OBSERVER DESIGN BASED IN THE MATHEMATICAL MODEL OF A WIND TURBINE

JOSE DE JESUS RUBIO1, MARICELA FIGUEROA1, JAIME PACHECO1
AND MANUEL JIMENEZ-LIZARRAGA2

1Instituto Politécnico Nacional – ESIME Azcapotzalco
Seccion de Estudios de Posgrado e Investigacion
Av. de las Granjas No. 682, Col. Santa Catarina, Mexico D.F. 02250, Mexico
jrubioa@ipn.mx

2School of Physical and Mathematical Sciences
Autonomous University of Nuevo Leon
San Nicolas de los Garza Nuevo Leon, Mexico
manalejimenez@yahoo.com

Received July 2010; revised November 2010

Abstract. As a result of increasing environmental concern, the impact of conventional electricity generation on the environment is being minimized and efforts are being made to generate electricity from renewable sources. One way of generating electricity from renewable sources is to use wind turbines that convert the energy contained in flowing air into electricity. The major contributions of this paper are as follows: 1) The mathematical model of a prototype of a wind turbine is presented. This mathematical model is obtained by using the Euler Lagrange method and the circuits theory. The data of a prototype are used for the simulations of the obtained model. The prototype is a windward wind turbine of three blades. 2) An observer to see the angular position of a blade and the angular velocity of a blade using the armature current in the wind turbine is proposed, and this observer is important because it is easy to have the measure of the third, but it is difficult to have the measure of the first and the second. 3) It is proven that the state error of the observer applied to the nonlinear model is exponentially stable. The stability of the proposed observer is based on the solution of the Lyapunov method.

Keywords: Mathematical model, Wind turbine, Observer, Stability

1. Introduction. As a result of increasing environmental concern, the impact of conventional electricity generation on the environment is being minimized and efforts are being made to generate electricity from renewable sources. One way of generating electricity from renewable sources is to use wind turbines that convert the energy contained in flowing air into electricity [26].

Up to this moment, the amount of wind power generation integrated into large-scale electrical power systems only covers a small part of the total power system load. However, a tendency to increase the amount of electricity generated from wind can be observed. Therefore, the penetration of wind turbines in electrical power systems will increase and they may begin to influence overall power system behavior [26].

Researchers are often trying to improve the total power. The dynamic model of a wind turbine plays an important role in the control of this system, and the control plays an important role to improve the total power of the wind turbine. However, these control systems only perform to their potential if they have access to the accurate information about the wind turbine behavior in real time.
Some authors have proposed the equations to model the dynamic behavior of the wind turbine. The paper of [1] deals with the control of variable-speed wind energy conversion systems in the context of linear parameter varying systems, a recent formulation of the classic gain scheduling technique. In [20], the modeling of wind turbines for power system studies is investigated. [6] presents a wind-turbine model for energy capture and switching simulation at lull wind conditions. In [7], a dynamic model has been derived, which can be used to simulate the doubly fed induction generator wind turbine using a single-cage and double-cage representation of the generator rotor. The research of [13] investigates how methods of learning influenced the emerging wind power industries in the Netherlands and Denmark. In [17], a simple control method is proposed that will allow an induction machine to run a turbine at its maximum power coefficient. The paper of [18] focuses on a control application of optimization in wind power systems. An optimal control structure for variable speed fixed pitch wind turbines is presented. The paper of [19] proposes a pitch-control technique for grid-connected wind turbines in a small power system with low system inertia. The work of [25] examines full-scale turbine blade aerodynamic measurements and current modeling methodologies to better understand the physical and numerical attributes that determine model performance. In [26], the focus is on fundamental frequency simulations, also known as electromechanical transient simulations for a wind turbine. In [28], dynamic models for the decentralized simulation of the wind speed, of the dynamic behavior of individual wind power plants and of entire wind farms were developed and parameterized. The aforementioned research is interesting, but most of the mathematical models are complex because they have many equations, and the equations are similar in almost all the research. The mathematical model presented in this paper is simple compared with the others because it only considers the dynamics of the wind turbine and not the dynamics related with the air and the power. In addition, the model presented in this paper is completely different to the models used by the aforementioned research, and it is based in the Euler Lagrange method and in the Kirchhoff voltage law.

