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ROBUST H_∞ CONTROL FOR NETWORKED SYSTEMS WITH PARAMETER UNCERTAINTIES AND MULTIPLE STOCHASTIC SENSORS AND ACTUATORS FAULTS

Songlin Hu[1], Dong Yue[1], Jinliang Liu[2] and Zhaoping Du[3]

[1]Department of Control Science and Engineering
Key Laboratory of Ministry of Education for Image Processing and Intelligent Control
Huazhong University of Science and Technology
No. 1037, Luoyu Road, Hongshan Dist., Wuhan 430074, P. R. China
songlin621@126.com; medongy@vip.163.com

[2]Department of Applied Mathematics
Nanjing University of Finance and Economics
No. 3, Wenyuan Road, Nanjing 210046, P. R. China
ljl9@163.com

[3]College of Electronics and Information
Jiangsu University of Science and Technology
No. 2, Mengxi Road, Zhenjiang 212003, P. R. China
duzhaoping99@163.com

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Abstract. *In this paper, the reliable robust H_∞ control problem is investigated for a class of uncertain discrete-time networked systems with network-induced delays and simultaneous consideration of multiple sensors and actuators failures. The failure rate for each individual sensor/actuator is described by an individual random variable satisfying a certain probabilistic distribution in the interval $[0, \theta]$ ($\theta \geq 1$). Attention is focused on the analysis and design of a reliable controller such that the closed loop control system is stable in the mean-square sense and preserves a guaranteed H_∞ performance index in the presence of network-induced delays, stochastic faults of multiple sensors and actuators. A sufficient condition is obtained for the existence of admissible controller by using Lyapunov functional method, and the cone complementarity linearization algorithm is employed to cast the controller design problem into a sequential minimization one subject to linear matrix inequalities. Moreover, the control design method is further extended to more general cases, where the system matrices of the considered plant contain parameter uncertainties, represented in either norm-bounded or polytopic frameworks. Finally, a simulation example is provided to illustrate the effectiveness of the proposed method.*
Keywords: Networked systems, Stochastic sensor faults, Stochastic actuator faults, Reliable control, Mean square stability, Linear matrix inequality (LMI)

1. **Introduction.** Networked control system (NCS) has a relatively new system structure in which the control loop is closed through a shared communication network [1]. Nowadays, networked control systems (NCSs) have gained a great deal of research attention in both theoretical analysis and practical engineering applications for their great advantages over traditional systems such as low cost, reduced weight and power requirements and simple installation and maintenance. However, the insertion of communication networks in the feedback control loop makes the analysis and design of NCSs difficult. This is because the communication networks, sometimes, are unreliable and may be subject to undesirable packet losses and network-induced delays, which may significantly degrade the system performance or even destabilize the NCSs. In the past few decades,

the stability analysis and control synthesis of linear or nonlinear NCSs in the presence of network-induced delays and/or data packet dropouts have attracted considerable research interest and lots of important results have been reported in the literature, to name a few, [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13].

It is worth mentioning that, almost in all the literature mentioned above, it has been implicitly assumed that all sensors, control components and actuators are in good working condition. However, due to the sensors or actuators aging, electromagnetic interference, network congestion, external disturbance et al., sensor and/or actuator failures and data distortion are usually unavoidable, which may degrade the system performance [14]. Therefore, it is important and meaningful to design a reliable controller against the sensor and/or actuator failures and data distortion. Note that, in the past few years, in the case of only considering the actuator failures, the reliable control problems have been extensively investigated for a variety of complex dynamical systems. For example, the reliable design problem for linear systems with actuator failures was investigated in [15]. In [16], reliable control problem was studied for fuzzy dynamic systems with time-varying delay. In [17], reliable robust H_∞ fuzzy control problem for uncertain nonlinear systems with Markovian jumping actuator faults has been investigated. [18] studied the reliable H_∞ controller design problem for a class of uncertain linear systems with actuator failures. Reliable guaranteed cost control problem with the actuator faults has been coped with in [19]. Based on the switching method, hybrid output feedback controller design problem has been addressed in [20] for an uncertain delay system with actuator failures. Moreover, sensors break down as frequently as actuators, and sensor failures often bring serious and even disastrous situations. In view of this, in most relevant literature, state estimation problems have been put forward and then extensively studied for various systems with sensor faults in [21, 22, 23, 24, 25, 26]. However, there is very few publications on reliable H_∞ control problem for various dynamical systems with sensor failures except for [27], where reliable H_∞ control against sensor failures was addressed for linear time-invariant multiparameter singularly perturbed systems.

