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A SIMPLIFIED RECURSIVE DYNAMIC PCA BASED MONITORING SCHEME FOR IMPERIAL SMELTING PROCESS

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Abstract. *Imperial Smelting Process (ISP) is one of the main methods for Zinc and Lead smelting. Its operating conditions change very frequently due to the changes of work points, which always lead to false alarms. We focus on this issue and present a recursive Dynamic PCA (RPCA) based monitoring scheme for ISP to adapt process changes. We present a simplified RPCA algorithm based on first-order perturbation analysis (FOP), which is a rank-one update of eigenvalues and their corresponding eigenvectors of an observation covariance matrix. The computation cost is greatly decreased. We also present two new statistics for process monitoring in ISP to avoid numerical computation difficulty induced by the traditional statistics. Finally, we apply the proposed method to real data from ISP. The results show that the proposed scheme can be able to eliminate false alarms and detect faults efficiently.*
Keywords: Recursive principal component analysis, Imperial smelting process, Fault detection, First-order perturbation analysis

1. **Introduction.** Imperial Smelting Process (ISP) is one of the main methods for Zinc and Lead smelting, the objective of which is to obtain maximal output, especially the maximal zinc output because zinc is more expensive than lead [1]. There are so many abnormal conditions or faults in the process because its operational conditions change very frequently, so it is very important to online detect the abnormal conditions or faults of the process. Over the past two decades, model based fault diagnosis techniques have made significant progress and received considerable attention in both research and application domains. It is most important for model based fault diagnosis to obtain a mathematic model, for example observer based scheme or Markove model based scheme for fault detection [2, 3]. However, ISP is such a complex process that we cannot build mathematical model for process monitoring and fault diagnosis (PM-FD). Fortunately, there is huge data collected and stored in this process, and lots of data-driven methods are used for

PM-FD and have attracted more and more attention such as AI based methods [4] and statistics methods [5].

Principal component analysis (PCA) is a most popular way for this purpose, which has been successfully applied to PM-FD of many industrial processes. The central idea of PCA is to reduce the dimensionality of a data set consisting of a large number of interrelated variables, while retaining as much as possible of the variation present in the data set. To use PCA for PM-FD, a PCA model is firstly established based on collected process data under normal operating conditions. Then, the control limits of monitoring statistics (e.g., T2, SPE) are calculated and thus the process can be online monitored by these statistics [6]. Despite its tremendous success, standard PCA-based monitoring technique has a few major drawbacks. First of all, it could not be used to deal with processes with time-varying parameters. When natural drifting behavior or changing of operation region happens, there will be more false alarms; thus more reasonable PCA model and control limits for monitoring statistics are obtained in an adaptive manner. A promising technique combining moving window (MW) with recursive updating of PCA model was proposed [7]. As a result, recursion with a window sliding along the data is more appropriate for time-varying processes. This adaptation approach is so-called as moving window PCA (MWPCA). The fast MWPCA enables online application of generic moving window based recursive PCA with a larger window size, but the efficient algorithm for updating PCA model was not addressed. Recently, the moving window kernel PCA has been proposed by Liu et al. for adaptive monitoring of nonlinear process [8], but the method is with much computation so that it is very difficult for online application.

Changes and drifts of many industrial processes like ISP would be frequently reflected in the process variables, which would lead to false alarms. Moreover, it is also a typical complex dynamic system due to so many complex chemical and physical reactions involved. Recent development of Dynamic PCA (DPCA) based process monitoring is focused on achieving adaptive process monitoring and dealing with process dynamics. To this end, Rigopoulos [9] presented a moving window scheme to identify an adaptive model in a simulated paper machine profile. Qin [10] proposed several recursive partial least squares (RPLS) algorithms for online process modeling to adapt process changes and off-line modeling to deal with a large number of data samples, and [11] discussed a recursive PCA in adaptive industrial process monitoring using rank-one modification and Lanczos tridiagonalization. Elshenawy [12] presented two RPCA algorithms aiming to reduce the computation cost without computing the correlation matrix. But it also endures a lot of computation cost because the recursive computation and indices computation of DPCA are very complex, or may be faced with some ill cases based on such complex real data obtained from ISP.

Motivated by the problems encountered when RPCA is solely implemented for process monitoring, the study reported in this paper focuses on developing a novel adaptive monitoring scheme for time-varying processes by taking advantage of Dynamic PCA (DPCA). A simplified algorithm is proposed based on first-order perturbation theory (FOP) [13] in order to decrease the computation cost clearly. We have also noticed that real data obtained from ISP is so complex, there are some ill cases in the computation process of statistics indices, and slight modifications on the PCA methods might result in a performance improvement in detecting and identifying process faults. So, we introduce two new statistics for alternative and apply them to the recursive PCA method.