From the aforementioned research, [1, 6, 7, 17, 18, 19] propose a control to improve the behavior of the wind turbine, but to implement most of the controls for any system, it is necessary to know all the dynamic parameters, and in many cases, one cannot have the measure of all the dynamic parameters; that is why it is important to use an observer to have an approximation of the unknown dynamic parameters. A control is very important to reach that any system obeys to what one wants.

Some authors have proposed some observers as are [4, 12, 23, 29].

Some authors proposed observers based on the solution of an algebraic Riccati equation, but the solution of this equation is the solution of nonlinear equations. The solution of an algebraic Riccati equation remains as an active area of research [4]. In [12], a linear matrix inequality based observer design approach is presented to guarantee the satisfaction of a variety of performance criteria ranging from simple estimation error boundedness to dissipativity. In [23], they propose a high gain observer for robotic arms. In [29], they propose observers that are used in linear systems.

The aforementioned research is interesting, but none of them proposes an observer which is applied to a wind turbine.

In this paper, the mathematical model of a prototype of a wind turbine is presented. This mathematical model is obtained by using the Euler Lagrange method and the circuits theory. The data of a prototype are used for the simulations of the obtained model where the proposed prototype is a windward wind turbine of three blades. An observer is proposed to see the angular position of a blade and the angular velocity of a blade using the armature current in the wind turbine, and this observer is important because it is easy to have the measure of the third, but it is difficult to have the measure of
the first and the second. It is proven that the state error of the observer applied to the proposed mathematical model is exponentially stable. The proposed observer is based in the solution of a Lyapunov equation. The solution of this equation is the solution of linear equations. A simulation shows the effectiveness of the suggested observer.

2. The Wind Turbine System. A wind turbine is an electric generator moved by the action of the wind. In the first case, the blades represent the change from the wind energy to the mechanical energy, in the second case, the turn of a rotor inside of a generator represents the change from the mechanical energy to the electrical energy. This fact can be seen in Figure 1 [17].

3. Windward Wind Turbines. The wind turbine of windward have the rotor or helix facing to the wind, it means, in front of the tower. The main advantage of this type of machine is that it avoids the influence of the aerodynamic shadow of the tower. This type of sensible rotors requires a system for the direction of the rotor to maintain it facing the wind direction. Such systems can be active or passive. A system of active direction needs to use motorized sensors of direction and this system guides the rotor automatically facing the wind direction. A passive system of direction needs that the blades use a stabilizing fin.

4. Mathematical Model of a Wind Turbine. The mathematical model is to find the mathematical equations that represent the change from the wind energy to the mechanical energy, later the mathematical equations that represent the change from the mechanical energy to the electrical energy as is given in Figure 1. A windward wind turbine of three blades is considered. First, the Euler Lagragian method [16, 27] is used to obtain the model that represents the change from the mechanical energy to the electrical energy. Let us consider Figure 2.
From Figure 2, it can be seen that:

\begin{align*}
  x_b &= -l_b \sin(\theta_b) \\
  y_b &= l_b \cos(\theta_b)
\end{align*}

where \( \theta_b \) is the angular position of a blade in rad, \( l_b \) is the length of the blade in m. Then, the kinetic energy \( K_b \) and the potential energy \( V_b \) are given as:

\begin{align*}
  K_b &= \frac{1}{2} m_b v_b^2 = \frac{1}{2} m_b l_b^2 \dot{\theta}_b^2 \\
  V_b &= L_b + m_b g y_b = L_b + m_b g l_b \cos(\theta_b)
\end{align*}

where \( m_b \) is the mass given in Kg, \( g \) is the gravity acceleration in m/sec\(^2\), \( L_b \) is the constant length of the tower in m. The total torque \( \tau_T \) is given as:

\begin{align*}
  \tau_T &= \tau_{b1} - k_b \dot{\theta}_b - b_b \ddot{\theta}_b
\end{align*}

where \( \tau_{b1} \) is the torque applied to move the blade in Kgm\(^2\)rad/sec\(^2\), \( k_b \) is a spring effect presented when the blade is near to stop the motion in Kgm\(^2\)/sec\(^2\), \( b_b \) is the shock absorber in Kgm\(^2\)rad/sec. Then, using the Euler Lagrange method [16, 27] gives the following equation:

\begin{align*}
  m_b l_b^2 \ddot{\theta}_b + b_b \dot{\theta}_b + k_b \theta_b - m_b g l_b \sin(\theta_b) &= \tau_{b1}
\end{align*}