On the other hand, we notice that, in most existing literature concerning sensor failures, a common assumption is that the measurement signal of the sensor is either completely missing or is completely accessible, and all sensors have identical failure characteristics [28]. Such an assumption, however, does have its limitation since it cannot cover some practical cases such as that the sensor fails partly or the failure rate for each individual sensor is different, see, e.g., [29]. Moreover, in comparison with the fruitful literature available for robust reliable control problem for various systems with actuator failures, there have been very few results published on reliable control problem for networked systems with multiple sensors failures, not to mention the simultaneous consideration of the multiple actuators failures and the H_∞ performance analysis. Taken a step further, although having considered multiple sensors failures in [21, 22, 23, 25, 26, 30], only robust filtering problems have been studied for both continuous-time and discrete-time networked systems. To the best of authors' knowledge, the reliable robust H_∞ control problem for uncertain linear discrete-time NCSs with multiple probabilistic sensors and actuators faults and network-induced delays has not been fully investigated, which constitutes the main motivation of the present study.

In this paper, we aim to solve the problem of robust H_∞ control for a class of uncertain linear discrete-time NCSs with network-induced delays, stochastic faults of the multiple sensors and actuators that are modeled by a set of random variables taking values in $[0, \theta]$ $(\theta \geq 1)$ and satisfying a certain probabilistic distribution. The main contribution of this paper is twofold: (1) The problem of robust H_∞ control for linear discrete-time NCSs with parameter uncertainties and simultaneous consideration of the stochastic faults of the sensors and actuators have been addressed. Moreover, the failure rate for each individual sensor/actuator satisfies individual probabilistic distribution; and (2) A set of random variables is introduced to describe the case of the multiple probabilistic sensors and/or actuators faults as well as the measurement distortion in a unified framework. The objective of this paper is to design a reliable controller such that the closed-loop control system is stable in the mean-square sense and preserves a guaranteed H_∞ performance index. Sufficient conditions are obtained for the existence of an admissible controller. Finally, a numerical example is given to demonstrate the effectiveness of the proposed design method.

The remainder of this paper is organized as follows. Section 2 formulates the problem under consideration. H_∞ performance analysis of the closed-loop control system is given in Section 3. The problem of H_∞ controller design is solved in Section 4. Based on Section 3 and Section 4, the problem of robust H_∞ performance analysis and controller design is solved. Section 5 gives an illustrative example and we conclude the paper in Section 6.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n \times m$ matrices. $\mathbb{Z}^+$ is the set of positive integers. $I_{n\times n}$ denotes n-dimensional identity matrix. The notation $X > 0$ ($X < 0$) for any $X \in \mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (negative definite). For a real matrix B and two real symmetric matrices A and C of appropriate dimensions, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a real symmetric matrix, where $*$ denotes the entries implied by symmetry. $diag\{\cdots\}$ stands for a block-diagonal matrix. The superscript "T" stands for matrix transposition. Let $\mathcal{E}\{\cdot\}$ stand for the mathematical expectation operator. Throughout this paper, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. **Problem Statement and Preliminaries.**

In this paper, we consider the following linear discrete-time system:

$$x(k+1) = Ax(k) + Bu(k) + B_w w(k) \tag{1}$$

$$z(k) = Cx(k) + Du(k) \tag{2}$$

$$x(0) = \phi \tag{3}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $w(k)$ is an external input, $z(k)$ is the controlled output vector, $\phi \in \mathbb{R}^n$ is the initial condition, A, B, B_w, C and D are known matrices with appropriate dimensions.

Throughout this paper, we assume that system (1)-(5) is controlled through a network, which consists of the following components: a nominal plant to be controlled, multiple sensors and actuators, communication networks and a controller. Moreover, it is assumed that the sampling data in the sensor node will take $\tau_{sc}(k)$ to reach the controller, and the control signal will take $\tau_{ca}(k)$ to reach the actuator. The sum of $\tau_{sc}(k) + \tau_{ca}(k)$ is denoted by d_k satisfying $0 \le d_k \le d_M$, where $d_M \in \mathbb{Z}^+$.

Considering the effect of the network-induced delays (or network transmission delays) from the sensor to controller and controller to actuator, a linear state feedback controller can be described as

$$u(k) = Kx(k - d_k) \tag{4}$$

where $K \in \mathbb{R}^{m\times n}$ is a feedback gain to be determined later.

However, due to various reasons such as sensor aging or sensor temporal failure, sometimes, all or part of the actual measured output of the sensors could be received by the controller. In this case, (4) can be described by

$$u(k) = K\Theta_1 x(k - d_k) \tag{5}$$

where $\Theta_1 = diag\{\theta_{11}, \theta_{12}, \cdots, \theta_{1n}\}$. For the random variables θ_{1i} $(i = 1, 2, \dots, n)$, we give the following assumption.

A1: θ_{1i} $(i = 1, 2, \dots, n)$ are n unrelated random variables which are also unrelated with $w(k)$. θ_{1i} has the probabilistic density function $f_i(s_i)$ $(i = 1, 2, \dots, n)$ on the interval $[0, \theta]$ with mathematical expectation μ_i and variance σ_i^2, where θ is a given constant satisfying $\theta \geq 1$.

Similarly, considering the stochastic faults of the actuator, all or part of the output of the controller can be received by the actuators at one moment; hence, (5) can be further rewritten as

$$u(k) = \Theta_2 K \Theta_1 x(k - d_k) \tag{6}$$

where $\Theta_2 = diag\{\theta_{21}, \theta_{22}, \cdots, \theta_{2m}\}$, θ_{2j} $(j = 1, 2, \dots, m)$ are random variables. Liking A1, we make the following assumption about the random variables θ_{2j} $(j = 1, 2, \dots, m)$.