The paper is organized as follows. In Section 2, we present a simplified FOP based PCA to recursively update the PCA model and introduce two new statistics. Section 3 gives the integrated RDPCA based process monitoring scheme. Imperial Smelting Process monitoring scheme and some simulation results are also shown in Section 3. The paper ends with conclusions in the last section.

2. Recursive Dynamic Principal Component Analysis.

2.1. Principal component analysis.

Similar to standard PCA, dynamic PCA consists of several steps and can be formulated as follows:

1) Data collection and pre-processing: consider a process with variables, let

$$\mathbf{X}_s = \begin{bmatrix} x_1^T & x_2^T & \dots & x_s^T \\ x_2^T & x_3^T & \dots & x_{s+1}^T \\ \vdots & \vdots & \ddots & \vdots \\ x_N^T & x_{N+1}^T & \dots & x_{N+s-1}^T \end{bmatrix} \in \Re^{N \times sm} \tag{1}$$

where $\mathbf{x}_i \in \mathbf{R}^m$, $i = 1, \dots, N+s-1$, denotes a sample vector of m sensors, which always is scaled to zero mean and unit variance. A time lag shift s is always assumed to be some integer and small. The normalization equation as follows:

$$\mathbf{x}_j = \frac{\left(\mathbf{x}_j^0 - b_j^0\right)}{\eta_j^0} \tag{2}$$

where b_j^0 and η_j^0 are the corresponding mean and variance of the jth variable $\mathbf{x}_j^0$ respectively, $\mathbf{x}_j^0$ is the raw data, and $j = 1, \dots, m$.

2) Decomposition of correlation matrix and thresholds calculation: The correlation matrix is constructed as:

$$\mathbf{R} \approx \frac{1}{N-1}\mathbf{X}_s^T\mathbf{X}_s = \mathbf{P\Lambda P}^T \tag{3}$$

with

$$\begin{aligned} \mathbf{\Lambda} &= \begin{bmatrix} \mathbf{\Lambda}_{pc} & 0 \\ 0 & \mathbf{\Lambda}_{res} \end{bmatrix} \\ \mathbf{\Lambda}_{pc} &= diag(\sigma_1^2, \dots, \sigma_{l_k}^2) \in \Re^{l_k \times l_k} \\ \mathbf{\Lambda}_{res} &= diag(\sigma_{l_k+1}^2, \dots, \sigma_{sm}^2) \in \Re^{(sm-l_k)\times(sm-l_k)} \\ \mathbf{P} &= [\mathbf{P}_{pc} \quad \mathbf{P}_{res}] \in \Re^{sm \times sm} \\ \mathbf{P}_{pc} &\in \Re^{sm \times l_k}, \quad \mathbf{P}_{res} \in \Re^{sm \times (sm-l_k)} \end{aligned}$$

where $\sigma_1, \sigma_2, \dots, \sigma_{l_k}$ $(\sigma_1 \geq \dots \geq \sigma_{l_k})$ are the l_k largest (principal) singular values and

$$\sigma_{l_k} \geq \sigma_{l_k+1} \geq \dots \geq \sigma_{sm}$$

For process monitoring purpose, two indices Hotelling's T^2 and SPE are calculated on the assumption that the sample number N is large enough. In order to simply our study and notation, thus χ^2-distribution instead of F-distribution can be adopted. So that T^2 satisfies with l_k degrees of freedom. For a given significance level α, the corresponding thresholds are set to be:

$$J_{th,T^2} = \frac{l_k(N-1)(N+1)}{N(N-l_k)}F_\alpha(l_k, N-l_k) \approx \chi_\alpha(l_k) \tag{4}$$

$$J_{th,SPE} = \theta_1\left(\frac{c_\alpha\sqrt{2\theta_2 h_0^2}}{\theta_1} + 1 + \frac{\theta_2 h_0(h_0-1)}{\theta_1^2}\right)^{1/h_0} \tag{5}$$

with

$$\theta_i = \sum_{j=l_k+1}^{sm} (\sigma_j^2)^i, \quad i = 1, 2, 3, \quad h_0 = 1 - \frac{2\theta_1\theta_3}{3\theta_2^2}$$

3) Online process monitoring: For a new scaled measurement $x \in \Re^m$, T^2 and SPE can be calculated online as follows:

$$T^2 = x^T \mathbf{P}_{pc} \mathbf{\Lambda}_{pc} \mathbf{P}_{pc}^T x \tag{6}$$

$$SPE = ||(\mathbf{I} - \mathbf{P}_{pc}\mathbf{P}_{pc}^T)x||^2 = x^T(\mathbf{I} - \mathbf{P}_{pc}\mathbf{P}_{pc}^T)^2 x \tag{7}$$