For the blades 2 and 3, it is considered that the angular position of the blade \( \theta_b \) is related with the angular position of the blades \( \theta_{b2} \) and \( \theta_{b3} \) as follows:

\begin{align*}
  \theta_{b2} &= \theta_b + 120^\circ \\
  \theta_{b3} &= \theta_b + 240^\circ
\end{align*}

where \( \theta_{b2} \) and \( \theta_{b3} \) are the angular position for the blades 2 and 3, respectively. So, for the blades 2 and 3, using Equation (4) provides us the equations for blades 2 and 3 in function of \( \theta_b \) as follows:

\begin{align*}
  m_b l_b^2 \ddot{\theta}_b + b_b \dot{\theta}_b + k_b (\theta_b + 120^\circ) - m_b g l_b \sin(\theta_b + 120^\circ) &= \tau_{b2} \\
  m_b l_b^2 \ddot{\theta}_b + b_b \dot{\theta}_b + k_b (\theta_b + 240^\circ) - m_b g l_b \sin(\theta_b + 240^\circ) &= \tau_{b3}
\end{align*}

where \( \tau_{b2} \) and \( \tau_{b3} \) are the torques applied to move the blades 2 and 3, respectively.
Let us use the following identity:
\[ \sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b) \]  
(7)

Applying (7) to \( \sin(\theta_b + 120^\circ) \) gives:
\[ \sin(\theta_b + 120^\circ) = \sin(\theta_b) \cos(120^\circ) + \cos(\theta_b) \sin(120^\circ) \]
\[ = -0.5 \sin(\theta_b) + 0.866 \cos(\theta_b) \]
(8)

Applying (7) to \( \sin(\theta_b + 240^\circ) \) gives:
\[ \sin(\theta_b + 240^\circ) = \sin(\theta_b) \cos(240^\circ) + \cos(\theta_b) \sin(240^\circ) \]
\[ = -0.5 \sin(\theta_b) - 0.866 \cos(\theta_b) \]
(9)

Adding \( - \sin(\theta_b) - \sin(\theta_b + 120^\circ) - \sin(\theta_b + 240^\circ) \) gives:
\[ - \sin(\theta_b) - \sin(\theta_b + 120^\circ) - \sin(\theta_b + 240^\circ) \]
\[ = - \sin(\theta_b) + 0.5 \sin(\theta_b) - 0.866 \cos(\theta_b) + 0.5 \sin(\theta_b) + 0.866 \cos(\theta_b) \]
\[ = 0 \]
(10)

\( 120^\circ \) and \( 240^\circ \) are rewritten in rad as follows:
\[ 120^\circ = \frac{2}{3} \pi \text{ rad} \]
\[ 240^\circ = \frac{4}{3} \pi \text{ rad} \]
(11)

Now, adding the three equations of (4) and (6) and using (10) and (11) gives:
\[ 3m_b l_1^2 \ddot{\theta}_b + 3b_l \dot{\theta}_b + 3k_l \theta_b + 2\pi k_b = \tau_b \]
(12)

where \( \tau_b = \tau_{b1} + \tau_{b2} + \tau_{b3} \).

Considering Figure 2, \( \tau_{b1}, \tau_{b2} \) and \( \tau_{b3} \) are given as follows:
\[ \tau_{b1} = \tau_{b0} e^{-t^{1/16}} \forall t \in \left[2\pi n, \left(2n + \frac{2}{3}\right) \pi\right] \quad n = 0, 1, 2, \ldots \]
\[ \tau_{b2} = \tau_{b0} e^{-t^{1/16}} \forall t \in \left[(2n + \frac{2}{3}) \pi, (2n + \frac{4}{3}) \pi\right] \quad n = 0, 1, 2, \ldots \]
\[ \tau_{b3} = \tau_{b0} e^{-t^{1/16}} \forall t \in \left[(2n + \frac{4}{3}) \pi, (2n + 2) \pi\right] \quad n = 0, 1, 2, \ldots \]
(13)

where \( \tau_{b0} \) is the maximum constant torque applied to move each blade in Kgm² rad/sec².