A2: θ_{2j} $(j = 1, 2, \dots, m)$ are m unrelated random variables which are also unrelated with $w(k)$. θ_{2j} has the probabilistic density function $g_j(\nu_j)$ $(j = 1, 2, \dots, m)$ on the interval $[0, \theta]$ with mathematical expectation λ_j and variance δ_j^2, where θ is a given constant satisfying $\theta \geq 1$.

Remark 2.1. *It can be seen from (6) that the stochastic matrices Θ_1 and Θ_2 are introduced to reflect the unreliable sensors and actuators, respectively. Generally speaking, different sensor or actuator has different failure rate. Hence, it is more reasonable to assume that the failure rate for each individual sensor or actuator satisfies individual probabilistic distribution. The stochastic matrix Θ_1 describes the the status of the whole sensors, and the elements θ_{1i} $(i = 1, 2, \dots, n)$ of the random matrix Θ_1 correspond to the status of the ith sensor. Therefore, from **A1**, we know that, at one moment, if $\theta_{1i} = 0$ $(i = 1, 2, \dots, n)$, it indicates that the ith sensor fails completely or data missing in the sensor-to-controller channel; if $\theta_{1i} = 1$, it means that the ith sensor is in good working condition, while if $\theta_{1i} \in (0, 1)$, it means that the ith sensor fails partly. When $\theta_{1i} \in (1, \theta)$, it means that the measurement distortion with the case of the sensor output larger than the real measurement at one moment. In addition, for a fixed i, $\sum f_i(s_i) = 1$. It can be seen that the widely used Bernoulli distribution is included as a special case. Similar discussion can be applied to the random variables ν_j $(j = 1, 2, \dots, m)$ in **A2**, and here it is omitted for simplicity.*

Remark 2.2. *Employing a Bernoulli random binary distributed white sequence to describe the status of the sensor has been extensively studied in the literature such as* [28, 31, 32, 33, 34, 35, 36, 37]*, in which a random variable δ_k taking values in $\{0, 1\}$ is used to show that the sensor fails completely for $\delta_k = 0$ and works well for $\delta_k = 1$. Very recently, an minor extension is developed in* [26]*, where a more general random variable δ_k taking values in the interval $[0, 1]$ was introduced. When $\delta_k \in (0, 1)$, it means the partial failure of sensor. Subsequently, a big extension is proposed in* [29, 30, 35]*, in which a set of new random variables is employed to describe the status of multiple sensors and actuators with different failure rates. However, in real systems, the measurement data may be transferred through multiple sensors and actuators. When some of the sensors and/or actuators go out of order, the actual measurement signal of sensors and/or actuators may be larger or smaller than what it should be at one moment, which is called "measurement distortion" as in* [14]*. The measurement distortion may degrade the system performance or even destabilize the system. Therefore, it is necessary to take the measurement distortion into consideration when studying the stability and performance of NCSs. In view of this, to describe this situation, in this paper, like in* [14]*, it is assumed that the random variables θ_{1i} $(i = 1, 2, \dots, n)$ and θ_{2j} $(j = 1, 2, \dots, m)$ can take values in the interval $[0, \theta]$ $(\theta \geq 1)$. In comparison with above-mentioned literatures, the main difference lies in: we employ $0 < \theta_{1i} < 1$, $0 < \theta_{2j} < 1$ and $\theta_{1i} > 1$, $\theta_{2j} > 1$ to represent the measurement distortion of* *the sensors and actuators. Obviously, our model can cover the case obtained in* [35, 36, 37]. *Furthermore, so far, there has been no research on the reliable robust H_∞ control problem for uncertain linear discrete-time NCSs in the presence of multiple sensors and actuators with different stochastic failure rates, and the objective of this paper is therefore to shorten such a gap.*

Remark 2.3. *From (6), it can be seen that, if the sensors and actuators are reliable, that is, $\Theta_1 = I_{n\times n}$, $\Theta_2 = I_{m\times m}$, then the controller $u(k) = \Theta_2 K \Theta_1 x(k-d_k)$ reduces to the common case, i.e., $u(k) = Kx(k-d_k)$, which has been extensively studied in* [2, 9].

