RESEARCH ON OBJECTIVE SORTING ALGORITHM OF GEO COMMUNICATION SATELLITE BASED ON FUZZY THEORY

CHENGSHENG PAN¹,², NA WANG¹,³, HAIYAN LIU¹,² AND XIULI DU²,³

¹University Key Laboratory of Communication and Signal Processing
²Key Laboratory of Communication Networks and Information Processing
³College of Information Engineering
Dalian University
No. 10, Xuefu Street, Jinzhou Xinqu, Dalian 116622, P. R. China
pcs@dlu.edu.cn

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ABSTRACT. GEO (Geostationary Earth Orbit) communication satellites are important guarantee for the war. We should research the sorting for attack if we want to win the war. Performance parameters of GEO communication satellites are analyzed, and definitions of threat index are established. Subsequently, objective sorting algorithm of GEO communication satellites based on fuzzy theory is proposed, and a mathematical model is established. Processing factors by the fuzzy theory, determining the weight of each threat index, obtaining integrated values of the satellite threat, and the satellites sorting are also included in this algorithm. The simulations in weights completely unknown and partly unknown are shown that the proposed algorithm has high intelligence and provides referenced value for anti-satellite campaign in space.

Keywords: GEO communication satellite, Threat index, Objective sorting

1. Introduction. Space environment is playing more and more important role in the modern war. Therefore, all great countries have paid high attention to anti-satellite. It is critical to evaluate the satellite threat and determine the sorting for attack, which would guarantee the final victory. GEO communication satellites can provide data transmission for the war, such as command issuance, information feedback and image transmission, which has cardinal significance to the war. For more results on this topic, we refer readers to [1,2]. Therefore, we should reduce enemy capability of tactic guarantee recurring to GEO communication satellite. If there is a communication system constituted by several satellites, we need to evaluate threat capability of each satellite, and then make sure the sorting. However, it is difficult to identify the threat factors even if the parameters are abundant for their incompatibility, so it is difficult to make sort of threat degree for space war. For more results on this topic, we refer readers to [3]. This problem belongs to multi-objective decision-making. To solve it we need to do research on objective sorting of satellites, which depends on establishing a reasonable mathematical model to calculate the integrated value of satellite threat that based on definition of each threat index, and then the sorting can be got by the integrated values. While the sorting is a complex task, many factors that affect sorting can be considered, such as satellite type, coverage, channel capacity, working frequency, data rate, communication type. In principle, the influence degree of these factors is different, and these factors are not only quantitative, but also qualitative. Accordingly, we adopt an approach with quantitatively analyzing and qualitatively analyzing. Quantitative analysis is to quantize the relevant factors directly, while qualitative analysis is to evaluate factors with fuzzy theory which cannot be quantized directly, which remains objective and fair without subjectivity and
unilateralism. In order to synthesize these factors, the sorting algorithm of satellite threat based on fuzzy theory is proposed, and the weights are determined by subjective or objective methods. Ultimately, the sorting can be obtained in the two different cases in which the weights are completely and partly unknown.

2. Threat Index Model Based on Fuzzy Theory. There are too many factors that affect threat sorting of GEO communication satellites, but considering the threat evaluation and the main performance parameters of satellite, the threat indexes include 4 parts, there are type, coverage, channel capacity and working frequency.

2.1. Type threat index. GEO communication satellites can be divided into relay satellites and non-relay satellites. For more results on this topic, we refer readers to [4]. Relay satellites are also called “satellites’ satellites”, which can provide monitoring services and data relay for other satellites and spacecraft, and greatly improve efficiency and emergency capacity. The definition of the type threat index is shown in Table 1.

2.2. Coverage threat index. The coverage of GEO satellite is 42.4% earth’s surface, and the longitude span is about 162.62°. For more results on this topic, we refer readers to [5]. This threat index is the longitude span’s percentage that the GEO satellite covered important area, and its definition is shown in Formulae (1) and (2).

\[ T_c = \sum_{i=1}^{n} T_{c_i} \]  

where \( T_{c_i} \) is the coverage threat index for an important area.

\[ T_{c_i} = \begin{cases} 
0 & x \in Z_4 \\
\frac{x - (a - 81.31)}{\text{mod}360} & x \in Z_2 \\
\frac{-x + (b + 81.31)}{\text{mod}360} & x \in Z_3 \\
\frac{b - a}{1} & x \in Z_1 
\end{cases} \]  

(2)

where \( x \) is the longitude where the GEO communication satellite lies in, \((a, b)\) is the longitude span where the important area are covered.