**Remark 4.1.** Equation (12) is the main equation of the mathematical model that represents the change from the wind energy to the mechanical energy.

**Remark 4.2.** From (13), when the first blade is moved by the first torque, the second blade and the third blade are moved by the first torque, but they are not moved by the second torque and the third torque. Later, the things change because the third blade takes the place of the first blade, the first blade takes the place of the second blade and the second blade takes the place of the third blade.

Now, let us analyze the change from the mechanical to the electrical energy. It can be seen in Figure 3.

In [8, 27], they presented the model of a motor, similarly, from Figure 3, a model for the generator is obtained using the Kirchhoff voltage law represented as the addition of voltages as follows:
\[ k_g \dot{\theta}_b = R_g i_g + L_g i_g + V_g \]
(14)

where \( k_g \) is the back emf constant in volts/sec/rad, \( R_g \) is the armature resistance in the generator in \( \Omega \), \( L_g \) is the armature inductance in the generator in \( H \), \( V_g \) is the armature voltage in the generator in volts and \( i_g \) is the armature current in the generator in amperes.

For the generator of this paper \( V_g = i_g \), thus (14) becomes to:
\[ k_g \dot{\theta}_b = (R_g + 1) i_g + L_g i_g \]
(15)
Remark 4.3. The condition $V_g = i_g$, gives that the resistance of the armature voltage in the generator is one, it is assumed because $k_g$ is unknown for the used generator. If $k_g$ was known, the value of the resistance of the armature voltage in the generator change, but Equation (14) is preserved in both cases.

Thus, Equations (12) and (13) are the mathematical model that represents the change from the wind energy to the mechanical energy, and (15) is the mathematical model that represents the change from the mechanical energy to the electrical energy and these equations represent the mathematical model of the wind turbine.

Let us define the new state variables as $x_1 = i_g$, $x_2 = \theta_b$, $x_3 = \dot{\theta}_b$, the input is $u = \tau_b$ and the output is $y = i_g$. Then, the model of Equations (12) and (15) becomes to:

\[
\begin{align*}
\dot{x}_1 &= \frac{(R_g + 1)}{L_g} x_1 + \frac{k_g}{L_g} x_3 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= -\frac{k_b}{m_b l_b^2} x_2 - \frac{b}{m_b l_b^2} x_3 - \frac{2\pi k_b}{3m_b l_b^2} + \frac{1}{3m_b l_b^2} u \\
y &= x_1
\end{align*}
\]

where only the armature current in the generator $y = i_g$ is seen, it is important to know the angular position of a blade $x_2 = \theta_b$ and the angular velocity of a blade $x_3 = \dot{\theta}_b$ using the armature current $y = i_g$ in the wind turbine.

Remark 4.4. Some authors have proposed the equations to model the dynamic behavior of the wind turbine as are [1, 6, 7, 13, 17, 18, 19, 20, 25, 26, 28], but most of the mathematical models are complex because they have many equations, and the equations are similar in almost all the research. The mathematical model presented in this paper is simple compared with the others because it only considers the dynamics of the wind turbine and not the dynamics related with the air and the power. In addition, the model presented in this
paper is completely different to the models used by the aforementioned research, it is based on the Euler Lagrange method and in the Kirchhoff voltage law.

Remark 4.5. To make the result of the proposed mathematical model less conservative, maybe some parameters should be considered as is the mathematical model of the sensors and the actuators to be included in the proposed mathematical model.