Substituting (6) into (1) and (2) yield

$$x(k+1) = Ax(k) + B\Theta_2 K\Theta_1 x(k-d_k) + B_w w(k) \tag{7}$$

$$z(k) = Cx(k) + D\Theta_2 K\Theta_1 x(k-d_k) \tag{8}$$

In the following, for the sake of presentation simplicity, we denote $\bar{\Theta}_1 = \mathcal{E}\{\Theta_1\}$, $\hat{\Theta}_1 = \Theta_1 - \bar{\Theta}_1$, $\bar{\Theta}_2 = \mathcal{E}\{\Theta_2\}$, $\hat{\Theta}_2 = \Theta_2 - \bar{\Theta}_2$. Then, according to Assumption 1 and Assumption 2, we have

$$\bar{\Theta}_1 = \sum_{l=1}^{n} \mu_l \Theta_1^l, \quad \mathcal{E}\left\{\hat{\Theta}_1\right\} = diag\left\{\underbrace{0,0,\cdots,0}_{n}\right\} \tag{9}$$

$$\bar{\Theta}_2 = \sum_{s=1}^{m} \lambda_s \Theta_2^s, \quad \mathcal{E}\left\{\hat{\Theta}_2\right\} = diag\left\{\underbrace{0,0,\cdots,0}_{m}\right\} \tag{10}$$

where

$$\Theta_1^l = diag\left\{\underbrace{0,0,\cdots,0}_{l-1},1,\underbrace{0,\cdots,0}_{n-l}\right\}, \quad \Theta_2^s = diag\left\{\underbrace{0,0,\cdots,0}_{s-1},1,\underbrace{0,\cdots,0}_{m-s}\right\}$$

According to the above analysis, we have the following systems to be investigated:

$$x(k+1) = Ax(k) + B\bar{\Theta}_2 K\bar{\Theta}_1 x(k-d_k) + B\Pi x(k-d_k) + B_w w(k) \tag{11}$$

$$z(k) = Cx(k) + D\bar{\Theta}_2 K\bar{\Theta}_1 x(k-d_k) + D\Pi x(k-d_k) \tag{12}$$

$$x(k) = \phi(k), \quad k = -d_M, -d_M+1, \ldots, 0 \tag{13}$$

where $\phi(k)$ is the initial condition of the state, and $\prod = \hat{\Theta}_2 K\bar{\Theta}_1 + \bar{\Theta}_2 K\hat{\Theta}_1 + B\hat{\Theta}_2 K\hat{\Theta}_1$.

Our objective in this paper is to develop techniques to deal with the robust reliable H_∞ controller problem for discrete-time NCSs with probabilistic sensors and actuators faults, measurement distortion, network-induced delay, and parameter uncertainties. For this purpose, the following definition and Lemma are needed for the derivation of out main results.

Definition 2.1. *System (11)-(13) with a given K is said to be mean-square stable (MSS) with an H_∞ norm bound γ if the following two conditions hold: (1) System (11)-(13) with $w(k) = 0$ if there exists a scalar $c > 0$ such that $\mathcal{E}\left\{\sum_{k=0}^{\infty}\|x(k)\|^2\right\} \le c \sup_{-d_M\le s\le 0} \mathcal{E}\left\{\|\phi(s)\|^2\right\}$ and (2) Under the zero initial condition, the controlled output $z(k)$ satisfies $\mathcal{E}\{\sum_{k=0}^{\infty} \|z(k)\|^2\} \le \gamma^2\mathcal{E}\left\{\sum_{k=0}^{\infty}\|w(k)\|^2\right\}$, where $w(k) \in l_2[0,\infty) = \{w(k) \mid \mathcal{E}\{\sum_{k=0}^{\infty}\|w(k)\|^2\} < \infty\}$.*

Lemma 2.1. [14] *Ξ_1, Ξ_2 and Ω are matrices with appropriate dimensions, $d(k)$ is function of k and $0 \le d_m \le d(k) \le d_M$, then the inequality $(d(k)-d_m)\,\Xi_1 + (d_M - d(k)) + \Omega < 0$ holds if and only if $(d_M-d_m)\,\Xi_1 + \Omega < 0$ and $(d_M-d_m)\,\Xi_2 + \Omega < 0$.*

3. $\boldsymbol{H_\infty}$ Performance Analysis. In this section, we are concerned with the problem of H_∞ performance analysis. More specifically, assuming that the matrices A, B, B_w, C, D and the feedback gain K are known, we shall study the conditions under which the closed-loop system (11)-(13) achieves the H_∞ performance γ. The following theorem shows that the H_∞ performance can be guaranteed if there exist some matrices satisfying certain matrix inequalities. Note that this theorem will play an instrumental role in the controller design problem.

Theorem 3.1. *System (11)-(13) with a given K is MSS with a guaranteed H_∞ performance γ if there exist matrices $P > 0$, $R > 0$, $Z > 0$, M_j, N_j $(j = 1, 2, 3)$ with appropriate dimensions such that*

$$\begin{bmatrix} \Omega & * & * & * & * & * \\ \Gamma_{21}^l & -Z & * & * & * & * \\ \Gamma_{31} & 0 & \Gamma_{33} & * & * & * \\ \Gamma_{41} & 0 & 0 & \Gamma_{44} & * & * \\ \Gamma_{51} & 0 & 0 & 0 & \Gamma_{55} & * \\ \mathcal{C} & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \quad l = 1, 2 \tag{14}$$

where

$$\Omega = \begin{bmatrix} -P + R + M_1 + M_1^T & * & * & * \\ M_2 - M_1^T + N_1^T & -M_2 + N_2 - M_2^T + N_2^T & * & * \\ M_3 - N_1^T & -M_3 + N_3 - N_2^T & -R - N_3 - N_3^T & * \\ M_4 & -M_4 + N_4 & -N_4 & -\gamma^2 I \end{bmatrix}$$

