International Journal of Innovative Computing, Information and Control
Volume **9**, Number **1**, January **2013**
ICIC International ©2013 ISSN 1349-4198
pp. 139–152

ROBUST H_∞ CONTROL WITH SELECTION OF SITES FOR APPLICATION OF DECENTRALIZED CONTROLLERS IN POWER SYSTEMS

Alexandre Cézar de Castro[1], Clivaldo Silva Araújo[1]
José Mário Araújo[2], Eduardo Telmo Fonseca Santos[2]
and Fernando Augusto Moreira[3]

[1]Departamento de Engenharia Elétrica
Universidade Federal da Paraíba
Campus Universitário I, S/N, João Pessoa-PB, 58051-900, Brazil
{ castro; clivaldo }@cear.ufpb.br

[2]Departamento de Tecnologia em Eletro-Eletrônica
Instituto Federal de Educação, Ciência e Tecnologia da Bahia
Rua Emídio dos Santos, S/N, Barbalho, Salvador-BA, 40301-015, Brazil
{ jomario; eduardot }@ifba.edu.br

[3]Departamento de Engenharia Elétrica
Universidade Federal da Bahia
Rua Aristides Novis, 2, Federação, Salvador-BA, Brazil
moreiraf@ufba.edu.br

Received October 2011; revised February 2012

Abstract. *Eigenanalysis and frequency domain techniques are used for selection of the best sites for application of controllers in multivariable power systems and to design robust H_∞ controllers, which are constrained to be decentralized with reduced order and fixed structure. Parameters of the controllers are tuned by using a reliable optimization method to result into a robust control in damping of the critical oscillation modes. Interactions among controllers are analyzed. A good coordination of control is achieved by simultaneous design and application of all controllers in the selected sites. The proposed techniques are applied to design H_∞-PSS in a multimachine power system.*
Keywords: Power systems, Singular values, Robust control, Decentralized control, Frequency response, Interactions, Optimization

1. **Introduction.** The electric power systems are generally large and complex with units separated by hundreds of kilometers. A common concern in these systems is about the electromechanical oscillations, which need controllers to be damped [1,2]. In power systems, some electromechanical oscillation modes (EOM) are usually weakly damped and a certain number of generators and FACTS (Flexible AC Transmission Systems) should be used for application of these controllers to damp the EOM. The places where the controllers will be applied must be previously selected. The problem of coordinated application of these controllers has been usually treated with the use of eigenvectors [1].

The analysis of the critical EOM in power systems is usually difficult due to the structure of the system, in which the EOM result with characteristics and complex interactions. Thus, a controller implemented in a generator can affect a number of EOM and an EOM can be observable in a group of generators, however, more controllable in other generators. Therefore, the preliminary analysis of controllability and observability of the EOM and their interactions is essential for the selection of generators for more effective application of controllers [3-5].

Traditional techniques used to design these controllers do not take into account the uncertainties due to changes in operating conditions, the neglected dynamic components in the model, the nonlinearities, etc. Since the last decade, some researchers have concerned with the application of robust controllers to damp the EOM on all common operating conditions [6,7].

The technical design of robust H_∞ controllers is based on the system represented in the time domain [8] or in the frequency domain [9]. The resulting controllers are centralized with orders higher than those of the systems themselves, not allowing the direct application of these techniques in power systems with multiple units. It is known, however, that for a controller system becomes well accepted by engineers of power systems and even other applications, it should be decentralized, low order, simple structure and easy of tuning. To meet these requirements, the order of the system model is first reduced and then after design the controller order is also reduced [10]. In these techniques, to design decentralized controllers, the project is done individually for each controller [10]. However, the controller designed for the reduced order model may not be robust or may even make the system unstable [11].

In order to achieve robust control with decentralization constraints, with low order controllers and with a reduced number of controllers, it is required to perform the design of the control structure in advance, which consists of selecting the best signals and locations for implementation of controllers and selection of control configuration, and in the particular case of this article, control is decentralized with power system stabilizers (PSS) [12,13] to control the excitation of generators. The proposed techniques can be applied also in the design of FACTS, SVC (Static Var Compensator), etc., which are commonly used to dampen the inter-area EOM [10]. Conventional PSS has been successfully applied for decades to damp all types of EOM [14]. However, the application of these stabilizers has been performed with independent or sequential designs, where the interactions are not taken into account. Although satisfactory results could be obtained, the stabilizers designed this way are not necessarily robust.

In this paper, a new procedure for analysis and coordinated application of PSS in a robust power system of multiple generators is proposed. Modal and frequency domain techniques are used for reliability analysis and design of PSS. The use of singular values and a method of optimization results decentralized PSS with configuration previously established to be similar to the stabilizers found in power industry. For a good control coordination, PSS are all fitted simultaneously to take into consideration the effects of interactions between generators and stabilizers.

The analysis of interactions in the frequency domain, which is a contribution of the paper, is performed in the selection of generators to have controllers applied. It is shown to be better than the usual technique using eigenvectors alone [4,15].

2. **Electromechanical Mode Analysis.** The electromechanical oscillations are common and critical in the interconnected power systems [15]. Two types of EOM are of greater interest: *local* EOM, with frequencies typically between 0.8 to 1.8 Hz. Usually, a local EOM is strongly controllable and strongly observable in a single generator and *inter-area* EOM have typical frequencies in the range of 0.1 to 0.8 Hz. Inter-area oscillation is a complex phenomenon involving many generators of different areas. An inter-area EOM can be moderately observable and moderately controllable in different areas or weakly observable and strongly controllable in one area and strongly observable and weakly controllable in another area. In addition, the characteristics of observability and controllability of an inter-area EOM are different in each generator of one area.