5. The Observer Design. In this section, an observer will be designed based on the assumption that only the armature current in the generator \( y = x_1 \) considered as the output of the system is available. Let \( \hat{x}_1, \hat{x}_2 \) and \( \hat{x}_3 \) be the estimation of the states \( x_1, x_2 \) and \( x_3 \). Let us define the output error as:

\[
e = y - \hat{y} = C\hat{x}
\]

where \( y = x_1, \hat{y} = \hat{x}_1, C = [1 0 0] \), \( \hat{x} = x - \hat{x} \) is the state error, \( x = [x_1 x_2 x_3]^T \), \( \hat{x} = [\hat{x}_1 \hat{x}_2 \hat{x}_3]^T \). The following observer is proposed:

\[
\begin{align*}
\dot{\hat{x}}_1 &= -\frac{(R_g + 1)}{L_g} \hat{x}_1 + \frac{k_g}{L_g} \hat{x}_3 + k_1 e \\
\dot{\hat{x}}_2 &= \hat{x}_3 + k_2 e \\
\dot{\hat{x}}_3 &= -\frac{k_b}{m_b l_b^2} \hat{x}_2 - \frac{b_b}{m_b l_b^2} \hat{x}_3 - \frac{2\pi k_b}{3m_b l_b^2} + \frac{1}{3m_b l_b^2} u + k_3 e
\end{align*}
\]

where the parameters \( k_1, k_2 \) and \( k_3 \) are scalar constants chosen by the designer.

The objective of the observer is that the states of the observer \( \hat{x}_1, \hat{x}_2 \) and \( \hat{x}_3 \) of (18) may follow the states of the plant \( x_1 = i_g, x_2 = \theta_b, x_3 = \dot{\theta}_b \) (16) using the armature current in the generator \( y = x_1 \) in the output error (17).

From (16)-(18), the dynamic equation of the estimation error dynamics can be written as:

\[
\begin{align*}
\dot{\hat{x}} &= A_0 \hat{x}
\end{align*}
\]

where \( A_0 = A - KC \), \( A = \begin{bmatrix} 0 & 0 & k_g \\ L_g & 0 & \frac{k_g}{L_g} \\ 0 & -\frac{k_b}{m_b l_b^2} & \frac{b_b}{m_b l_b^2} \end{bmatrix} \), \( K = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} \), \( C \) is given in Equation (17). Note that the matrix \( A \) is the matrix of the system (16). The pair \((A, C)\) of the system (16) is observable, it is proven in the following lemma.

Lemma 5.1. \((A, C)\) of the system (16) is an observable pair.

Proof: Considering \( A \) of the system (19) and \( C \) of (17) and the parameters of Table 1 (please see the next section) gives:

\[
\begin{align*}
\begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} 1 & 0 & 0 \\ -1492.3 & 0 & 0.74614 \\ 2.2269 \times 10^6 & -1.4923 \times 10^{-2} & -1128.4 \end{bmatrix} = 3
\end{align*}
\]

As \( rankO = 3 \) and \( A \in \mathbb{R}^{3 \times 3} \), thus the pair \((A, C)\) of the system (16) is observable [3, 14].

Now, in this paper, it will be proven that the state error of the observer is exponentially stable.
Theorem 5.1. If for the the estimation error dynamics (19) there exist positive and symmetric matrices $P$ and $Q$ such that the following Lyapunov equation holds:

$$PA_0 + A_0^TP = -Q$$

(21)

where $A_0$ is given in (19), then the state error $\tilde{x}$ of the observer (18) is exponentially stable and the following is satisfied:

$$\|\tilde{x}\|^2 \leq \alpha \|\tilde{x}(0)\|^2 e^{-\gamma t}$$

(22)

where $\tilde{x}(0)$ is the initial state error, $\alpha = \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}$ and $\gamma = \lambda_{\min}(Q)\lambda_{\min}(P^{-1})$.

Proof: Let us define the following Lyapunov function:

$$V = \tilde{x}^TP\tilde{x}$$

(23)

Using (19), the derivative of $V$ is:

$$\dot{V} = \tilde{x}^TP\dot{\tilde{x}} + \dot{\tilde{x}}^TP\tilde{x} \leq \tilde{x}^T[P A_0 + A_0^TP]\tilde{x}$$

(24)

Considering the Lyapunov Equation (21) [14, 16, 23, 27], (24) becomes to:

$$\dot{V} \leq -\tilde{x}^TQ\tilde{x} = -\tilde{x}^TQ\tilde{x} = -\gamma V$$

(25)

where $\gamma$ is given in (22) and $V$ is given in (23). The solution of (25) is:

$$V = V(0)e^{-\gamma t}$$

(26)

where $V(0)$ is the initial condition of $V$. Using the definition of $V$ of (23) in (26) gives:

$$\lambda_{\min}(P)\|\tilde{x}\|^2 \leq \tilde{x}^TQ\tilde{x} = V = V(0)e^{-\gamma t} = \tilde{x}(0)^TP\tilde{x}(0)e^{-\gamma t} \leq \lambda_{\max}(P)\|\tilde{x}(0)\|^2 e^{-\gamma t}$$

(27)

Equation (22) is obtained using Equation (27). Thus, state error $\tilde{x}$ is exponentially stable and it will converge to zero [11, 14, 16, 27].

