A DESIGN METHOD FOR 1-D IIR FILTERS WITH A NECESSARY AND SUFFICIENT STABILITY CRITERION

TOMA MIYATA\textsuperscript{1}, NAOYUKI AIKAWA\textsuperscript{1}, YASUNORI SUGITA\textsuperscript{2} and TOSHINORI YOSHIKAWA\textsuperscript{2}

\textsuperscript{1}Tokyo University of Science
2641 Yamazaki, Noda, Chiba 278-8510, Japan
\{j8109702@ed; ain@te\}.noda.tus.ac.jp

\textsuperscript{2}Nagaoka University of Technology
1603-1 Kamitomioka-machi, Nagaoka, Niigata 940-2188, Japan
\{sugita; tyoshi\}@nagaokaut.ac.jp

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ABSTRACT. In general, it is necessary to guarantee stability when designing of One-dimensional (1-D) infinite impulse response (IIR) filters. Methods for guaranteeing stability by using Rouché’s theorem, the positive realness condition, or a method based on the positive realness have previously been proposed for defining the necessary iterative approximation algorithm. In these cases, the conventional stability criteria become the sufficient condition to guarantee stability. As a result, the stability domains obtained using these criteria are narrow and variable. In the present paper, we propose a design method of 1-D IIR filters, which applies a stability criterion based on the system matrix to the successive projection (SP) method. The stability criterion based on the system matrix in the proposed method becomes the necessary and sufficient condition for guaranteeing stability. Therefore, the stability domain does not depend on denominator polynomial coefficients, and the domain is not variable. Moreover, the stability domain is wider than that by the conventional stability criteria. As a result, 1-D IIR filters obtained using the proposed method have a smaller ripple than those from using conventional methods. In addition, the proposed design method realizes faster design times than those from using the conventional design methods.

Keywords: IIR filter, Filter stability, System matrix, Successive projection method

1. Introduction. Recently, digital signal processing is in use in various various fields (e.g., information and communication, measurement and control, and medical fields) in [13-16]. In short, digital filters technology is important. One-dimensional (1-D) digital filters can be classified as finite impulse response (FIR) filters or infinite impulse response (IIR) filters. IIR filters can be implemented with less memory and with fewer computations per output sample than equivalent FIR filters. However, unlike FIR filters, the design problem for IIR filters is nonlinear because of the nonlinear interdependence of the filter coefficients and the frequency response. In addition, the stability of IIR filters must be guaranteed.

In the past decade, a number of methods for designing 1-D IIR filters using semidefinite programming (SDP), linear programming, least-squares, the $p$-th norm optimal method or the successive projection (SP) method have been proposed in [1-8]. SDP, linear programming, least-squares, and the $p$-th norm optimization method, all require large amounts of memory to solve the design problem. In contrast, using the SP method has the advantage that it uses less memory and the algorithm is a simple iterative approximation technique [5]. In the field of control regulation, there have been many stability-related studies in...
The positive realness condition, Lyapunov’s stability criterion, Rouché’s theorem, and the stability criterion based on the system matrix are proposed methods of checking system stability. Recently, Y. Sugita et al. [5] proposed a design method of 1-D IIR filters using the SP method, and a design method using Rouché’s theorem to guarantee the stability. However, Rouché’s theorem is merely a sufficient (but not a necessary) condition for guaranteeing stability. Therefore, the stability domain obtained using Rouché’s theorem is narrow and variable. Also, in some cases, the design method excludes good designs. B. Dumitrescu et al. [3,4] proposed design method of 1-D IIR filters using the $p$-th norm optimal method and a method based on the positive realness to guarantee filter stability. In this case, the stability domain is wider than obtained by Rouché’s theorem, and the frequency response of the filter obtained by the design method in [4] is better than that from using the conventional design method. The method based on the positive realness checks filter stability using before updating the stable filter coefficients and the updating values of coefficients, which are the same as those in Rouché’s theorem. This stability criterion is also merely a sufficient (but not a necessary) condition for guaranteeing stability. Then, the stability domain obtained using this method differs depending on before updating the filter coefficients and the updating values of filter coefficients. T. Yamazaki et al. [8] proposed a design method of 1-D IIR filters using linear programming and the positive realness condition to guarantee filter stability. The stability domain obtained using the positive realness condition is not changed by after updating the filter coefficients. However, this stability criterion is also merely a sufficient (but not a necessary) condition for guaranteeing stability. Hence, this stability criterion is not able to identify the complete stable domain. As a result, good designs can be excluded by using these stability criteria.