2.2. **Recursive DPCA based on FOP.** Standard PCA (DPCA) based process monitoring is efficient to deal with time-invariant process. However, it is hard to monitor the normal process variations caused by external disturbances and conditions change, which would lead to the changes of work points. To deal with this problem, an adaptive model should be constructed to update the PCA (DPCA) model to adapt to the time-variant sceneries. To this end, the correlation matrix $\mathbf{R_k}$ at time instant k is required to be computed and update its eigen-structure, however, the calculation costs of $\mathbf{R_k}$ would be so tremendous that it is even impossible for on-line application. Therefore, a recursive algorithm based on FOP is introduced to update the fixed model with lower computation costs as follows.

Let $\mathbf{x}_k^0 \in \Re^{sm}$ be the raw data at time instant k, the mean value at time instant $k-1$ is:

$$\mathbf{b}_{k-1} = \sum_{i=1}^{k-1} \mathbf{x}_i^0/(k-1) \tag{8}$$

So similarly, $\mathbf{b}_k$ presents the mean value at time instant k.

$$\mathbf{b}_k = \sum_{i=1}^{k} \mathbf{x}_i^0/k \tag{9}$$

It is deserved to note that

$$k\mathbf{b}_k = (k-1)\mathbf{b}_{k-1} + \mathbf{x}_k^{0T} \tag{10}$$

For further discussion, Equation (10) can be rewritten as:

$$\mathbf{b}_{k-1} = \frac{k}{k-1}\mathbf{b}_k - \frac{1}{k-1}\mathbf{x}_k^{0T} \tag{11}$$

Let

$$\begin{aligned}\Delta\mathbf{b}_k = \mathbf{b}_k - \mathbf{b}_{k-1} &= \mathbf{b}_k - \frac{k}{k-1}\mathbf{b}_k + \frac{1}{k-1}\mathbf{x}_k^{0T} \\ &= \frac{1}{k-1}\left(\mathbf{x}_{k+1}^{0T} - \mathbf{b}_{k+1}\right) = \frac{1}{k-1}\left(\mathbf{x}_{k+1}^{0} - \mathbf{b}_{k+1}^T\right)^T\end{aligned} \tag{12}$$

Recall following equation from (11)

$$\mathbf{R}_k = \mu\mathbf{\Lambda}_k^{-1}\left(\mathbf{\Lambda}_{k-1}\mathbf{R}_{k-1}\mathbf{\Lambda}_{k-1} + \Delta\mathbf{b}_k\Delta\mathbf{b}_k^T\right)\mathbf{\Lambda}_k^{-1} + (1-\mu)\mathbf{x}_k^T\mathbf{x}_k \tag{13}$$

If the variance does not change significantly, we can use the initial variance to scale the data and do not update the variance. Equation (13) can be reduced into,

$$\mathbf{R}_k = \mu\mathbf{R}_{k-1} + \mu\mathbf{\Lambda}_k^{-1}\Delta\mathbf{b}_k\Delta\mathbf{b}_k^T\mathbf{\Lambda}_k^{-1} + (1-\mu)\mathbf{x}_k^T\mathbf{x}_k \tag{14}$$

where $\mu = \frac{k-1}{k}$, $\mathbf{x}_k \in \Re^{sm}$ denotes the measurement vector at time instant k. For further discussion, Equation (14) can be regenerated by:

$$\mathbf{R}_k = \frac{k-1}{k}\mathbf{R}_{k-1} + \frac{1}{k-1}\mathbf{x}_k^T\mathbf{x}_k \tag{15}$$

According to the algorithm proposed by [13], if the correlation matrix can be recursively based on first-order perturbation analysis (FOP) updated by

$$\mathbf{R}_k = \varepsilon \mathbf{R}_{k-1} + (1-\varepsilon)\mathbf{x}_k^T \mathbf{x}_k \tag{16}$$

where forgetting factor ε is a small positive number ($\varepsilon \to 0$). And then, the singular values and eigenvectors of the updated matrix can be described by:

$$\sigma_{k,i}^2 = \varepsilon \sigma_{k-1,i}^2 + (1-\varepsilon)\mathbf{P}_{k-1,i}^T \mathbf{x}_k \mathbf{x}_k^T \mathbf{P}_{k-1,i} \tag{17}$$

$$\mathbf{P}_{k,i} = \mathbf{P}_{k-1,i} + \sum_{j=1}^{m} \lambda_{ji} \mathbf{P}_{k-1,i} \tag{18}$$