Only using modal analysis it is possible to identify the different characteristics of EOM and its observability and controllability. Two types of modal analysis are suggested in the following, aiming the selection of generators for application of controllers.

2.1. **Modal analysis using eigenvectors.** The nominal model of a power system composed of p generators is represented in linearized form, in the time domain by

$$\dot{x} = Ax + Bu \tag{1}$$

$$y = Cx \tag{2}$$

The transient response to a vector of step-type inputs is described by

$$y_p = \sum_{i=1}^{n} \left(\frac{C g_i v_i^T B e^{\lambda_i t}}{\lambda_i} \right) u_p \tag{3}$$

where n is the system order, and g_i e v_i are the right and left eigenvectors, respectively, associated to the eigenvalue λ_i and T indicates transposition of vector.

The matrix, $p \times p$, $R_i = C g_i v_i^T$ from Equation (2) represents the coupling between the input vector and the output vector, through the i-th mode. R_i is called the "matrix of residue" of i-th mode. The generic element of R_i, r_{jk}, represents the coupling of the k-th input with i-th mode and j-th output. If the modulus of r_{jk} is high, it means that y_j, u_k and the i-th mode are strongly coupled. Considering that only decentralized PSS are of interest, only the diagonal elements of each matrix R_i are needed for analysis of residues. The residue r_{kk} with high modulus is desirable, but not sufficient. The ideal is to have r_{kk} associated with the EOM of interest with high modulus and favorable interactions with other EOM [4].

The residue r_{kk} indicates both controllability and observability of the mode in the generator k. It is of interest in this study to separately analyze the observability and controllability of the EOM in each generator. It appears that elements with high modulus in a column of R_i means that the corresponding input is very effective to control the i-th mode and that elements with high modulus in a row i of R_i means that the i-th mode is strongly observable in the corresponding output. Thus, two coupling factors in the following will be used for the analysis:

$$I_{oj}^i = \sqrt{\sum_{k=1}^{p} r_{jk} r_{jk}^*} \quad \text{and} \quad I_{oj}^i = \sqrt{\sum_{j=1}^{p} r_{jk} r_{jk}^*}$$

where $(.)^*$ denotes complex conjugate. I_{oj}^i represents the coupling of the output j with all inputs through i-th mode, i.e., it is a measurement of the observability of i-th mode in the j-th response. Similarly, I_{ck}^i is a measure of controllability of i-th mode by k-th input [19].

Modal techniques that use eigenvectors treat each mode separately, without taking into account the beneficial and adverse interactions with other modes. In fact, a PSS applied to a generator can affect various EOM and can cause significant effects in another generators.

2.2. **Modal analysis in frequency domain.** The power system of Equation (1) is described by

$$y(j\omega) = G(j\omega) u(j\omega) \tag{4}$$

where $G(j\omega)$ is the transfer function matrix of frequency responses (MFTfr).

For analysis of multivariable systems, the singular values of the MFTfr are used. The singular values of interest are: the maximum singular value defined by $\overline{\sigma}(G)$ and the

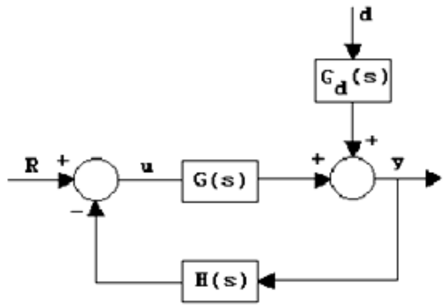

FIGURE 1. Power system with controllers

minimum, defined by $\underline{\sigma}(G)$. The retio $\gamma(G) = \frac{\overline{\sigma}(G}{\underline{\sigma}(G)}$ is defined as the *condition number*. The following properties of interest are described [15,17]:

1. $\overline{\sigma}$ in the frequency of an EOM represents the degree of observability of the mode in system response and $\underline{\sigma}$ represents the degree of controllability of the mode. EOM weakly damped and strongly observable in the response signals show large peaks in the graphs of $\overline{\sigma}(j\omega)$;
2. A depression in the graph of $\underline{\sigma}$ indicates the existence of a complex zero in the system with significant effect on the response and possibly on the controller design;
3. High condition number ($\gamma > 10$) indicates control difficulty, mainly if $\underline{\sigma}(G) \ll 1$.

The effect of $\underline{\sigma}(G)$ in the system performance with respect to the variation in the reference and disturbance rejection is studied, considering the power system $G(s)$ with controllers $H(s)$, reference inputs R and disturbances d, as shown in Figure 1.

Considering $G_d = I$, the following relationship is obtained

$$y = (I + GH)^{-1}GR + (I + GH)^{-1}d \tag{5}$$

where $S = (I + GH)^{-1}$ is the matrix of sensitivity and $T = SG$ is the transfer function matrix of the closed-loop system.

Consider a variation in the reference R, assuming $d = 0$. Then, results $\frac{\|y\|}{\|R\|} = \sigma(\bar{T}) \leq \frac{\bar{\sigma}(G)}{\underline{\sigma}(GH+I)}$. Knowing that $\underline{\sigma}(I + GH) \geq \underline{\sigma}(GH - I)$ and that $\underline{\sigma}(GH) \geq \underline{\sigma}(G)\underline{\sigma}(H)$, results that $\bar{\sigma}(T) \leq \frac{\bar{\sigma}(G)}{\underline{\sigma}(G)\underline{\sigma}(H)-1}$. Similarly, considering only the effect of disturbance on the output, it appears that $\frac{\|y\|}{\|d\|} = \overline{\sigma}(S) \leq \frac{1}{\underline{\sigma}(G)\underline{\sigma}(H)-1}$. These results show that $\underline{\sigma}(G)$, which depends on the selection of inputs and outputs, must be large to reduce $\overline{\sigma}(T)$ and $\overline{\sigma}(S)$ and hence the effect of disturbances.