One related study [9] presents a method based on the system matrix. In this case, the stability criterion consists of inequality constraints, which depends only on denominator coefficients. The proposed method gives a necessary and sufficient condition for guaranteeing stability, and the stability domain does not depend on the coefficients of the denominator polynomial. In addition, the obtained stability domain is wider than that based on the conventional methods (Rouché’s theorem, the positive realness condition and a method based on the positive realness). Design methods for IIR filters using stability criteria with a necessary and sufficient condition have not proposed previously.

In the present paper, we propose a design method for 1-D IIR filters, which applies a stability criterion based on the system matrix to the SP method. This stability criterion is a necessary and sufficient condition. However, this stability criterion does not guarantee stability by limiting the amount of the updating values of filter coefficients. Because the denominator polynomial may become unstable when its coefficients are updated, we propose a method of correcting the coefficients to give a stable polynomial: if the system obtained is unstable, the problematic eigenvalues are reduced in magnitude to make the system stable, and the coefficients of the denominator polynomial are derived from the adjusted eigenvalues. The proposed design method realizes faster design times and smaller ripples than the conventional design method using the SP method with Rouché’s theorem described in [5]. In addition, 1-D IIR filters obtained by the proposed design method have as good as or better ripples than those obtained by the conventional methods using linear programming with the positive realness condition [8] or the $p$-th norm optimal method with a method based on the positive realness [4].

2. Preparation. In this section, we describe a method for designing 1-D IIR filters using the SP method, which is a type of projection onto convex sets (POCS).
In general, the frequency response of 1-D IIR filters is expressed as:

\[
H(\omega) = \frac{\sum_{i=0}^{M} a_i e^{-j\omega}}{1 + \sum_{i=1}^{N} b_i e^{-j\omega}} = \frac{N(\omega)}{D(\omega)},
\]

where \(a_i\) and \(b_i\) are real filter coefficients, and \(M\) and \(N\) are the orders of the numerator and denominator, respectively.

Next, we define a frequency response with the desired amplitude characteristics \(A\) and phase characteristics \(\theta\) by

\[
F(\omega) = A(\omega)e^{j\theta(\omega)}.
\]

Then, the design problem is to find filter coefficients \(a_i\) and \(b_i\) satisfying

\[
\gamma(\omega) = |F(\omega) - H(\omega)| \leq \lambda(\omega),
\]

where \(\lambda(\omega) > 0\) is the maximum allowable deviation from the desired frequency response.

In general, an approximation problem in the complex domain is a nonlinear optimization problem in the real domain. However, by introducing a simple transformation from [5], it can be converted to a linear optimization problem in the real domain. After this transformation, the error function in the \(n\)-th iteration step at a frequency point \(\omega\) is given by

\[
e^n(\omega, t_1, t_2, a^n, b^n) = \left| \frac{P_n(\omega, t_1)}{Q_{n-1}(\omega, t_2)} \right| \leq \lambda(\omega),
\]

where

\[
P_n(\omega, t_1) = A(\omega) \cos(\theta(\omega) + 2\pi t_1) + \sum_{i=1}^{N} A(\omega) b^n_i \cos(\theta(\omega) - i\omega + 2\pi t_1) - \sum_{i=0}^{M} a^n_i \cos(-i\omega + 2\pi t_1),
\]

\[Q_{n-1}(\omega, t_2) = \cos(2\pi t_2) - \sum_{i=1}^{N} b^{n-1}_i \cos(-i\omega + 2\pi t_2),
\]

and \(t_1\) and \(t_2\) are rotation parameters for the transformation.

The SP method minimizes the function \(G\) given by

\[
G = \|a^{n+1} - a^n\|^2 + \|b^{n+1} - b^n\|^2 + \alpha \left\{ e^n(\omega_M, t_{1M}, t_{2M}, a^{n+1}, b^{n+1}) - \lambda(\omega_M) \right\},
\]

where \(\alpha\) is the Lagrange multiplier, \(a^{n+1}\) and \(b^{n+1}\) are the projections of \(a^n\) and \(b^n\), respectively, and

\[
\|a^{n+1} - a^n\|^2 = \sum_{i=0}^{M} (a^{n+1}_i - a^n_i)^2,
\]

\[
\|b^{n+1} - b^n\|^2 = \sum_{i=1}^{N} (b^{n+1}_i - b^n_i)^2.
\]