where $i, j = 1, \ldots, m$, $b_{ii} = 0$ and

$$\lambda_{ji} = \frac{\mathbf{P}_{k-1,i}^T \mathbf{x}_{k-1} \mathbf{x}_{k-1}^T \mathbf{P}_{k-1,i}}{\sigma_{k-1,i}^2 - \sigma_{k-1,j}^2} \tag{19}$$

$$\lambda_{ij} = -\lambda_{ji} \tag{20}$$

It is deserved to note that if the difference between two successive eigen-values is small, there will be undesirable behavior of the algorithm. To overcome this problem, Equation (19) can be reconstructed without altering the complexity of the algorithm with following modifications:

$$\lambda_{ji} = \frac{\mathbf{P}_{k-1,i}^T \mathbf{x}_{k-1} \mathbf{x}_{k-1}^T \mathbf{P}_{k-1,i}}{\max(\gamma \sigma_{k-1,1}^2, \sigma_{k-1,i}^2 - \sigma_{k-1,j}^2)} \tag{21}$$

2.3. **Two new statistics.** Assume that the process is normal, $\mathbf{x} \sim N(0, \mathbf{R})$, and it holds that

$$\mathbf{z}_{pc} = \mathbf{P}_{pc}^T \mathbf{x} \in \Re^{l_k}, \quad \mathbf{z}_{pc} \sim N\left(0, \mathbf{P}_{pc}^T \mathbf{R} \mathbf{P}_{pc}\right) = N\left(0, \mathbf{\Lambda}_{pc}\right) \tag{22}$$

thus T^2 satisfies χ^2-distribution with l_k degrees of freedom when N is large enough. Note that,

$$\mathbf{z}_{res} = \mathbf{P}_{res}^T \mathbf{x} \in \Re^{(sm-l_k)}, \quad \mathbf{z}_{res} \sim N\left(0, \mathbf{P}_{res}^T \mathbf{R} \mathbf{P}_{res}\right) = N(0, \mathbf{\Lambda}_{res}) \tag{23}$$

As a result,

$$\mathbf{x}^T \mathbf{P}_{res} \mathbf{\Lambda}_{res}^{-1} \mathbf{P}_{res}^T \mathbf{x}$$

satisfies χ^2-distribution with $sm - l_k$ degrees of freedom and would be a reasonable statistic for fault detection purpose. Actually the statistic mentioned above is called Hawkin's T_H^2, which is however less used in practice due to some drawbacks with $\mathbf{\Lambda}_{res}$ when some of the singular values $\sigma_{l_k+1}, \ldots, \sigma_{sm}$ are closed to zero. To avoid this difficulty, an alternative test statistic is proposed. Let

$$\mathbf{\Xi} = diag\left(\frac{\sigma_{sm}^2}{\sigma_{l_k+1}^2}, \ldots, \frac{\sigma_{sm}^2}{\sigma_{sm-1}^2}, 1\right) \in \Re^{(sm-l_k)\times(sm-l_k)} \tag{24}$$

It turns out that

$$\mathbf{\Xi}^{1/2} \mathbf{P}_{res}^T \mathbf{x} \sim N\left(0, \sigma_{sm}^2 \mathbf{I}_{(sm-l_k)\times(sm-l_k)}\right)$$

And moreover

$$\mathbf{x}^T \mathbf{P}_{res} \mathbf{\Xi} \mathbf{P}_{res}^T \mathbf{x} = \sigma_{sm}^2 \mathbf{x}^T \mathbf{P}_{res} \mathbf{\Lambda}_{res}^{-1} \mathbf{P}_{res}^T \mathbf{x} \tag{25}$$

Define a new test statistic as

$$SPE_{new} = \mathbf{x}^T \mathbf{P}_{res} \mathbf{\Xi} \mathbf{P}_{res}^T \mathbf{x} \tag{26}$$

Threshold for a given significance level α is equal to

$$J_{th,SPE,new} = \sigma_{sm}^2 \chi_\alpha^2 (sm - l_k) \tag{27}$$

Recall that

$$SPE = \mathbf{x}^T(\mathbf{I} - \mathbf{P}_{pc}\mathbf{P}_{pc}^T)^2\mathbf{x} = \mathbf{x}^T\mathbf{P}_{res}\mathbf{P}_{res}^T\mathbf{x} \tag{28}$$

The new statistic SPE_{new} is χ^2-distributed, and better than conventional SPE since the threshold associated with SPE is an approximation, while the threshold with SPE_{new} can be exactly determined by the available χ^2-distribution table. On the other hand, $J_{th,SPE,new}$ is much less complicated compared with $J_{th,SPE}$. For further discussion, let

$$\overline{\Xi} = diag\left(\frac{\sigma_{sm}^2}{\sigma_1^2}, \ldots, \frac{\sigma_{sm}^2}{\sigma_{sm-1}^2}, 1\right) \in \Re^{sm\times sm} \tag{29}$$