3. **Modal Interactions.** In the design of decentralized stabilizers some requirements need to be satisfied to have good coordination of the various EOM control, without affecting the damping of the "excitation modes" that are associated with fields and excitation systems of generators, whose dampings rapidly decreases with the increase in the PSS gains. Thus, it is required that the generators selected for the application of PSS have high values of $\overline{\sigma}(G(j\omega))$ in the frequency range of the EOM and adverse interactions that can interfere with the damping of some EOM, hampering the control coordination should be avoided, in order to satisfactorily damp all critical EOM.

It is known that a power system stabilizer, tuned to dampen a local EOM may decrease the damping of inter-area modes. In [4], it was pointed out that the arguments of the residue associated with the EOM must be analyzed to identify possible adverse interactions that can be compensated with modifications in the controller structure. With the proposed technique of eigenvectors analysis it can be verified that a stabilizer applied to a generator can affect many modes, indicating "modal interactions", but one cannot distinguish whether they are beneficial or adverse interactions.

Here we propose a new frequency domain technique for identifying generators where controllers may cause adverse interactions. Thus, to identify whether a generator can cause beneficial or adverse interactions, it is considered that $\overline{\sigma}(G_a)$ and $\underline{\sigma}(G_a)$ are the main singular values of a certain group of generators and that the generator g is included to this group. Let one consider this group $(G_a + g)$ called the group $G_{b.}$. Thus, if $\overline{\sigma}(G_b) > \overline{\sigma}(G_a)$ and $\underline{\sigma}(G_b) > \underline{\sigma}(G_a)$, the generator g may cause favorable interactions in observability and controllability. If $\overline{\sigma}(G_b) < \overline{\sigma}(G_a)$, the generator g may cause adverse interactions on the observability and if $\underline{\sigma}(G_b) < \underline{\sigma}(G_a)$ the generator g may cause adverse interactions in controllability.

Remembering that:

i) Lower $\underline{\sigma}(G)$ means that there will be a need for greater control effort, requiring greater gains for PSS with decreased the damping and even destabilization of excitation modes;

ii) $\overline{\sigma}(G)$ should be high in the frequencies of EOM in order that these modes can be well observable on the feedback signals;

iii) Lower $\underline{\sigma}(G)$ means that the system sensitivity to disturbances is increased;

iv) If $\underline{\sigma}(G) \ll 1$ in the frequency range of the EOM, it will be almost impossible to achieve robust control for a system with decentralized control.

Thus, generators that can cause adverse interactions in controllability should not be considered for application of PSS.

In power systems with many generators, usually there are some critical EOM that can be damped with a limited number of decentralized controllers. However, it is not possible to associate any EOM to only one specific generator.

The analysis of coupling between the controlled and uncontrolled generators can explain why the implementation of controllers in some generators can dampen the critical EOMs, even the ones more associated with other generators.

Consider the complete system of Equation (3), represented by

$$\begin{bmatrix} y_1(j\omega) \\ y_2(j\omega) \end{bmatrix} = \begin{bmatrix} G_{11}(j\omega) & G_{12}(j\omega) \\ G_{21}(j\omega) & G_{22}(j\omega) \end{bmatrix} \begin{bmatrix} u_1(j\omega) \\ u_2(j\omega) \end{bmatrix} \tag{6}$$

The indexes 1(2) denote the group of selected (not selected) generators for application of controllers. Thus, consider $u_1 = -Hy_1$. Then $y_1 = (I + G_{11}H)^{-1}G_{12}u_2$ and $y_2 = G_{22}u_2 - G_{21}Hy_1$. Substituting y_1 in this second equation, results that $y_2 = G'_{22}u_2$ where $G'_{22} = G_{22} - G_{21}H(I + G_{11}H)^{-1}G_{12}$. Now, assuming that $\underline{\sigma}(G_{11}H) \gg 1$ in the frequencies of critical EOM, results that:

$$\overline{\sigma}(G_{22}) - \overline{\sigma}(G'_{22}) \leq \frac{\gamma\overline{\sigma}(G_{21})\overline{\sigma}(G_{12})}{\underline{\sigma}(G_{11})} \tag{7}$$

The second member of Equation (6) represents the impact of controllers H in the output vector y_2.

Finally, assuming that with the effect of control, the peaks in the frequencies of critical EOM are so small that $\overline{\sigma}(G_{22}) - \overline{\sigma}(G'_{22}) \approx \overline{\sigma}(G_{22})$, then from Equation (6) results:

$$\overline{\sigma}(G_{22})\underline{\sigma}(G)_{11} \leq \gamma\overline{\sigma}(G_{21})\overline{\sigma}(G_{12}) \tag{8}$$

In these equations $\overline{\sigma}(G_{21})\overline{\sigma}(G_{12})$ represents the coupling between groups 1 and 2 of generators. This coupling is responsible for interactions between these two groups. It seems clear that the controllers applied in group 1 can only damp one EOM that is more associated with the group 2 if the coupling is strong between these groups in the frequency ω_i of mode. Thus, from Equation (7), if $\overline{\sigma}(G_{22}(j\omega_i)) \gg 1$ (high observability of the mode in group 2) and $\underline{\sigma}(G_{22}(j\omega_i)) \gg 1$ (good controllability of the mode in group 1), then $\overline{\sigma}(G_{21}(j\omega_i))\overline{\sigma}(G_{12}(j\omega_i))$ (strong coupling). In this situation, the controllers applied on the group 1 can damp the EOMs that are more associated with the generators of the group 2. On the other hand, if the coupling is weak and the EOM is observable in group 2 ($\overline{\sigma}(G_{22}) \gg 1$), then $\underline{\sigma}(G_{11}) \ll 1$ and thus the mode cannot be controlled by the controllers of group 1. Thus, in order to controllers of group 1 control the EOM more associated with group 2, which includes generators that can cause adverse interactions, these modes must be controllable in group 1 and observable in group 2.