In [5], the optimization problem is solved using the following updating of the filter coefficients:

\[
a^{n+1}_i = \begin{cases} 
a^n_i + \delta_i & \text{if } e^n(\omega_M, t_{1M}, t_{2M}, a^n, b^n) > \lambda(\omega_M) \\
a^n_i & \text{if } e^n(\omega_M, t_{1M}, t_{2M}, a^n, b^n) \leq \lambda(\omega_M)
\end{cases},
\]
\[ b_i^{n+1} = \begin{cases} b_i^n - \delta_{i,b} & \text{if } e^n (\omega_M, t_1 M, t_2 M, a^n, b^n) > \lambda (\omega_M) \\ b_i^n & \text{if } e^n (\omega_M, t_1 M, t_2 M, a^n, b^n) \leq \lambda (\omega_M) \end{cases} \]  

(11)

where

\[ \delta_{i,a} = \frac{\{e^n (\omega_M, t_1 M, t_2 M, a^n, b^n) - \lambda (\omega_M)\} \times Q_n (\omega_M, t_2 M) \cos (-i\omega_M + 2\pi t_1 M)}{R (\omega_M, t_1 M)}, \]

(12)

and

\[ \delta_{i,b} = \frac{A (\omega_M) \{e^n (\omega_M, t_1 M, t_2 M, a^n, b^n) - \lambda (\omega_M)\} \times Q_n (\omega_M, t_2 M) \cos (\theta (\omega_M) - i\omega_M + 2\pi t_1 M)}{R (\omega_M, t_1 M)}. \]

(13)

Here, \( \omega_M \) is the frequency point that gives the maximum deviation from the desired specification. In following section, we describe stability criteria.

3. Comparing Stability Criteria. In this section, we show stability criteria and compare the corresponding stability domains by using stability triangles.

3.1. Stability criteria. Now, we describe some stability criteria.

[Stability Criterion Based on the System Matrix]

The state space representation of 1-D IIR filters is

\[ \mathbf{q}(n+1) = \mathbf{Aq}(n) + \mathbf{Bx}(n), \]

(14)

\[ y(n) = \mathbf{Cq}(n) + d, \]

(15)

where \( x(n) \) is the input and \( y(n) \) is output. The various coefficients are given by

\[ \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots & \vdots \\ 0 & \cdots & \cdots & 0 & 1 & \vdots \end{bmatrix}, \]

(16)

\[ \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}^T, \]

(17)

\[ \mathbf{C} = [a_{N-a_0} b_N \ a_{N-1-a_0} b_{N-1} \ \cdots \ a_1-a_0 b_1], \]

(18)

\[ d = a_0 x(n) + a_{N+1} u(n-N-1) + \cdots + a_M u(n-M). \]

(19)

\( \mathbf{A} \) is the system matrix, \( \mathbf{q} \) is the state vector, \( u \) is the feed-back part of the output, and \( u = 0 \) if \( M \leq N \).

As shown in [9], the system is stable when all eigenvalues \( \nu_i \) (1 \( \leq \) \( i \) \( \leq \) \( N \)) of \( \mathbf{A} \) satisfy

\[ |\nu_i| < \rho, \]

(20)

where \( \rho \) is the prescribed maximum pole radius (0 \( < \rho \) \( \leq \) 1). That is, the eigenvalues \( \nu_i \) correspond to a pole of the transfer function. This criterion is a necessary and sufficient condition for guaranteeing stability.

Usually, poles of the transfer function are found by assuming that the denominator of the transfer function is zero and solving the resulting equation. General formulas for the solution of algebraic equations exist only for equations of fourth-order or less. In addition, the formulas for solutions of third- and fourth-order equations are useless because there can be a high sensitivity between the equation coefficients and the roots. Therefore, a number of numerical calculation methods for third-order and grate polynomials have been proposed [11,12]. One of the methods is to solve the equation as an eigenvalue problem.
using the companion matrix. Using this method, the stability criteria based on the system matrix have the advantage that they are not influenced by the order of the denominator polynomial of the transfer function.

[Rouché’s Theorem]

If \( f(z) \) and \( g(z) \) are analytic inside and on a closed contour \( C \), and \( |f(z)| < |g(z)| \) on \( C \), then \( f(z) \) and \( f(z) + g(z) \) have the same number of zeros inside \( C \) [7].