2.3. Channel capacity threat index. Channel capacity is referred to the maximum of information that the channel can transmit. For more results on this topic, we refer readers to [6]. This threat index is a sum, and definition is shown in Formula (3).

\[ T_v = \sum_{i=1}^{n} v_i \]  

(3)

where \( v_i \) is the maximum transmission rate of the channel \( i, i \in (1, n) \).

Table 1. Type threat index

<table>
<thead>
<tr>
<th>Satellite’s type</th>
<th>Relay</th>
<th>Non-relay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threat index ( T_t )</td>
<td>( T_{t_1} )</td>
<td>( T_{t_2} )</td>
</tr>
</tbody>
</table>
Table 2. Working frequency threat index

<table>
<thead>
<tr>
<th>Working frequency</th>
<th>UHF</th>
<th>SHF</th>
<th>EHF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threat index (T_f)</td>
<td>(T_{fU})</td>
<td>(T_{fS})</td>
<td>(T_{fE})</td>
</tr>
</tbody>
</table>

There are some satellites which not just work in one frequency, so that the additive method is used directly to calculate this threat index. Ultimate definition is shown in Formula (4).

\[
T_f = T_{fU} + T_{fS} + T_{fE}
\]  

3. Multi-objective Sorting Model. The sorting of satellite threat belongs to multi-objective sorting problem. In order to solve this problem, the sorting algorithm based on fuzzy theory is proposed, which includes six steps, and flow chart of the algorithm is shown in Figure 1.

3.1. Factors normalization. All factors are normalized in order to be compared in the same magnitude.

3.1.1. The objective decision-making matrix \(A\). There are limited decision schemes for multi-objective decision-making, and objective space can be denoted by decision-making matrix \(A\).

\[
A = \begin{pmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}
\]  

Decision space is constituted by limited decision variables in \(A\). Namely, \(X = \{x_1, x_2, \ldots, x_n\}\), \(x_j (j = 1, 2, \ldots, n)\) is variable set. Specifically, whether the scheme variable is one-dimensional or multi-dimensional is determined by practical problem, \(n\) is the number of variables. \(a(x_j) = (a_1(x_j), a_2(x_j), \ldots, a_m(x_j))^T, j = 1, 2, \ldots, n\) is the objective value of \(x_j\), note \(a_{ij} = a_i(x_j), (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)\), \(a_{ij}\) is the \(i\)th objective value of scheme \(x_j\).
3.1.2. The objective relative membership degree matrix. Before making decision we should normalize the factors. According to the different types, different methods can be used to normalize the factors. Usually, there exist four types, which are benefit, cost, fixed and interval, $a_1, a_2, a_3$ and $a_4$ belong to the type. That is, the bigger, the better. Therefore, $r_1, r_2, r_3$ and $r_4$ are determined by benefit.

To benefit, the relative membership degree can be given.

$$r_{ij} = \frac{(a_{ij} - a_{i \text{min}})}{(a_{i \text{max}} - a_{i \text{min}})}P_i$$

where $P_i$ is decision-makers parameter; the above formula's prerequisite is $a_{ij} \geq 0, (i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)$. Note $a_{i \text{max}} = \max_{1 \leq i \leq n} \{a_{ij}\}, a_{i \text{min}} = \min_{1 \leq i \leq n} \{a_{ij}\}$.