4. **Site Selection for Descentralized Controller Application.** In this paper, the site are generators more effective in the excitation control. Only the generators that can cause favorable interactions must be considered for possible application of PSS.

It is known that modern excitation systems with fast response and higher gains can deteriorate the damping of the EOM, yet they are the most effective for the application of controllers to damp these modes. It is also known that small generators are inefficient to damp the EOMs. Thus, the larger generators, with modern excitation systems with fast responses and higher gains, must be considered in the selection procedure.

In practice, it appears that a number of decentralized controllers equal to the number of critical EOM is usually sufficient to damp these modes [17].

The procedure for selection of generators consists of

1. *Identification of generators that can cause adverse interactions*: The identification of these generators is performed considering an initial group of two generators with good observability and good controllability in the frequency range of all critical EOM. These generators can be selected with the analysis of eigenvectors and residues. The other generators are included one by one to this group. The generators that cause decrease in $\underline{\sigma}$ are those which may cause adverse interactions and are therefore discarded;
2. *Final selection of the generators*: From the remaining generators, it is selected among the large generators with excitation systems having fast responses and high gains associated with the critical EOM, a group consisting of a number of generators equal to the number of these modes and with higher $\underline{\sigma}(G_{11}(j\omega))$ in the frequency region of the EOM. If necessary, other generators can be included to this group.

5. **Robust $\boldsymbol{H_\infty}$ Control Using Descentralized Controllers.** The controller has a fixed structure in the form $H(s) = diag(h_1(s), \ldots, h_p(s))$. The controllers $h_i(s)$ are designed simultaneously, taking into consideration the unstructured uncertainties of the model that include modeling errors, changes in operating conditions, exclusion of dynamics, nonlinearities, etc. The representation of uncertainties is shown in Figure 2.

In Figure 2, Δ is a matrix that includes all uncertainties of the system. The robustness can only be achieved for limited uncertainties. Thus, it is established that $\overline{\sigma}(\Delta) \leq 1$. The uncertainties are properly weighted with the functions $W_1(s)$ and $W_2(s)$ that are stable matrices that characterize the spatial and frequency structure of uncertainties. $W_1(s)$ somehow represents the magnitude of uncertainties, defining its limits. $W_2(s)$ represents the variation uncertainties with frequency, which have small values at lower frequencies and increase at higher frequencies. In fact, the construction of $W_1(s)$ and $W_2(s)$ for multivariable systems is not trivial. The usual procedure [17,18] that is adopted in this paper considers: $W_1(s) = \omega_1(s)I$ and $W_2(s) = \omega_2(s)I$ with $\omega_2(s) = \frac{\tau s+\tau_0}{\frac{\tau}{\tau_\infty}s+1}$, where $\frac{1}{\tau}$ is approximately the frequency at which the relative uncertainty reaches 100%, τ_0 is the

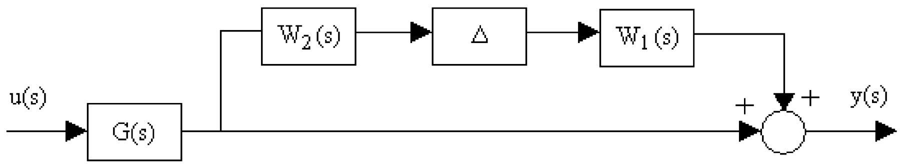

FIGURE 2. System model with uncertainties at output

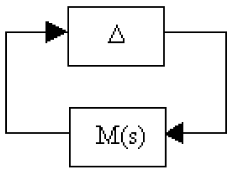

FIGURE 3. System structure M-Δ

relative uncertainty at steady state, τ_∞ is the weight magnitude in high frequencies and $\omega_1\omega_2$ is the unique upper limit associated to all control channels [17,19].

The goal is to design controllers to stabilize not only the nominal plant $G(s)$, but the set of all plants defined by $G'(s) = [I + W_1\Delta(s)W_2(s)]\, G(s)$. The sensitivity matrix of the real system is $S = (I + GH)^{-1}$. Thus $S' = (I + (I + W_1\Delta W_2)GH)^{-1}(I + GH)$. Assuming that $\underline{\sigma}(GH) \gg 1$ for the frequencies of EOM, it results that:

$$\overline{\sigma}(S') \leq \frac{\gamma(H)\gamma(G)}{\underline{\sigma}(I + W_1\Delta W_2)}\overline{\sigma}(S) \tag{9}$$

From Equation (8), the controller can limit the deterioration of sensitivity to the uncertainties if $\gamma(H) = 1$, meaning that identical controllers must be applied to all generators. Now, separating the block of the uncertainties, results the model M-Δ of Figure 3, where $M(s) = -\omega_1\omega_2 T(s)H(s)$.