A new combined index is proposed as follows:

$$T_{c,new}^2 = \mathbf{x}^T\mathbf{P}\overline{\Xi}\mathbf{P}^T\mathbf{x} = \sigma_{sm}^2 T^2 + T_{new}^2 \tag{30}$$

It is clear that the corresponding threshold for a given significance level α is

$$J_{th,c,new} = \sigma_{sm}^2\chi_\alpha^2(sm) \tag{31}$$

The detection logic is summarized as follows:

$$SPE \le J_{th,SPE}, \quad T^2 \le J_{th,T^2}$$

$$SPE_{new} \le J_{th,SPE,new} \text{ and } T_{c,new}^2 \le J_{th,c,new} \Rightarrow$$

Fault-free, otherwise there is a fault.

2.4. **Algorithm analysis.** We compare the computational costs of the proposed RDPCA algorithm, with other known algorithm, such as the standard singular value decomposition (SVD) algorithm, Lanzos tridiagonalization [11], inverse iteration approach and fast MWPCA [12]. For the sample-wise case, the overall complexity using these five algorithms is listed in Table 1.

Table 1. Comparison of computational complexity

Approach	Flops	Computation complexity
Standard SVD	$22m^3 + 4m^2 + 12m + 1$	$O(m^3)$
Lanczos approach	$(4l_k + 3)m^2 + (6.5l_k - 1.5l_k^2 + 12)4m + 1$	$O(l_k m^2)$
fast MWPCA	$6m^2 + 20m^2 + 11m + 17$	$O(m^3)$
Inverse iteration	$\frac{2}{3}m^3 l_k + 2l_k m^2 - (5l + 12)m + l_k + 1$	$O(m^3)$
RDPCA	$14s^2m^2 + 19sm + 1$	$O(m^2)$

Here m is the number of system variables and l_k are the PCs. Table 1 shows that the proposed recursive algorithm considerably reduces the computation cost. Because there is exists a time lag shift, the flops of the FOP based RDPCA approach is $14s^2m^2 + 19sm + 1$. As mentioned above, s is small and always assumed to be some integer, so the computation complexity of RDPCA is approximately $O(m^2)$. In the special case of updating, i.e., $l_k = 1$, flops of Lanczos approach is $7m^2 + 68m + 1$, which has the same computation complexity $O(m^2)$ with the proposed recursive PCA algorithm. In other cases, the computation complexity of FOP approach is the smallest. Since the proposed RDPCA does not need to decompose the covariance matrix every step, instead of recursive calculating the eigenpairs, a significant further reduction in computation cost is also available.

3. Adaptive Monitoring of Imperial Smelting Process.

3.1. **Description of process.** Blast smelting for Zinc and Lead is a very typical main complex metallurgical process firstly patented by Imperial Smelting Company in Britain. So it is also called Imperial Smelting Process (ISP). The smelting Equipment of ISP is called Imperial Smelting Furnace (ISF). Lead and Zinc are simultaneously and continuously smelted in a closed furnace in a series of complicated chemical reactions with little process details known.

The outstanding feature of ISP is to smelt two metals simultaneously in the same process, which can be divided into two sub-processes: sintering process and smelting process. At the beginning, a preparative sintering process is performed for smelting material preparation, and the agglomerate, which includes zinc and leads mine in this process is sintered and desulfurated under certain burning conditions. And then the agglomerate blended with coke in some ratio and flux is put into the top of ISF through two devices called bells. The structure of ISF is shown as Figure 1.