By calculating the relative membership degree, the decision matrix $A$ can be transformed into the objective relative membership degree matrix $R$.

$$R = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{pmatrix}$$

3.2. Principles and methods to establish objective weights. Assume, without loss of generality, that weights of the first $s$ objectives are unknown, while the rest ($m - s$) are already given, namely

$$\begin{cases} \omega_i = \omega_i^* \geq 0 & (i = s + 1, s + 2, \ldots, m) \\ \sum_{i=s+1}^{m} \omega_i^* \leq 1 \end{cases}$$

Note

$$\omega = (\omega_1^T, \omega_{s+1}^*, \omega_{s+2}^*, \ldots, \omega_m^*)^T$$

$$\varpi = (\omega_1, \omega_2, \ldots, \omega_s)^T$$

$$g = 1 - \sum_{i=s+1}^{m} \omega_i^*$$

$$h = \sum_{i=1}^{s} \omega_i^2$$

Thus, when the objective scheme $x_j \in X$, the linear weighted comprehensive value of the relative membership degree defined as

$$\rho_j(\varpi) = \sum_{i=1}^{m} \omega_i r_{ij} \quad (j = 1, 2, \ldots, n) \quad \omega \geq 0 \quad (i = 1, 2, \ldots, s)$$

The nonlinear programming problem can be conveniently solved by the method of Lagrange function. A Lagrange function can be constructed as follows:

$$L(\varpi, \lambda) = \sum_{j=1}^{n} \rho_j(\varpi) / n + \lambda \left( \sum_{i=1}^{s} \omega_i^2 - h \right)$$

where $\lambda$ is a Lagrange multiplier.

After calculating we can obtain

$$\omega_i = \frac{g \sum_{j=1}^{n} r_{ij}}{\left( \sum_{i=1}^{s} \sum_{j=1}^{n} r_{ij} \right)} \quad (i = 1, 2, \ldots, s)$$
Table 3. Parameters of the satellite

<table>
<thead>
<tr>
<th>Objective Satellite type</th>
<th>Longitude (°)</th>
<th>Channel capacity (MHz)</th>
<th>Working frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Relay E10</td>
<td>100 * 64</td>
<td>SHF</td>
</tr>
<tr>
<td>2</td>
<td>Non-Relay W12.9</td>
<td>120 * 125</td>
<td>SHF</td>
</tr>
<tr>
<td>3</td>
<td>Relay W135.5</td>
<td>80 * 256</td>
<td>EHF</td>
</tr>
<tr>
<td>4</td>
<td>Non-Relay E55.3</td>
<td>30 * 125</td>
<td>SHF, EHF</td>
</tr>
<tr>
<td>5</td>
<td>Non-Relay W72.6</td>
<td>50 * 64</td>
<td>UHF, EHF</td>
</tr>
<tr>
<td>6</td>
<td>Relay W8.7</td>
<td>60 * 125</td>
<td>SHF</td>
</tr>
<tr>
<td>7</td>
<td>Non-Relay E175.2</td>
<td>100 * 125</td>
<td>SHF</td>
</tr>
<tr>
<td>8</td>
<td>Non-Relay E160.1</td>
<td>80 * 64</td>
<td>UHF, EHF</td>
</tr>
</tbody>
</table>

Table 4. Threat index

<table>
<thead>
<tr>
<th>Threat index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tt</td>
<td>0.6000</td>
<td>0.4000</td>
<td>0.6000</td>
<td>0.4000</td>
<td>0.4000</td>
<td>0.6000</td>
<td>0.4000</td>
<td>0.4000</td>
</tr>
<tr>
<td>Tc</td>
<td>0.3630</td>
<td>0.4774</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.4037</td>
<td>1.0304</td>
<td>1.0115</td>
</tr>
<tr>
<td>Tf</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.5000</td>
<td>0.8000</td>
<td>0.7000</td>
<td>0.3000</td>
<td>0.3000</td>
<td>0.7000</td>
</tr>
</tbody>
</table>

Thus, all the weights of the factors are determined.

3.3. Fuzzy multi-objective sorting. The integrated value of the satellite threat is calculated by linear weighted method. This method makes satellite threat index multiplied by the corresponding weights directly, then summed. The total threat index model as follows:

\[ Q_k = \omega_1 r_{1k} + \omega_2 r_{2k} + \omega_3 r_{3k} + \omega_4 r_{4k} \]  

where \( Q_k \) is the integrated value of the satellite threat; \( r_{1k}, r_{2k}, r_{3k}, \) and \( r_{4k} \) is type, coverage, channel capacity, and working frequency threat index of the \( k \)th satellite respectively; \( \omega_1, \omega_2, \omega_3, \omega_4 \) are the weights.