$$\bar{\mathcal{A}} = \begin{bmatrix} A - I & B\bar{\Theta}_2 K \bar{\Theta}_1 & 0 & B_w \end{bmatrix}, \quad \Gamma_{21}^1 = \sqrt{d_M} M^T, \quad \Gamma_{21}^2 = \sqrt{d_M} N^T$$

$$\Gamma_{31} = \begin{bmatrix} \mathcal{A} \\ \sqrt{d_M}\bar{\mathcal{A}} \end{bmatrix}, \quad \Gamma_{33} = \begin{bmatrix} -P^{-1} & 0 \\ 0 & -Z^{-1} \end{bmatrix}, \quad \Gamma_{41} = \begin{bmatrix} \bar{\mathcal{B}} \\ \sqrt{d_M}\bar{\mathcal{B}} \end{bmatrix}, \quad \Gamma_{44} = \begin{bmatrix} \hat{P} & 0 \\ 0 & \hat{Z} \end{bmatrix}$$

$$\hat{P} = diag\Big\{\underbrace{-P^{-1}, -P^{-1}, \cdots, -P^{-1}}_{n\times m}\Big\}, \quad \hat{Z} = diag\Big\{\underbrace{-Z^{-1}, -Z^{-1}, \cdots, -Z^{-1}}_{n\times m}\Big\}$$

$$\Gamma_{51} = \begin{bmatrix} \Upsilon_1^T & \Upsilon_2^T & \cdots & \Upsilon_n^T \end{bmatrix}^T, \quad \Gamma_{55} = diag\Big\{\underbrace{-I, -I, \cdots, -I}_{n\times m}\Big\},$$

$$\begin{aligned} \bar{\mathcal{B}}^T &= \begin{bmatrix} \chi_1^T & \chi_2^T & \cdots & \chi_n^T \end{bmatrix} \\ \chi_l^T &= \begin{bmatrix} \mathcal{B}_{l1}^T & \mathcal{B}_{l2}^T & \cdots & \mathcal{B}_{lm}^T \end{bmatrix}, \quad l = 1, 2, \cdots, n \\ \Upsilon_l^T &= \begin{bmatrix} \mathcal{D}_{l1}^T & \mathcal{D}_{l2}^T & \cdots & \mathcal{D}_{lm}^T \end{bmatrix}, \quad l = 1, 2, \cdots, n \\ \mathcal{B}_{ls} &= \begin{bmatrix} 0 & \sqrt{\Delta_{ls}} B\Theta_2^s K \Theta_1^l & 0 & 0 \end{bmatrix}, \quad s = 1, 2, \ldots, m \\ \mathcal{D}_{ls} &= \begin{bmatrix} 0 & \sqrt{\Delta_{ls}} D\Theta_2^s K \Theta_1^l & 0 & 0 \end{bmatrix}, \quad s = 1, 2, \ldots, m \\ \Delta_{ls} &= \mu_l \delta_s^2 + \lambda_s \sigma_l^2 + \sigma_l^2 \delta_s^2, \quad \mathcal{C} = \begin{bmatrix} C & D\bar{\Theta}_2 K \bar{\Theta}_1 & 0 & 0 \end{bmatrix} \end{aligned}$$

Proof: For technical convenience, we rewrite (11) and (12) as

$$x(k+1) = \mathcal{A}\xi(k) + \mathcal{B}x(k - d_k) \tag{15}$$

$$z(k) = \mathcal{C}\xi(k) + \mathcal{D}x(k - d_k) \tag{16}$$

where

$$\begin{aligned}\xi(k) &= \begin{bmatrix} x^T(k) & x^T(k-d_k) & x^T(k-d_M) & w^T(k)\end{bmatrix}^T \\ \mathcal{A} &= \begin{bmatrix} A & B\bar{\Theta}_2 K\bar{\Theta}_1 & 0 & B_w\end{bmatrix} \\ \mathcal{B} &= B\Pi, \quad \mathcal{D} = D\Pi \\ \mathcal{C} &= \begin{bmatrix} C & D\bar{\Theta}_2 K\bar{\Theta}_1 & 0 & 0\end{bmatrix}\end{aligned}$$

and Π is defined in (11).