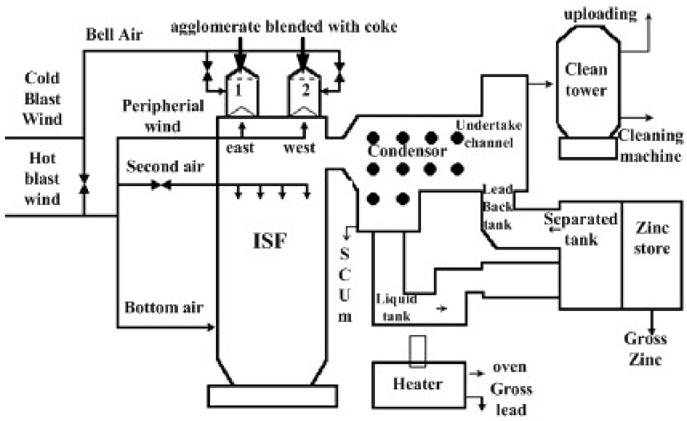

FIGURE 1. Imperial smelting furnace structure

Blasting hot airs in more than 800°C and oxygen are blasted into ISF for speeding the chemical reactions. The majority of hot airs are mainly put into the bottom of ISF and the rest part, as secondary air, is sent into the upper part near fading bell of the surface. When the Redox-reaction is taking place, the sulfates of raw metals are reduced due to carbons in the coke. Zinc is gasified because of a lower boiling point and passes through the throat of furnace, to a condenser where plenty of tiny drops of liquid lead are sprayed to absorb zinc gas with high temperature. An important technical detail is that the liquid of lead is not directly received from smelting furnace but an independent solvent of zinc recycling from spraying device to a lead pool. After passing through a series of devices for segregating, the mixture of liquid zinc and lead is separated. Zinc is lighter and accumulates in the upper layer while lead is heavier and accumulates in the lower layer. After abstracting, refining and purifying successively, we can obtain raw zinc of 98% and pure zinc of 99.995%. The exhaust gas released from furnace throat is put into a washing tower for dust removal.

In ISF, the inputs are coke, agglomerate, some flux and air, and the outputs are zinc, lead, residue and exhaust gas. There are almost 100 measurements of temperature, pressure; flow and level variables to be monitored in the whole ISF. Here, 22 main process variables are selected and listed in Table 2.

There are four types of common faults available in this process, which are slag leakage in lead pool, nodulation, hang-up and device fault in electrical heating bed. It is obvious

Table 2. Some important measurements

Number	Name	Number	Name
1	Hot air temperature	2	Hot air pressure
3	Hot air flow	4	Cold air pressure
5	Surrounding air flow	6	Secondary air flow
7	Southeast air flow	8	Northeast air flow
9	Southwest air flow	10	Northwest air flow
11	Base air flow	12	Base air pressure
13	East top temperature	14	West top temperature
15	Chamber pressure	16	Lead pool temperature
17	Zinc pool temperature	18	Electrical hot bed temperature
19	Mixed air flow	20	No. 1 exhaust gas temperature
21	No. 2 exhaust gas temperature	22	No. 3 exhaust gas temperature
23	Hearth box temperature		

that these faults may lead many of these variables to change. The agglomerate and hot coke from the preheating furnace are periodically blended in a specified ratio and put into ISF through so-called feeding bells and meanwhile some of the manipulating variables such as hot air temperature and blasting air flow must be pre-set. One of the difficulties in designing monitoring scheme is some normal process changes and multiple operating conditions are mistaken as abnormal deviations that may cause false alarms. Therefore, an adaptive process monitoring scheme should be designed. To this end, we apply the proposed RDPCA scheme to monitoring the process.

3.2. **Data preprocessing.** To effectively extract the information in the data relevant to process monitoring, it is necessary to pretreat the data. One of the pretreatment procedures is the so-called autoscaling, which normalizes the process variables in a way that each variable has equal weight before applying the process monitoring method. Autoscaling consists of two steps, i.e., centering the variables to zero mean and then dividing these variables by their standard deviations. For recursive implementation of PCA, the mean and variance should be updated according to the following equations:

$$b_{j,k} = \alpha b_{j,k-1} + (1-\alpha)x^0_{j,k} \tag{32}$$

$$\eta^2_{j,k} = \alpha R^2_{j,k-1} + (1-\alpha)(x^0_{j,k} - b_{j,k})^2 \tag{33}$$

where $x^0_{j,k}$ is the jth measurement variable before normalization, $\mathbf{b}_k = [b_{j,k}, \ldots, b_{sm,k}]^T \in \Re^{sm}$ and $\eta^2_{j,k}$ are the update mean vector and variances of measurement data, and $j = 1, \ldots, sm$. According to these updated relations, the normalized data is calculated as Equation (2).

A complete implementation of RPCA also requires recursive calculation of the number of PCs. There are many approaches to calculate the number of PCs, but most of them use monotonically increasing or decreasing indices. The decision to choose the index is very subjective and depends on application requirements. Several comparative studies have been conducted on these approaches. Unfortunately, not all of the approaches are suitable for the recursive determination of the number of PCs. The CPV approach is implemented in our application study [11].