It is assumed that the nominal system $M(s)$ is stable and that the uncertainties Δ are stable. Thus, the real system M-Δ is stable for all uncertainties, satisfying $\overline{\sigma}(\Delta) \leq 1$, if and only if [15]:

$$\overline{\sigma}(M(j\omega)) \leq 1 \quad \forall\omega \tag{10}$$

Thus, to achieve robustness, the parameters of the controllers $h_i(s)$ are tuned between practical limits to solve the following optimization problem:

$$\min\ \sup(\overline{\sigma}(M(j\omega))) \tag{11}$$

Substituting $M(j\omega)$ into Equation (9) results $\overline{\sigma}(M) \leq |\omega_1\omega_2|\overline{\sigma}(T)\overline{\sigma}(H) < 1$, or

$$\overline{\sigma}(T) < \frac{1}{|\omega_1\omega_2|}\frac{1}{\overline{\sigma}(H)} \quad \forall\omega \tag{12}$$

For the particular case of identical controllers, $\overline{\sigma}(H) = \underline{\sigma}(H) = \frac{1}{|h_i(j\omega)|}$. Then, Equation (10) reduces to:

$$\overline{\sigma}(T) < \frac{1}{|\omega_1\omega_2|}\frac{1}{|h_i(j\omega)|} \quad \forall\omega \tag{13}$$

Thus, the most practical procedure for designing robust decentralized controllers consists of adjusting all the $h_i(s)$ to minimize $\overline{\sigma}(T)$, satisfying Equation (10) (general) or Equation (11) (identical controllers).

It is noteworthy that in selecting the generators with $\max(\underline{\sigma}(j\omega))$, the sensitivity is minimized and while minimizing $\overline{\sigma}(T)$ it results small $\gamma(G)$. Thus, the resulting system has

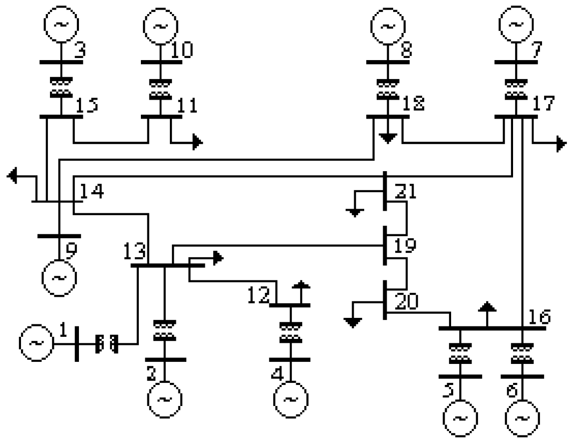

FIGURE 4. Power system configuration

robust stability, low sensitivity to small disturbances and low deterioration of sensitivity by uncertainties, resulting in a good control performance.

It is observed that the objective function $\overline{\sigma}(T)$ to be minimized is not an explicit expression. Thus, an optimization technique to minimize without calculation of function derivatives is recommended. The direct technique of optimization called pattern search, from Hooke and Jeeves [20] is used due to its adaptation to this problem. This direct search technique was successfully tested by Gottfried and Weisman [20] in several test functions with notoriously difficult convergence. The limitation of the technique is that it converges more slowly than the derivative methods. In the case of tuning of PSS, in which adverse reactions are excluded, it can be said that the technique of Hooke and Jeeves is highly reliable.

It is recognized the difficulty of obtaining an H_∞ optimal controller with the available techniques, due to the singularity phenomenon for the factorization technique called J-spectral [9] and due to the state condition fault in H_∞ control problem [18], which occur in the vicinity of the optimal solution in the iterative process of these techniques. These limiting behaviors do not exist in the proposed technique in which an H_∞ optimal controller can be smoothly achieved.

6. **Application to a Power System.** The power system of ten generators shown in Figure 4, will be used for analysis and application of PSS. Full details of this system are presented in [4], and it can also be obtained from the authors. The system is sufficiently large and complex to illustrate the main topics presented in this paper. The generator #10 is a large equivalent machine, representing the remaining system. The speed signals are used as outputs and the input voltages of excitation systems are used as the control inputs. The system without PSS has 36 states.

In Figure 5, the graphs for $\overline{\sigma}(T)$ and $\underline{\sigma}(T)$ of the complete system are presented. The peaks in the graph $\overline{\sigma}(T)$ show that this system has four insufficiently damped EOM. These modes are critical, being called: Mode 1 (0.46 Hz), Mode 2 (0.86 Hz), Mode 3 (0.89 Hz) and Mode 4 (0.92 Hz). The first two modes are inter-area and the last two modes are local ones. In the graph of $\underline{\sigma}(T)$ shown in Figure 5, it appears that the modes 1, 3 and 4 have low controllability in the complete system, anticipating the probable existence of adverse interactions.

For the selection of generators, it will be initially conducted an analysis of the critical EOM. The couplings of these modes with inputs and outputs of the generators and the main residues are respectively shown in Tables 1 and 2.