Remark 5.1. The authors [1, 6, 7, 17, 18, 19] propose a control to improve the behavior of the wind turbine, but to implement most of the controls for any system, it is necessary to know all the dynamic parameters, and in many cases one cannot have the measure of all the dynamic parameters, that is why it is important to use an observer to have an approximation of the unknown dynamic parameters.

Remark 5.2. Some authors have proposed some observers as are [4, 12, 23, 29]. The aforementioned research is interesting, none of them proposes an observer which is applied for a wind turbine.

Remark 5.3. To make the proposed observer less conservative, maybe some dynamics of the mathematical model of the wind turbine could be considered unknown in the observer, but the observability and the stability results would be lost.

6. Simulation of the Proposed Mathematical Model. A prototype of a wind turbine is considered for the simulation of the proposed mathematical model, the prototype of the wind turbine is seen in Figure 4.

This prototype has three blades and it does not use a gear box. The mathematical model of the wind turbine is given by Equations (13), (15) and (16). The parameters of this prototype are given in Table 1.

Equation (16) is the proposed mathematical model, this mathematical model uses the parameters of Table 1. 0, 0 and 0 are considered as the initial conditions for the states of the plant $x_1 = i_g$, $x_2 = \theta_b$, $x_3 = \theta_b$.

Figure 5 shows the states when torques (inputs) are not considered.
From Figure 5, it can be seen that the mathematical model has good behavior when torques are not considered, because in this case, the parameters $x_1 = i_g$, $x_2 = \theta_b$ and $x_3 = \dot{\theta}_b$ describe that the wind turbine is not in motion.

Figure 6 shows the states when an exponential torque (input) $\tau_{b1}$ is considered where $\tau_{b0} = 3 \times 10^{-1}$.

From Figure 6, it can be seen that the mathematical model has good behavior because as the torque $\tau_{b1}$ decreases, the current $x_1 = i_g$ decreases, the position of the blade $x_2 = \theta_b$ reaches a constant and the angular velocity of the blade $x_3 = \dot{\theta}_b$ decreases.

Figure 7 shows the states when the torque inputs of (13) are considered where $\tau_{b0} = 3 \times 10^{-1}$.

From Figure 7, it can be seen that the mathematical model has good behavior because from (12) $\tau_b = \tau_{b1} + \tau_{b2} + \tau_{b3}$, $\tau_b$ is almost constant, then the current $x_1 = i_g$ is almost constant, the position of the blade $x_2 = \theta_b$ grows and the angular velocity of the blade $x_3 = \dot{\theta}_b$ is almost constant.

**Remark 6.1.** The simulations presented in this paper are completely different to the presented in [1, 6, 7, 13, 17, 18, 19, 20, 25, 26, 28], because they present the output power
or the wind speed or the voltage while in this paper the torque, the position of the blade, the angular velocity of the blade, and the current are presented.

7. Simulation of the Proposed Observer. In this section, the behavior of the proposed observer is shown in simulations. The objective of the observer is that the states of the observer $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$ of (18) may follow the states of the plant $x_1 = i_g$, $x_2 = \theta_b$, 

Figure 5. Behavior of the proposed mathematical model when torques are not considered

Figure 6. Behavior of the proposed mathematical model when a torque is considered
In this paper, the root mean square error (RMSE) for the states $x_2$ and $x_3$ [23, 24] is used, it is given as:

$$\text{RMSE} = \left( \frac{1}{T} \int_0^T e_1^2 \, dt \right)^{\frac{1}{2}}$$  \hspace{1cm} (28)$$

Please note that $x_2$ and $x_3$ are the most important states because they are unknown.

