random SPS is outstanding. Thus, it can be analyzed that the pseudo-random SPS shows the best PAPR reduction performance as expected. However, the PAPR reduction performance and the computational complexity of PTS scheme depend on the method of subblock partitioning. In other words, there is a trade-off between PAPR reduction performance and computational complexity in OPTS scheme.

![Figure 6. CCDF of MC-CDMA signal relatives to the sub-block partition method](image)

**Figure 7.** CCDF of MC-CDMA signal relatives to the sub-block partition method

As to Figure 7, CCDF of the PTS system with both the number of sub-blocks $V$ and the fixed phase weighting factor $W = 2$ have been illustrated. Note that varying the number of sub-blocks gives little effect. In contrast, increasing the numbers of sub-blocks and admitted angles number of sub-blocks exhibits better a result. It shows that $V = 4$ and the number of admitted (weight phase) angles of 2 are reasonable, in terms of computational complexity, because the number of computations is $(\text{number of admitted angles})^{V-1}$. The exponent $V - 1$ arises from the fact the bit can be fixed without any performance loss. There is no analytical way so far to compute the optimal factors except by an exhaustive search procedure that makes optimal binary phase sequence unsuitable for large number of sub-blocks.
Figure 8. Comparison of the PAPR CCDF of several PTS methods

Figure 9. BER performance of MC-CDMA system with EM method

Figure 8 depicts the CCDF of the PAPR with the PTS sequence search by PSO technique and GA with $V = 8, 16$ and $W = 4$. The results presented in Figure 8 imply that the proposed method and GA can provide the same performance, but with much lower computational complexity. As the number of sub-blocks and the set of phase weighting factor are increased, the performance of the PAPR reduction becomes better. However, the processing time gets longer because of much iteration. For a fair comparison, both the PSO-based and the GA-based optimization run 20 times. Just as expected, the PAPR performance of our proposed EM-based PTS scheme with $(V, W) = (2, 2)$, is not only almost the same as that of the GA-based PTS scheme with $(V, W) = (4, 4)$, but also having much lower computational complexity. In general, in order to obtain optimal PAPR search for the number of sub-blocks and phase weight factor must be accomplished. As the number of sub-blocks and phase weight factors increases, PAPR reduction improves.
The number of calculation increases as the number of sub-blocks increases, such that complexity increases exponentially and process delay occurs simultaneously. As one can see, the PTS with EM technique has almost the same performance of PAPR reduction as that of the optimal PTS scheme. Finally, this paper presents that investigates the trade-off between number of sub-blocks and phase weight factors for reduce PAPR.

Figure 9 depicts the performance of BER versus SNR of actual MC-CDMA signals with PAPR reduction based on different schemes over the AWGN channel, in which the typical HPA of the solid state power amplifier (SSPA) has been considered [25]. As seen in the Figure 9, to meet BER $10^{-4}$ at IBO = 7 dB, the proposed method can achieve 0.7 dB SNR gain than PSO-PTS method and can achieve 1 dB SNR gain than GA-PTS method, respectively. Consequently, it is clear than in the case using at IBO = 7 dB, BER performance closely matches with linear amplifier.

5. **Conclusions.** This paper presented an EM-like method that was used to obtain the optimal phase weighting factor for the PTS technique to reduce computational complexity and improve PAPR performance. We formulated the phase weighting factors search of the PTS technique as a global optimization problem with bound constraints. The computer simulation results showed that compared with the various stochastic search techniques developed previously, the proposed EM-like method obtained the desirable PAPR reduction with low computational complexity. By the sake of its flexibility, i.e., the number of sub-blocks, the number of admitted angles and SPSs, the resulting peak power reduction can be achieved with several levels of acceptable PAPR. Since the computational complexity reduction ratio increases as the number of sub-carriers increases, the proposed scheme becomes more suitable for the high data rate multi-carrier transmission systems.

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