Construct a Lyapunov functional as $V(k) = \sum_{n=1}^{3} V_n(k)$, where $V_1(k) = x^T(k)Px(k)$, $V_2(k) = \sum_{i=k-d_M}^{k-1} x^T(i)Rx(i)$, $V_3(k) = \sum_{i=-d_M}^{-1}\sum_{j=k+i}^{k-1} y^T(j)Zy(j)$ with $P > 0$, $R > 0$, $Z > 0$ and

$$y(k) = x(k+1) - x(k) = \bar{\mathcal{A}}\xi(k) + \mathcal{B}x(k-d_k) \tag{17}$$

where $\bar{\mathcal{A}}$ is defined in (14). Calculating the forward difference of $V(k)$, we can obtain

$$\Delta V(k) = \sum_{n=1}^{3} \Delta V_n(k) \tag{18}$$

where

$$\Delta V_1(k) = \xi^T(k)\mathcal{A}^T P\mathcal{A}\xi(k) + \mathcal{E}\left\{x^T(k-d_k)\mathcal{B}^T P\mathcal{B}x(k-d_k)\right\} - x^T(k)Px(k)$$

$$\Delta V_2(k) = \mathcal{E}\left\{V_2(k+1) - V_2(k)\right\} = x^T(k)Rx(k) - x^T(k-d_M)Rx(k-d_M) \tag{19}$$

$$\Delta V_3(k) = \mathcal{E}\left\{V_3(k+1) - V_3(k)\right\} = \mathcal{E}\left\{d_M y^T(k)Zy(k) - \sum_{i=k-d_M}^{k-1} y^T(i)Zy(i)\right\} \tag{20}$$

From (15), we have

$$\begin{aligned}&\mathcal{E}\left\{x^T(k-d_k)\mathcal{B}^T P\mathcal{B}x(k-d_k)\right\} \\ &= \sum_{l=1}^{n}\sum_{s=1}^{m} \Delta_{ls} x^T(k-d_k)\left(B\Theta_2^s K\Theta_1^l\right)^T P\left(B\Theta_2^s K\Theta_1^l\right)x(k-d_k) \\ &= \sum_{l=1}^{n}\sum_{s=1}^{m} \xi^T(k)\mathcal{B}_{ls}^T P\mathcal{B}_{ls}\xi(k)\end{aligned} \tag{21}$$

where $\mathcal{B}_{ls}$ $(l = 1, 2, \ldots, n; s = 1, 2, \ldots, m)$ are defined in (14). Then following the similar lines of [14], the rest of proof can be completed easily. Due to page consideration, it is omitted here.

4. H_∞ Controller Design.

In this section, let us consider the stabilizing controller design. As we know, if the stabilization condition is given in the form of LMI, a commonly used approach to obtaining the feedback gain matrix is that by pre- and post multiplying with a certain diagonal matrix, and then defining some new matrix variables such that the new matrix variables are linearly related to the system matrices. Finally, the feedback gain matrix can be obtained by using the obtained new matrix variables. However, the condition given in Theorem 3.1 is a set of matrix inequality with some matrix inverse constraints. Therefore, by experience, we know that the feedback gain K cannot be directly obtained by the commonly used method. Specifically, just the term $B\bar{\Theta}_2 K\bar{\Theta}_1$ in Theorem 3.1 makes it difficult to design the feedback gain K, by pre- and post multiplying $B\bar{\Theta}_2 K\bar{\Theta}_1$ with a suitable invertible matrix X; then there exists a term $XB\bar{\Theta}_2 K\bar{\Theta}_1 X$, obviously, it is a nonlinear term, of which the feedback gain matrix cannot be obtained directly. To solve the feedback gain matrix effectively, defining new matrix variables $\bar{P} = P^{-1}$, $\bar{Z} = Z^{-1}$, and then replacing P^{-1} and Z^{-1} in (14) with $\bar{P}$ and $\bar{Z}$, respectively, and the obtained condition is denoted by (14). Now, the following algorithm is suggested to obtain the feedback gain matrix.

Algorithm (H_∞ controller design): Given positive integer d_M and a scalar $\gamma > 0$, let N denote the maximum number of iterations allowed.

Step 1: Find a feasible set $\left\{P_0, \bar{P}_0, Z_0, \bar{Z}_0\right\}$ satisfying (14) and

$$\begin{bmatrix} P & I \\ I & \bar{P} \end{bmatrix} \geq 0, \quad \begin{bmatrix} Z & I \\ I & \bar{Z} \end{bmatrix} \geq 0 \tag{22}$$

If there is no feasible solution, Exit. Otherwise, set $k = 0$.

Step 2: According to (14) and (22), solve the LMI problem:

$$mim\ tr\left\{P_k\bar{P} + \bar{P}_kP + Z_k\bar{Z} + \bar{Z}_kZ\right\}$$

Step 3: Substituting the obtained matrix variables $\left\{P, \bar{P}, Z, \bar{Z}\right\}$ into the following matrix inequality

$$P - \bar{P} > 0, \quad Z - \bar{Z} > 0 \tag{23}$$

If condition (23) is satisfied, then output the feedback gain K. Exit.

Step 4: If $k > N$, Exit. Otherwise, set $k = k + 1$ and go to Step 2.