Assume that \( f(z) = D(z) \) and \( g(z) = \delta_b(z) \), where \( D(z) \) is the denominator polynomial of the transfer function to be designed, and \( \delta_b(z) \) is the solution update added to the actual denominator polynomial \( D(z) \) in an iteration step. The function \( f(z) \) and \( g(z) \) are analytic everywhere except at \( z = \infty \). According to Rouché’s theorem, if we choose \( C \) to be a circle with radius \( \rho \) (\( 0 < \rho < 1 \)) centered at origin of the complex domain, and if \( D(z) \) has all its zeros inside this circle, then the new polynomial \( \tilde{D}(z) = D(z) + \delta_b(z) \) is stable, when combination of \( D(z) \) and \( \delta_b(z) \) satisfy the following condition on the circle \( |z| = \rho \):

\[
|D(z)| > |\delta_b(z)|
\]  

(21)

[Positive Realness Condition]

As shown in [8], an arbitrary polynomial \( \tilde{D}(z) \) is stable when it satisfies the following condition:

\[
\text{Re}\{\tilde{D}(z)\} > \epsilon,
\]

(22)

where \( \epsilon \) is a small positive value.

In this case, the denominator polynomial of the transfer function in designing the filter is \( D(z) \), the updating value is \( \delta_b(z) \), and an arbitrary polynomial is \( \tilde{D}(z) = D(z) + \delta_b(z) \).

Then, stability of the arbitrary polynomial \( \tilde{D}(z) \) is checked using positive realness after the updating polynomial coefficients.

[Method Based on Positive Realness]

In [3], positive realness is solved as follows:

If an arbitrary polynomial \( J(z) \) is positive realness condition as (22), then \( J(z) \) is stable in all combinations \( \tilde{D}(z) \) and \( D(z) \) in [3], where

\[
J(z) = \frac{\tilde{D}(z)}{D(z)} = 1 + \frac{\delta_b(z)}{D(z)},
\]

(23)

\( D(z) \) is an arbitrary polynomial and \( \tilde{D}(z) = D(z) + \delta_b(z) \).

Now, the denominator polynomial of transfer function in the designing filter is \( D(z) \), its updating value is \( \delta_b(z) \), and \( \tilde{D}(z) \) is denominator polynomial after updating the coefficients. According to the positive realness condition, if \( J(z) \) is stable, then combination of \( D(z) \) and \( \tilde{D}(z) \) also are stable.

3.2. Comparing stability domains. We compare the stability domains obtained using these methods by using stability triangles [3]. Figures 1 and 2 show simulation results of stability domains obtained by these methods, where the initial stable second-order polynomials in Figures 1 and 2 are respectively constructed using \( D(z) = 1 - 0.5z^{-1} + 0.6z^{-2} \) and \( D(z) = 1 - 0.3z^{-1} - 0.4z^{-2} \), and \( z \) transformation of the updating value is \( \delta_b(z) = \delta_1 z^{-1} + \delta_2 z^{-2} \). These initial polynomials are same as those in [3]. The system matrices used in Figures 1 and 2 are constructed using \( D(z) = 1 - (0.5 - \delta_1)z^{-1} + (0.6 + \delta_2)z^{-2} \) and \( D(z) = 1 - (0.3 - \delta_1)z^{-1} - (0.4 - \delta_2)z^{-2} \), respectively.

In Figures 1 and 2, stability domains obtained using a stability criterion based on the system matrix are wider than those obtained using Rouché’s theorem, positive realness condition or a method based on the positive realness. On the other hand, stability criterion based on the system matrix includes the complete stable domain. In addition, stability
domains obtained using Rouché’s theorem and a method based on the positive realness shown in Figures 1 and 2 depend on initial stable second-order polynomials, whereas the stability criterion based on the system matrix and the positive realness condition produce constant stability domains. However, the positive realness condition does not obtain the complete stable domains. Consequently, there is the case that the design methods for 1-D IIR filters using Rouché’s theorem, the positive realness condition, or based on the positive realness cannot be achieved optimum solutions (good design results). Therefore, 1-D IIR filters obtained by using a stability criterion based on the system matrix have the possibility of smaller amplitude ripples or group delay errors than those obtained by using the conventional methods.
4. **New Design Algorithm for 1-D IIR Filters.** In this section, we describe our proposed method for designing 1-D IIR filters which applies a stability criterion based on the system matrix to the SP method.