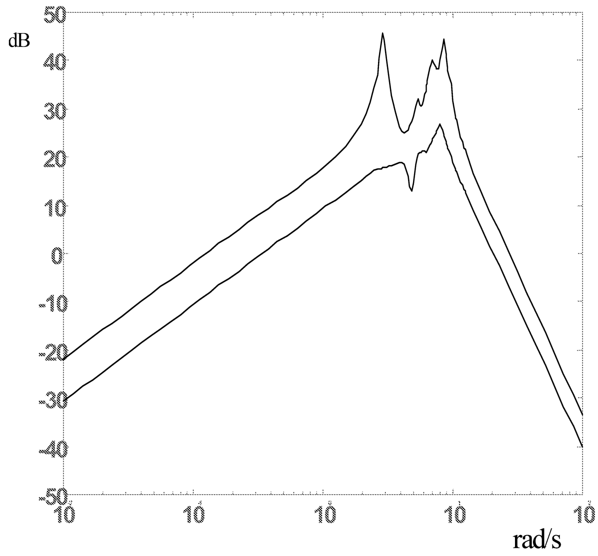

FIGURE 5. Graphs of $\overline{\sigma}(T)$ and $\underline{\sigma}(T)$ for the complete system

TABLE 1. Critical mode coupling of generators

Generator	Mode 1		Mode 2		Mode 3		Mode 4	
	I_{01}	I_{c1}	I_{02}	I_{c2}	I_{03}	I_{c3}	I_{04}	I_{c4}
1	12.6	12.8	6.6	10.6	0.8	1.7	2.0	4.5
2	12.3	16.0	5.9	10.4	0.7	1.7	1.8	4.2
3	6.7	2.5	7.1	4.6	1.0	2.0	10.6	9.8
4	13.8	2.1	11.6	3.2	1.6	0.6	4.5	1.8
5	12.0	22.4	2.1	6.9	0.6	2.4	1.2	0.7
6	12.3	15.8	2.0	6.9	0.6	2.7	1.4	0.9
7	15.7	2.9	12.1	4.5	3.9	3.3	2.6	0.8
8	14.8	3.4	9.4	2.3	6.4	4.5	1.0	0.6
9	11.5	15.3	5.2	12.5	0.5	2.7	1.0	3.7

TABLE 2. Main diagonal element modulus of matrixes R_i

Generator	1	2	3	4	5	6	7	8	9
Mode									
1	4.2	5.2	0.4	0.8	7.1	5.2	1.2	1.3	4.7
2	3.0	2.7	1.4	1.6	0.6	0.6	2.4	0.9	2.8
3	0.2	0.2	0.2	0.1	0.2	0.2	1.6	3.6	0.2
4	0.7	0.6	8.4	0.6	0.1	0.1	0.2	0.1	0.3

Table 1 shows that mode 1 is fairly observable in all generators and more controllable in generators 1, 2, 5, 6 and 9. Mode 2 is more observable in generators 4, 7 and 8 and more controllable in generators 1, 2 and 9. The mode 3 is more observable and more controllable in generators 7 and 8, and the mode 4 is more controllable and more observable in the generator 3.

The joint analysis of observability and controllability from Table 2 reaches similar results. Based on these results, [4] suggested the application of PSS in generators 2, 3, 5 and 8.

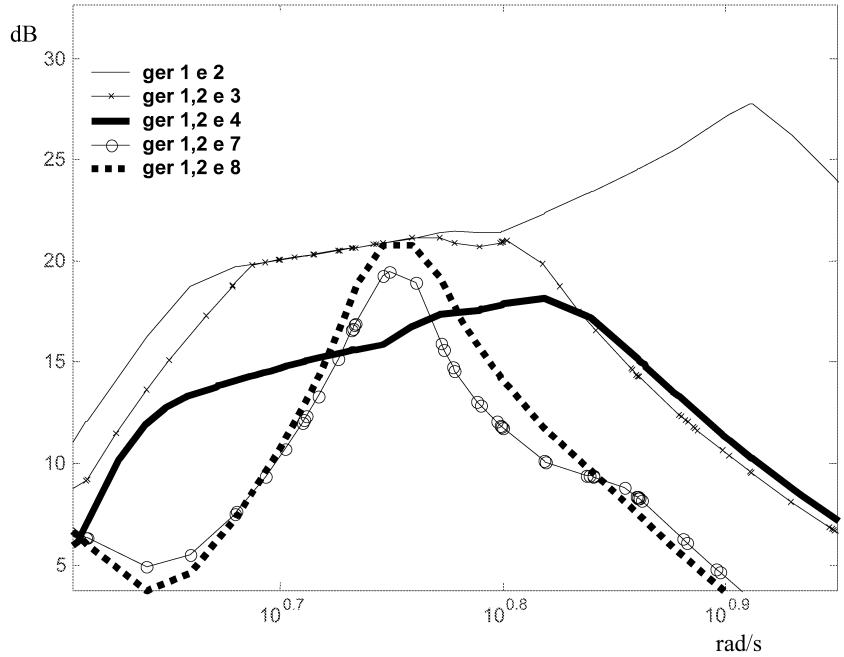

Figure 6. Graphs of $\underline{\sigma}(G)$ showing the generators that cause adverse interactions

Table 3. Values of H for generators, and K_A and T_A for excitation systems

Generator	1	2	3	4	5	6	7	8	9
H(s)	11.0	11.5	10.0	6.4	9.9	9.9	9.0	9.0	15.0
K_A	50.0	50.0	20.0	15.0	100	100	10.0	10.0	50.0
T_A(s)	0.05	0.05	0.05	0.05	0.05	0.05	0.1	0.1	0.1

Now, the proposed procedure is followed for the selection of generators. Initially, from Table 1, it appears that generators 1 and 2 show reasonable observability and controllability in the frequency range of the EOM, therefore they are selected to form a group of two generators. After that, the remaining generators are included one by one. It was found that in addition to the generators 1 and 2, the generators 5, 6 and 9 have favorable interactions of observability and controllability and the generators 3, 4, 7 and 8 have adverse interactions of controllability. Figure 6 shows the graphs of $\underline{\sigma}(G(j\omega))$ from the set of generators 1 and 2 and with the inclusion of generators 3, 4, 7 and 8, showing adverse interactions.