Remark 4.1. *Note that Theorem 3.1 provides an efficient way for H_∞ performance analysis for deterministic systems. However, parameter uncertainties often exist in practical applications. In fact, following the similar line as in* [38]*, based on Theorem 3.1, the corresponding robust stability results and controller design can be obtained easily (including the norm-bounded uncertainty and polytopic uncertainty), it is omitted here for simplicity.*

Remark 4.2. *As a byproduct, when we only consider the actuators failures, (11)-(13) is described by*

$$\begin{aligned} x(k+1) &= \hat{\mathcal{A}}\xi(k) + B\hat{\Theta}_2Kx(k-d_k) \\ z(k) &= Cx(k) + D\hat{\Theta}_2Kx(k-d_k) \\ x(k) &= \phi(k), \quad k = -d_M, -d_M+1, \ldots, 0 \end{aligned} \tag{24}$$

where $\xi(k)$ is defined as in (15), $\hat{\mathcal{A}} = \begin{bmatrix} A & B\bar{\Theta}_2K & 0 & B_w \end{bmatrix}$, following a similar proof line as in Theorem 3.1, the corresponding robust stability results and controller design can be obtained easily, it is omitted here for simplicity.

Remark 4.3. *It is worth mentioning that the results given in this paper are not seen in the existing literatures. The main advantage of our approach is that we consider the probabilistic sensors and actuators failures and data distortion simultaneously. This is different from the existing works* [20, 26, 35]*, which only consider the NCSs with either actuator failures or sensor failures. Moreover, though the similar idea has been used in* [29]*, only the analysis and design of an observer-based feedback controller has been studied, and the state feedback control design problem has not been discussed. Also, the measurement distortion has not been considered in* [29]*. Furthermore, the underling problem in this paper does have been investigated in* [14]*; however, only the continuous-time networked systems was involved; moreover, the control design method proposed in this paper is different from that in* [14]*.*

5. **Numerical Example.** In this section, we will use an example to illustrate the usefulness of the controller design method developed in this paper.

Example 5.1. *Consider the following open-loop unstable system:*

$$x(k+1) = \begin{bmatrix} 0.7566 & -0.0591 \\ 0.6149 & 1.1704 \end{bmatrix} x(k) + \begin{bmatrix} 0.1126 & 0.4238 \\ -0.2959 & 0.9067 \end{bmatrix} u(k) + \begin{bmatrix} -0.6522 \\ 0.1915 \end{bmatrix} w(k) \quad (25)$$
$$z(k) = \begin{bmatrix} -1.1074 & -0.6290 \end{bmatrix} x(k) + \begin{bmatrix} -0.3473 & 0.9067 \end{bmatrix} u(k)$$

For this example, to illustrate the effectiveness of the theoretical results developed in this paper, we can consider the following two cases:

Case 1: Assume that all the sensors and actuators have no faults, that is, they are reliable, in this case, the stochastic matrices Θ_1 and Θ_2 of system (11)-(13) have expectation and variance

$$\bar{\Theta}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad V_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{\Theta}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

respectively. Suppose $d_m = 0$, $d_M = 2$, the prescribed performance index $\gamma^* = 6.25$. Applying Theorem 3.1 and Algorithm, the state feedback gain matrix is given by

$$K = \begin{bmatrix} -0.2326 & 0.3229 \\ -0.1744 & -0.1071 \end{bmatrix} \quad (26)$$

To illustrate the disturbance attenuation performance of the designed the reliable controller, we now assume the zero initial condition and set the external disturbance $w(k) = \begin{cases} 1, & 10 \le k \le 30 \\ 0, & \text{otherwise} \end{cases}$. In this case, the state responses of system (25) are depicted in Figure 1. By calculation, we obtain that $\|z(k)\|_2 = 19.1512$ and $\|w(k)\|_2 = 4.5826$, and therefore, $\gamma = 4.1791$, which stays below the prescribed performance index $\gamma^* = 6.25$.

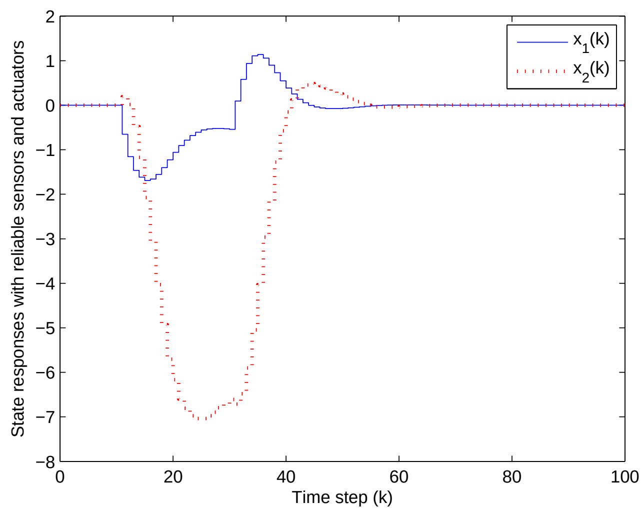

FIGURE 1. State responses when $w(k) \neq 0$

Case 2: Assume that the faults have been allowed to occur, simultaneously, in both the sensors and actuators. In this case, suppose that the probabilistic density function of the random variables θ_{11} and θ_{12} in $[0, \theta]$ (θ can be larger than 1) are described by

$$f_1(s_1) = \begin{cases} 0.2, & s_1 = 0 \\ 0.6, & s_1 = 0.5 \\ 0, & s_1 = 1 \\ 0.2, & s_1 = 1.5 \end{cases}, \quad f_2(s_2) = \begin{cases} 0.1, & s_2 = 0 \\ 0.6, & s_2 = 0.5 \\ 0, & s_2 = 1 \\ 0.3, & s_2 = 2 \end{cases}$$

from which the expectations and variances can be easily calculated as $\mu_1 = 0.75$, $\mu_2 = 0.9$, $\sigma_1^2 = 0.0375$, $\sigma_2^2 = 0.54$. In this case,

$$\bar{\Theta}_1 = \begin{bmatrix} 0.75 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad V_1 = \begin{bmatrix} 0.1936 & 0 \\ 0 & 0.7348 \end{bmatrix} \tag{27}$$