If the system is unstable after the denominator polynomial coefficients are updated, the coefficients are corrected by the following method:

1. Find the largest eigenvalues satisfying $|\nu_i| \geq \rho$ ($1 \leq i \leq N$).
2. Replace eigenvalues with corrected eigenvalues $\nu'_i$ given by
   \[ \nu'_i = \frac{\nu_i \rho}{\max_{i} |\nu_i|}. \]  
   (24)
3. Adjust the stable denominator polynomial coefficients by using the corrected eigenvalues.

The proposed algorithm for designing 1-D IIR filters using the stability criterion based on the system matrix is shown as a flow chart in Figure 3. The proposed method is an iterative approximation method, the same as the conventional SP method described in [5].

![Flow chart](image)

**Figure 3.** New design algorithm for 1-D IIR filters

5. **Design Examples.** In this section, we demonstrate the effectiveness of the proposed design method for 1-D IIR filters.

**Example 5.1.** The filter specifications to compare the conventional method using linear programming and the positive realness condition to guarantee filter stability described in [8] are as follows.

**Filter orders:**
- $M = 12$, $N = 6$

**Desired amplitude characteristics:**
\[ F(\omega) = \begin{cases} e^{-j\omega} & \text{for } 0 \leq \omega \leq 0.5\pi \\ 0 & \text{for } 0.6\pi \leq \omega \leq \pi \end{cases} \]

Prescribed maximum pole radius: \( \rho = 1.0 \)

Figure 4 shows amplitude response, group delay and amplitude error in the obtained filter, Figure 5 shows pole location, and Table 1 shows the filter performance. Amplitude response of the obtained filter shown in Figure 4 has equi-ripple characteristics. Next, we compare the proposed design method with the conventional method described in [8] using the maximum amplitudes of the error (Table 1). As shown, it is evident that the proposed method produces smaller amplitudes of error than the conventional method. We compare the group delay error at the passband in the table, and the maximum pole radius is determined from specifications. It is clear from that the proposed method produces smaller group delay error than that produced by the conventional method. The obtained maximum pole radius of the filter in Figure 5 is 0.9934, which meets the specification. Therefore, the obtained filter is stable.

**Figure 4.** The amplitude response, group delay, and amplitude error of the obtained filter in Example 5.1

**Table 1.** Performance in Example 5.1

<table>
<thead>
<tr>
<th></th>
<th>Proposed method</th>
<th>Conventional method in [8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum amplitude of the error in the passband</td>
<td>0.010 (0.086dB)</td>
<td>0.0142 (0.122dB)</td>
</tr>
<tr>
<td>Maximum amplitude of the error in the stopband</td>
<td>0.010 (−40.00dB)</td>
<td>0.0142 (−36.95dB)</td>
</tr>
<tr>
<td>Maximum group delay error</td>
<td>0.0733</td>
<td>0.8701</td>
</tr>
</tbody>
</table>

**Example 5.2.** To compare a conventional method using the \( p \)-th norm optimal method and a method based on the positive realness to guarantee filter stability describe in [4],
we design a 1-D IIR filter using the proposed method with the same specifications as in Example 5.1, along with a maximum allowable pole radius of $\rho = 0.9440$.

Figure 6 shows amplitude response, group delay, and amplitude error in the obtained filter, Figure 7 shows pole location, and Table 2 shows the filter performance. Amplitude response of the obtained filter shown in Figure 6 has equi-ripple characteristics. Next, we compare the proposed design method described with the conventional method described in [4] using the maximum amplitudes of the error (Table 2). As shown, the proposed method produces smaller amplitudes of error than the conventional method. Comparing the group delay error at the passband, it is clear that the proposed method produces smaller group delay error than that produced by the conventional method. The obtained maximum pole radius of the filter in Figure 7 is 0.9439, which meets the specification. Therefore, the obtained filter is stable. In general, amplitude and group delay error are small when the filter’s pole radius is large. In this example, the maximum allowable pole radius is set to $\rho = 0.9440$, the same as in the conventional method described in [4]. The proposed method realizes small amplitude and group delay error when the maximum allowable pole radius is limited to $\rho < 1$.

**Table 2. Performance in Example 5.2**

<table>
<thead>
<tr>
<th></th>
<th>Proposed method</th>
<th>Conventional method in [4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum amplitude of the error in the passband</td>
<td>0.0147 (0.127dB)</td>
<td>0.0153 (0.132dB)</td>
</tr>
<tr>
<td>Maximum amplitude of the error in the stopband</td>
<td>0.0147 (-36.65dB)</td>
<td>0.0153 (-36.31dB)</td>
</tr>
<tr>
<td>Maximum group delay error</td>
<td>0.940</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Example 5.3.** To compare the conventional method using the SP method used with Rouche’s theorem to guarantee filter stability described in [5], we design a 1-D IIR filter using the proposed method with the same specifications as in Example 5.1, along with a maximum allowable pole radius of $\rho = 0.850$.