In Figure 6, it becomes clear that the generators 3, 4, 7 and 8 may deteriorate the controllability of all EOM. Thus, these generators should not be considered for the application of PSS.

These conclusions cannot be obtained from the analysis with eigenvectors (Tables 1 and 2). This explains why the PSSs designed including generators 3 and 8 needed high gains, causing deterioration in the damping of the excitation modes [19].

For final selection of generators on the simultaneous application of PSS, it is presented Table 3, containing values of the constants of inertia H of the generators, gains K_A and of T_A time constants of excitation systems.

In Table 3, it appears that the generator 4 is relatively small, and has little K_A. Generators 3, 7 and 8, although not small, have small gains K_A.

The interaction analysis showed that the generators that have adverse interactions are the smallest of them, and the ones that have excitation systems with small gains and or slower speed responses. These results agree with the conclusions of the practice of power systems, where these generators are considered inefficient to damp EOM with the application of PSS. The analysis of interactions goes beyond that, showing that PSS applied to these generators can deteriorate the controllability of EOM in many other generators, deteriorating the effect of EOM damping from other PSS. Thus, the EOMs more associated with these generators must be damped by PSS applied to other generators that cause favorable interactions. All this explains why some PSS remained disconnected after unsuccessful tuning attempts.

Thus, the generators 1, 2, 5, 6 and 9 are pre-selected for analysis and final selection, which are the ones that cause favorable interactions and are the biggest generators with higher gains K_A. Assuming that four PSSs are sufficient to damp the critical EOM, one must select four of the five preselected generators. There are $\begin{pmatrix} 5 \\ 4 \end{pmatrix} = 5$ sets of generators for final analysis. The graphical analysis of $\overline{\sigma}(G)$ and $\underline{\sigma}(G)$ of these sets showed that the set with the generators 1, 2, 5 and 6 has the highest $\underline{\sigma}(G)$ in the frequency range of the EOM with good observability of the critical EOM. These results are shown in Figure 7. These generators were selected for the PSS application, being the generator 9 reserved for possible application of PSS, in case the robustness can not be achieved with only four PSS. It should be observed by comparing Figures 5 and 7 that the selected set provides better controllability of the critical EOM than the complete set of generators, due to adverse interactions.

This shows that, contrarily to what one might imagine, PSS applied to all generators of the system may provide lower damping of the EOM than those obtained with the application of PSS only on generators selected by the proposed technique.

As it can be seen in the graph of $\underline{\sigma}(G)$, there is a complex zero, which appears in multiple sets. However, analyzing the graphs $\overline{\sigma}(G)$ and $\underline{\sigma}(G)$, it appears that this zero does not affect the controllability of any EOM.

Finally, analyzing in Figure 7 the graph of $\underline{\sigma}(G)$ from the selected set of generators (Group 1) and the graph $\overline{\sigma}(G)$ of the set of non-selected generators (Group 2), it appears that all critical EOM in Group 1 are controllable and the EOM more associated with

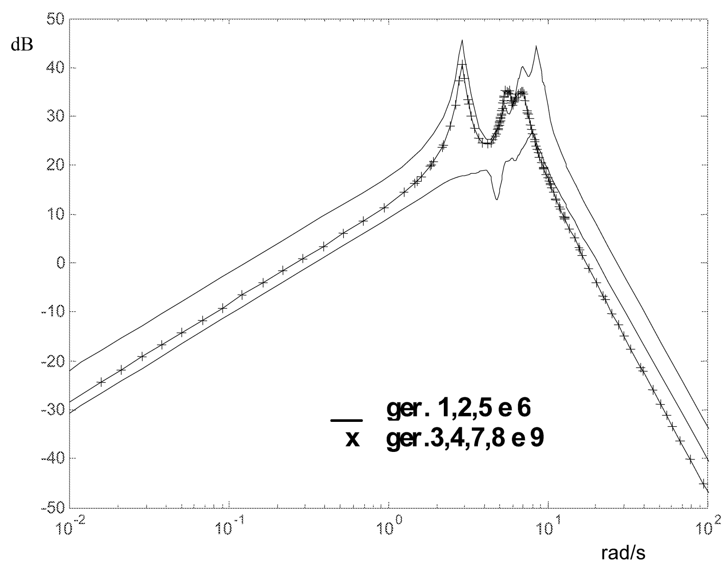

FIGURE 7. Graphs of $\overline{\sigma}(G)$ and $\underline{\sigma}(G)$ from the set of generators 1, 2, 5 and 6; and $\underline{\sigma}(G)$ from the set of generator 3, 4, 7, 8 and 9

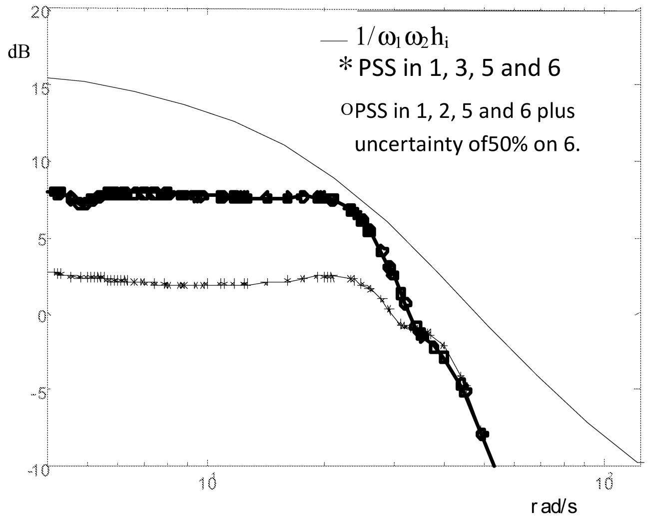

FIGURE 8. Graphs of $\overline{\sigma}(T)$ from the set of generators 1, 2, 5 and 6 with and without an uncertainty of 50% in the input control of the generator 6 and $\frac{1}{\|\omega_1\omega_2 h_i\|}$

TABLE 4. Damping ratio of critical EOMs

Modes	without PSS	with PSS
1	0.0395	0.0767
2	0.0405	0.0758
3	0.0296	0.0779
4	0.0354	0.0808

Group 2 are observable in this group. This assures that PSS applied to selected generators will damp all EOM.