3.3. **Test results and analysis.**

3.3.1. *Algorithm.* In this section, FOP-based recursive DPCA algorithms are illustrated in ISP for adaptive monitoring including following steps:

a) Construct the initial DPCA model according to Equations (1) and (3). Time lag shift s is chosen to be 1 and significance level α is selected as 0.98.
b) Determine the numbers of the principal components using recursive approach.
c) Collect the raw data $\mathbf{x}_k^0 \in \Re^{1\times sm}$, normalize it using Equation (2), and the forgetting factor ε here is selected as 0.01.
d) Initial conditions are calculated for sample time instant $k-1$, including σ_{k-1}^2, $\mathbf{P}_{k-1,i}$, $\mathbf{b}_{k-1}$.
e) According to Equations (17) and (18), the eigen-values σ_k^2 and eigen-vectors $\mathbf{P}_k$ at time instant k are updated respectively.
f) Online calculating four statistics T^2, SPE, SPE_{new} and $T_{c,new}^2$ using Equations (6), (7), (26) and (30) respectively.
g) On the basis of Equations (4), (5), (27) and (31), calculating 4 corresponding thresholds, the computational sequence is identical with above procedure.
h) Using the detection logic, which is summarized in part 2.3, to monitor the process.

3.3.2. *Simulations under the fault-free condition.* Statistics indices from real data of ISP under the fault-free condition are shown in Figures 2-5. It can be seen from Figure 2 and Figure 3 that the process monitoring cannot work using standard DPCA because there exist so many fault alarms, although it is very clear that there exist less fault alarm in Figure 3 than in Figure 2. To cope with this problem that change of work points leads to fault alarms, recursive DPCA is introduced in Figures 4 and 5. It can be seen from Figures 4 and 5 that the proposed scheme is more effective under the fault-free condition. And it is also very clear that there exist less fault alarms in Figure 5 than in Figure 4.

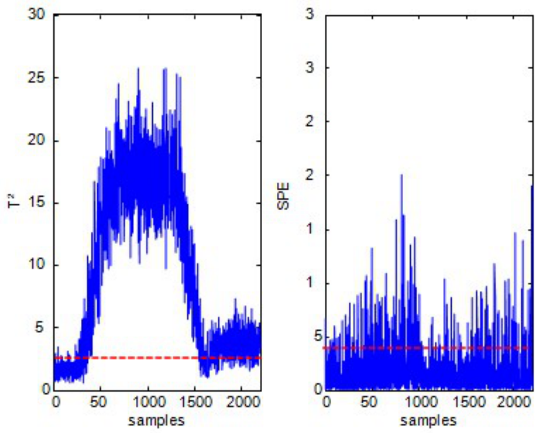

FIGURE 2. Normal variations by DPCA with conventional indices

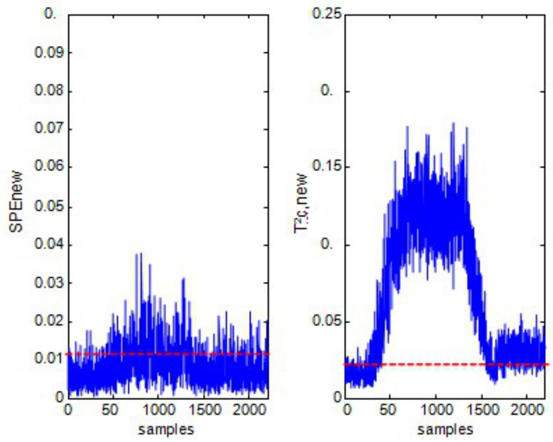

FIGURE 3. Normal variations by DPCA with two new indices

3.3.3. *Simulations under the fault condition.* Statistics indices from real data of ISF under the fault condition are shown in Figures 6-9.

It can be seen from Figures 6 and 7 that the process monitoring can not work using standard DPCA under the fault condition. Now, we would like to use recursive DPCA for monitoring purpose shown in Figures 8 and 9 under fault condition. It can be seen from Figures 8 and 9 that the proposed scheme is more effective under the fault condition because it is very clear that there exist few fault alarms in Figures 8 and 9.

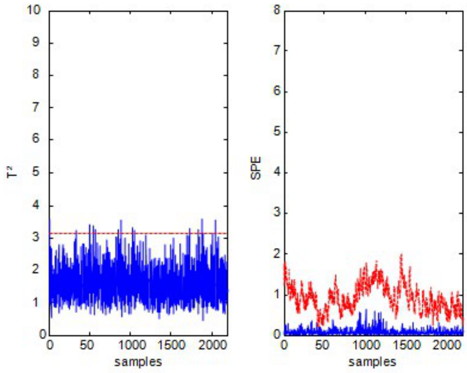

FIGURE 4. Normal variations by recursive DPCA (fault free) with conventional indices