Figure 8 shows amplitude response, group delay, and amplitude error in the obtained filter, Figure 9 shows pole location, and Table 3 shows the filter performance. Amplitude response of the obtained filter shown in Figure 8 has equi-ripple characteristics. Next, we compare the proposed design method with the conventional method described in [5] using
Figure 6. The amplitude response, group delay, and amplitude error of the obtained filter in Example 5.2

Figure 7. Pole location in Example 5.2

the maximum amplitude of the error for the stopband (Table 3). As shown, the proposed method produces smaller amplitudes of error in the stopband than the conventional method. Comparing the group delay error at the passband, it is clear that the proposed method produces smaller group delay error than that produced by the conventional method. The obtained maximum pole radius of the filter in Figure 9 is 0.849, which meets the specification. Therefore, the obtained filter is stable. Table 3 also shows design time. In this example, the IIR filter is designed using a workstation that runs 64-bit Linux (CentOS 5.2) and has a 2.4 GHz Intel Core2Quad Q6600 quad-core processor and 16GB of main memory. As shown, the proposed method is faster than the conventional method to design 1-D IIR filters. Thus, the proposed method realizes both smaller amplitude of error and less design time (computational complexity) of IIR filters by using a stability criterion based on the system matrix, as compared with the conventional method described in [5].
Figure 8. The amplitude response, group delay, and amplitude error of the obtained filter in Example 5.3

Figure 9. Pole location in Example 5.3

Table 3. Performance in Example 5.3

<table>
<thead>
<tr>
<th></th>
<th>Proposed method</th>
<th>Conventional method in [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum amplitude of the error in the passband</td>
<td>0.0171 (0.1473dB)</td>
<td>0.0171 (0.1473dB)</td>
</tr>
<tr>
<td>Maximum amplitude of the error in the stopband</td>
<td>0.0681 (−23.3371dB)</td>
<td>0.0750 (−22.4988dB)</td>
</tr>
<tr>
<td>Maximum group delay error</td>
<td>0.7166</td>
<td>0.7513</td>
</tr>
<tr>
<td>Design time</td>
<td>19sec</td>
<td>35sec</td>
</tr>
</tbody>
</table>
Example 5.4. The filter specifications to compare the conventional method using linear programming and the positive realness condition to guarantee filter stability described in [8] are as follows.

Filter orders:
\( M = 4, \ N = 4 \)

Desired amplitude characteristics:
\[
F(\omega) = \begin{cases} 
  e^{-j\omega} & \text{if } 0 \leq \omega \leq 0.2\pi \\
  0 & \text{if } 0.4\pi \leq \omega \leq \pi
\end{cases}
\]

Prescribed maximum pole radius:
\( \rho = 0.8940 \)

Figure 10 shows amplitude response, group delay, and amplitude error in the obtained filter, Figure 11 shows pole location, and Table 4 shows the filter performance. Amplitude response of the obtained filter shown in Figure 10 has equi-ripple characteristics. Next, we compare the proposed design method with the conventional method described in [8] using the maximum amplitudes of the error (Table 4). As shown, the proposed method produces almost the same amplitudes of error as produced by the conventional method. Comparing group delay error at the passband, it is clear that the proposed method produces smaller group delay error than the conventional method. In [8], T. Yamazaki et al. indicate a trade-off between amplitude and group delay error. Consequently, their conventional method is able to obtain slightly smaller amplitudes of error, and the proposed method is able to obtain slightly smaller group delay error. The obtained maximum pole radius of filter in Figure 11 is 0.8938, which meets the specifications. Therefore, the obtained filter is stable.

Figure 10. The amplitude response, group delay, and amplitude error of the obtained filter in Example 5.4

6. Conclusion. In this paper, we have proposed a method for designing 1-D IIR filters which applies a stability criterion based on the system matrix to the SP method. The stability criterion is a necessary and sufficient condition for guaranteeing stability. Therefore, the stability domain does not depend on the denominator polynomial coefficients and is
wider than that obtained by the conventional stability criteria. Moreover, we have also proposed a method of correcting coefficients if the obtained filter becomes unstable during updating. Finally, design examples have been provided to illustrate the effectiveness of the proposed design method.

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