For the design of robust and decentralized PSS, it is considered that $\omega_1 = 1$ and $\omega_2 = \frac{0.125s+0.019}{0.5s+1}$. For best performance result of the controlled system in the face of uncertainties, identical PSS are applied to selected generators. The adopted conventional structure of the PSS is $h_i(s) = K_s \frac{T_w s}{1+T_w s}\left(\frac{1+T_1 1s}{1+T_2 s}\right)^2$. Arbitrating $T_w = 20s$, the time constants T_1, T_2 and the gain K_s are tuned to min sup$(\overline{\sigma}(T(j\omega)))$ using the technique of direct optimization. After tuning, it was resulted $K_s = 0.7$, $T_1 = 0.05s$ and $T_2 = 0.009s$.

In Figure 8, the graphs of $\overline{\sigma}(T)$ are shown for generator set 1, 2, 5 and 6 with PSS and with a reduction of 50% in the gain of PSS for generator 6 (this reduction represents the uncertainty of the control input for this generator) compared with the graph of $\left|\frac{1}{\omega_1\omega_2 h_i}\right|$. The minimization resulted in a flat graph of $\overline{\sigma}(T)$, satisfying the required conditions from Equations (9) and (10) for control robustness. Since the peaks of $\overline{\sigma}(T)$ are associated with the lower damping of EOM, it becomes clear that the oscillations are damped and that the robustness is associated to the oscillations. Thus, some PSS that were designed using conventional techniques may be robust.

In Table 4, the damping ratios of critical EOM for the system with and without the four PSS are compared. It is interesting to note that for robust control, small increases in the damping ratios of EOM are sufficient. These small displacements of the critical eigenvalues are due to small gains K_s. With these PSS gains, good damping of the

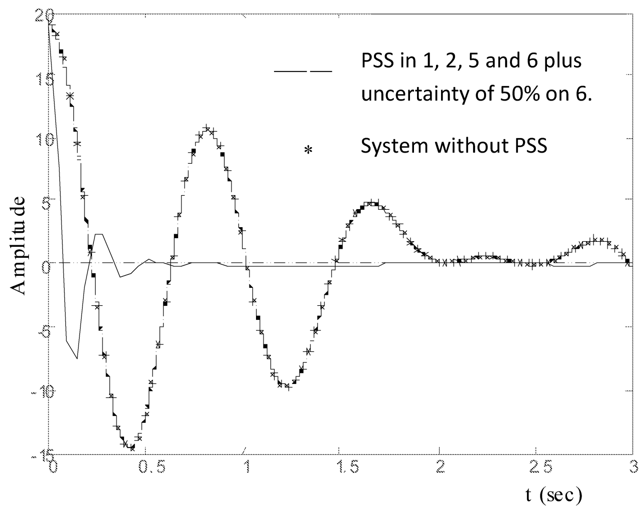

FIGURE 9. Response time curves of ω_6 from system without PSS and with PSS, with uncertainty of 50% in the input control of generator 6; impulse type disturbance in generator 6

excitation modes are preserved. This is an important result, since the excitation modes usually limit the robustness of the PSS.

Figure 9 are shown the time response of the output ω_6 of the generator 6 to the system without PSS and with PSS having uncertainty of 50% in the control input of the same generator. In both cases it is assumed an impulsive disturbance in mechanical torque of generator 6. As shown, there is a good performance of the controlled system in the face of disturbance and uncertainty.

7. **Conclusions.** New techniques for site selection and simultaneous design of all decentralized controllers are proposed. Although the techniques are of general application in power systems, this paper focuses on the problem of excitation control.

The controllers are designed to provide robust H_∞ control system using conventional stabilizers for power systems, without reducing the order of the system model.

Taking into account the couplings and interactions between generators and control, the proposed techniques were more efficient than techniques using eigenvectors for the selection of generators and to apply one by one the PSS, due to the previous elimination of adverse interactions and due to the visualization and simultaneous control of all critical modes of oscillation.

This paper also contributes by showing that conventional PSS can be tuned to provide the robust H_∞ control in power systems, if the best sites for application of PSS are pre-selected to avoid adverse interactions.

Due to the large time required for construction of singular value graphs in large power systems, the proposed techniques are more attractive for application in a few areas of a power system, in which the generators, FACTS, SVC, etc. in the areas of interest are represented by full models and simplified models for other areas.

The MATLAB software was used in the frequency domain and in the time domain analysis, and, since that all the computational procedure for site selection and controllers design is made *offline*, there is no penalty for potential practical applications of the approach.

Another contribution of the paper is that the system is not reduced in order and the controllers are of known structure and low order.

The unique condition imposed to develop the results is the representation of unstructured uncertainties with a single bound for all generators. This represents a conservative measure for all procedure. Lots of work for future intends to use new techniques to reduce conservation.

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