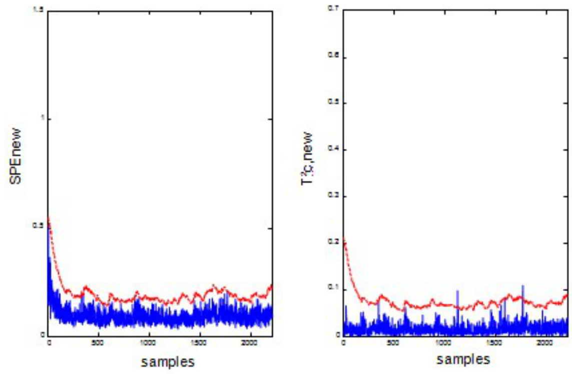

FIGURE 5. Normal variations by recursive DPCA (fault free) with new indices

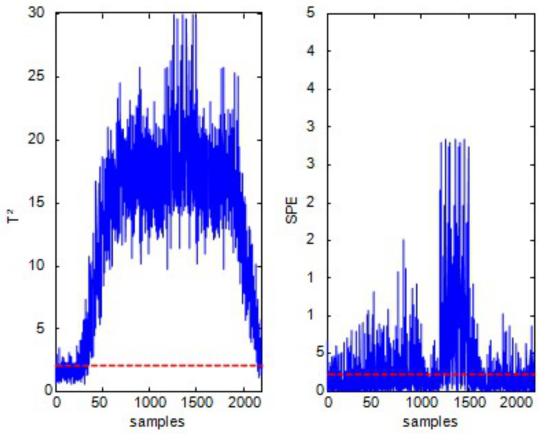

FIGURE 6. Monitoring by DPCA with conventional indices

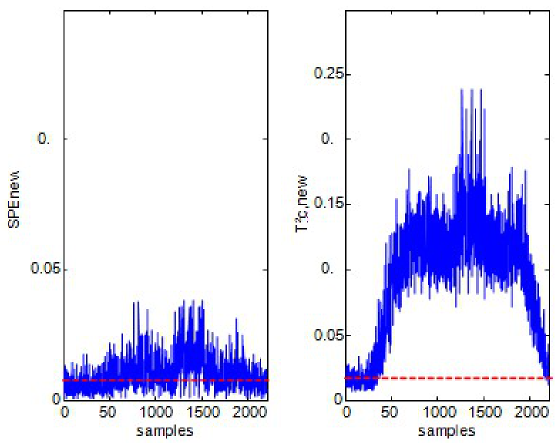

FIGURE 7. Monitoring by DPCA with new indices

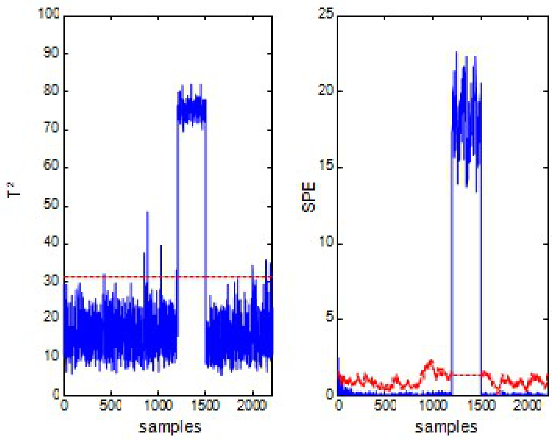

FIGURE 8. Monitoring by recursive DPCA with conventional indices

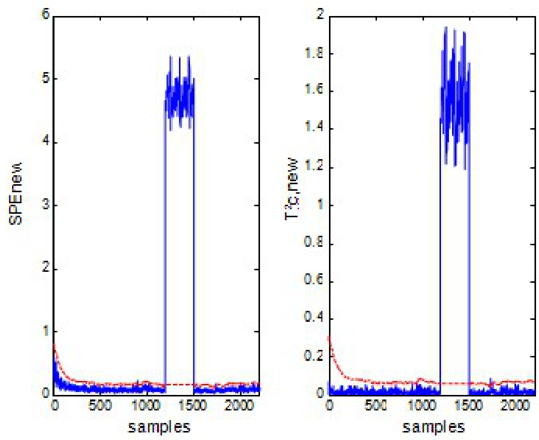

FIGURE 9. Monitoring by recursive DPCA with new indices

4. **Conclusions.** A recursive Dynamic PCA (RDPCA) based monitoring scheme for Imperial Smelting Process (ISP) is proposed to adapt the process changes. It includes a FOP based recursive PCA algorithm and two new statistics indices. The applications of the proposed scheme to ISP demonstrate the feasibility and effectiveness of the recursive algorithms for adaptive process monitoring. The algorithm significantly reduces the online computation cost, and have been derived based on rank-one matrix update of the covariance matrix. Because most industrial process experience slow and normal changes such as equipment aging, sensor drifting and periodic cleaning, the proposed adaptive monitoring scheme in this paper is expected to have wide applications in industry.

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