In simulations, the mathematical model of the wind turbine is given by Equation (16), 10, $-0.75$ and 10 are considered as the initial conditions for the states of the plant $x_1 = i_g$, $x_2 = \theta_b$ and $x_3 = \theta_b$. 0, 0 and 0 are used as the initial states for the observer $\hat{x}_1$, $\hat{x}_2$ and $\hat{x}_3$. Most of the parameters of the plant are given in Table 1, the parameter $u = \tau_b$ is given by Equation (13).

The observer is given by Equation (18). The parameters of the observer are $k_1 = -1$, $k_2 = -1 \times 10^{-5}$ and $k_3 = -1 \times 10^{-5}$. The Lyapunov Equation (21) is satisfied with

$$P = \begin{bmatrix} 3.7723 \times 10^{-4} & 1.9482 \times 10^{-4} & 2.6097 \times 10^{-6} \\ 1.9482 \times 10^{-4} & 2.5798 & 5.0958 \times 10^{-3} \\ 2.6097 \times 10^{-6} & 5.0958 \times 10^{-3} & 2.5255 \times 10^{-2} \end{bmatrix} \text{ and } Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  

Considering Theorem 5.1, the state error of the observer is exponentially stable. In addition, $\gamma = \lambda_{\min}(Q)\lambda_{\min}(P^{-1}) = (1)(1.995 \times 10^{-3}) = 1.995 \times 10^{-3}$ and $\alpha = \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} = \frac{2982.6}{1.995 \times 10^{-3}} = 1.495 \times 10^6$ are the parameters of the exponentially stability.

Figure 8 shows the behavior of the observer for 1 second. It can be seen that all the estimated states converge to the real states in 0.2 seconds.

Figure 9 shows the state errors for the proposed observer for 1 second. It can be seen that the state errors converge to zero in 0.2 seconds.

Figure 10 shows the behavior of the observer for 15 seconds. It can be seen that all the estimated states converge to the real states and this result is preserved.
Figure 8. Behavior of the proposed observer for 1 second

Figure 9. State errors for the proposed observer for 1 second

Figure 11 shows the state errors for the proposed observer for 15 seconds. It can be seen that the state errors converge to zero and this result is preserved.

Using (28), the RMSE for the states $x_2$ and $x_3$ for the proposed observer are given in Table 2.

From Table 2, it can be seen that the state errors will converge to zero because the RMSE are smaller for 15 sec than for 1 sec.

Thus, the proposed observer has a good behavior.
Figure 10. Behavior of the proposed observer for 15 seconds

Figure 11. State errors for the proposed observer for 15 seconds

Table 2. The RMSE for the states $x_2$ and $x_3$ for the proposed observer

<table>
<thead>
<tr>
<th></th>
<th>RMSE for $x_2$</th>
<th>RMSE for $x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 sec</td>
<td>0.0914</td>
<td>1.5694</td>
</tr>
<tr>
<td>15 sec</td>
<td>0.0245</td>
<td>0.4052</td>
</tr>
</tbody>
</table>
Remark 7.1. The authors [1, 6, 7, 17, 18, 19] show the simulations of a control applied to improve the behavior of the wind turbine, but none shows the simulations of an observer.

Remark 7.2. The authors [4, 12, 23, 29] show the simulations of an observer, but none shows the simulations of an observer applied to a wind turbine.

8. Conclusion. In this paper, the mathematical model of a prototype of a wind turbine was presented. This model was obtained using the Euler Lagrange method and the circuits theory. The data of a prototype were used for the simulations of the obtained model where the proposed prototype is a windward wind turbine of three blades. A nonlinear observer was proposed to see the angular position of a blade and the angular velocity of a blade using the armature current in the wind turbine. As a future research, a system with the control and the observer will be proposed to improve the behavior of the wind turbine, and the proposed algorithms will be mixed with the evolving systems theory [2, 9, 10, 15, 21, 22].

Acknowledgements. The authors are grateful to the editors and the reviewers for their valuable comments and insightful suggestions, which helped to improve this research significantly. The authors thank the Secretaria de Investigación y Posgrado and the Comisión de Operación y Fomento de Actividades Académicas del IPN and the Consejo Nacional de Ciencia y Tecnología for their help in this research.

REFERENCES