Calculate all integrated values of satellite threat \( Q_k \), and the bigger \( Q_k \), the greater the threat degree. So we can determine the sorting of satellite threat.

4. Simulation Results and Analysis. We have a warfare supposition which involves enemy, our army and the third side. Currently, there are several relative GEO communication satellites participating in the war, which may have threat. The relational parameters of each satellite are shown in Table 3. Latitude span of important areas are selected, they are (73.67°E, 135.05°E), (67°W, 124°W) and (122°E, 154°E). Namely, \{ (a, b) \} = \{ (73.67, 135.05), (67, -124), (122, 154) \}. Threat index of relay and non-relay is given by 0.6 and 0.4 respectively. That are \( Tt_1 = 0.6, Tt_2 = 0.4 \). Threat index of the satellite which is working in UHF, SHF and EHF is given 0.2, 0.3, and 0.5 respectively. Those are \( Tf_U = 0.2, Tf_S = 0.3, Tf_E = 0.5 \). Decision-makers parameter is selected as 1, namely, \( P_i = 1 \). The satellite type, coverage, channel capacity, working frequency threat index are shown in Table 4.

We can obtain relative membership degree matrix \( R \):

\[
R = \begin{bmatrix}
1.0000 & 0 & 1.0000 & 0 & 0 & 1.0000 & 0 & 0 \\
0 & 0.1714 & 0.9544 & 0.9544 & 0.9544 & 0.0610 & 1.0000 & 0.9717 \\
0.1852 & 0.6829 & 1.0000 & 0.0318 & 0.2488 & 0.5382 & 0.1111 \\
0 & 0 & 0.4000 & 1.0000 & 0.8000 & 0 & 0 & 0.8000
\end{bmatrix}
\]
4.1. The weights are completely unknown. In this case, \( s = 4 \), \( g = 1 \). According to Formula (11), weights of the threat index can be calculated as follows: \( \omega_1 = 0.2164 \), \( \omega_2 = 0.3655 \), \( \omega_3 = 0.2018 \), and \( \omega_4 = 0.2164 \). Thus, the weights are determined completely, \( \omega = (0.2164 \ 0.3655 \ 0.2018 \ 0.2164) \). According to Formula (12), calculate the integrated value of the satellite threat, which is, \( Q = (0.2537 \ 0.2004 \ 0.8535 \ 0.5716 \ 0.5219 \ 0.2889 \ 0.4741 \ 0.5506) \). Sorting result is shown in Figure 2. The sorting result is \( x_3 > x_4 > x_8 > x_5 > x_7 > x_6 > x_1 > x_2 \).

4.2. The weights are partly unknown. When weights are not completely known, we assume that the weights of \( Tt \) and \( Tc \) are known, namely, \( \omega_1 = 0.1 \), and \( \omega_2 = 0.4 \), while the rest weights are unknown. In this case, \( s = 4 \), \( g = 1 \). According to Formula (11), we can obtain the rest weights by satellite relative membership degree matrix \( R \), \( \omega_3 = 0.2413 \), and \( \omega_4 = 0.2587 \). Thus, the weights are completely determined, \( \omega = (0.1000 \ 0.4000 \ 0.2143 \ 0.2587) \). According to Formula (12) calculate the integrated value of the satellite threat, \( Q = (0.1447 \ 0.2333 \ 0.8265 \ 0.6481 \ 0.5887 \ 0.1844 \ 0.5299 \ 0.6224) \). Sorting result is shown in Figure 3, and the sorting result is \( x_3 > x_4 > x_8 > x_5 > x_7 > x_2 > x_6 > x_1 \).

This method can be applied to two cases that weights completely unknown and partly unknown, and the weights are obtained by objective data or decision-makers’ preference. According to the above two cases, the final weights are different, so the sorting result is not identical.
5. **Conclusions.** After analyzing threat index of GEO communication satellite, all factors can be compared in the same magnitude by normalization. Furthermore, the method that combines subjective with objective is used in weights calculation, and to a certain extent avoids the shortage that mere using subjective or objective method. Lastly, the optimal weights can be obtained by Lagrange function, and the sorting result based on fuzzy theory can also be determined. Simulation results show that the algorithm can determine the threat degree of GEO communication satellite, and can achieve the desired result.

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