Similarly, assume that the probabilistic density function of the random variables θ_{21} and θ_{22} in $[0, \theta]$ (θ can be larger than 1) are given as

$$g_1(\upsilon_1) = \begin{cases} 0.3, & \upsilon_1 = 0 \\ 0.5, & \upsilon_1 = 0.5 \\ 0, & \upsilon_1 = 1 \\ 0.2, & \upsilon_1 = 1.2 \end{cases}, \quad g_2(\upsilon_2) = \begin{cases} 0.3, & \upsilon_2 = 0 \\ 0.4, & \upsilon_2 = 0.5 \\ 0, & \upsilon_2 = 1 \\ 0.3, & \upsilon_2 = 1.1 \end{cases}$$

from which the expectations and variances can be easily calculated as $\lambda_1 = 0.49$, $\lambda_2 = 0.53$, $\delta_1^2 = 0.1729$, $\delta_2^2 = 0.1821$. In this case,

$$\bar{\Theta}_2 = \begin{bmatrix} 0.49 & 0 \\ 0 & 0.53 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0.4158 & 0 \\ 0 & 0.4267 \end{bmatrix} \tag{28}$$

Suppose $d_m = 0$, $d_M = 2$, and the prescribed performance index $\gamma^* = 4.25$. Applying Theorem 3.1 and Algorithm again, the reliable state feedback gain matrix can be obtained as

$$K = \begin{bmatrix} -0.3934 & 0.2632 \\ -0.4346 & -0.2331 \end{bmatrix} \tag{29}$$

Liking Case 1, to illustrate the disturbance attenuation performance, we choose the initial condition as $x_0 = \phi(0) = [0, 0]^T$ and the external disturbance $w(k)$ as the same in Case 1. Figure 2 shows the evolution of the state variables. It can be calculated that $\|z(k)\|_2 = 15.4315$ and $\|w(k)\|_2 = 4.5826$, and therefore, $\gamma = 3.3674$, which stays below the prescribed performance index $\gamma^* = 4.25$. Furthermore, from Figures 1 and 2, it can

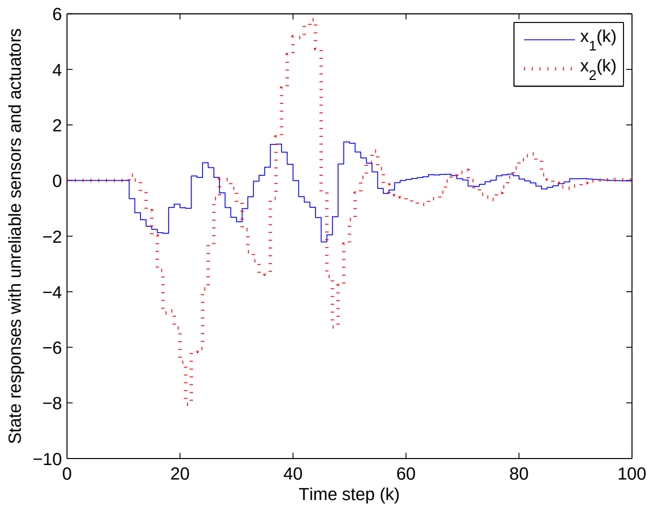

FIGURE 2. State responses when $w(k) \neq 0$

be seen that when there indeed exist the failures of sensors and actuators, although the designed controller can guarantee the asymptotical stability of the considered system in the mean-square sense, the dynamic performance, such as adjust time, overshoot and oscillating frequency is obviously worse than without taking into account the failures of the sensors and actuators. This demonstrates that the failures of sensors and/or actuators will degrade the system performance.

6. **Conclusions.** In this paper, the problem of robust H_∞ control problem has been discussed for a class of uncertain linear discrete-time NCSs with stochastic sensors and actuators failures and data distortion. The failures of the sensors and actuators are assumed to happen in a random way, and the failure rate for each individual sensor/actuator is governed by an individual random variable satisfying a certain probabilistic distribution in the interval $[0, \theta]$ $(\theta \geq 1)$. A sufficient condition for the existence of the reliable state feedback controller to stabilize the closed-loop control system and achieve the prescribed H_∞ performance index has been given in the terms of the feasibility of recursive linear matrix inequality. Simulation results have demonstrated the effectiveness of the proposed design method.

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