International Journal of Innovative Computing, Information and Control
Volume **9**, Number **2**, February **2013**

ICIC International ©2013 ISSN 1349-4198
pp. **667–677**

DESIGN OF OBSERVER BASED ADAPTIVE PID CONTROLLER FOR NONLINEAR SYSTEMS

Leehter Yao and Hong-Kang Wen

Department of Electrical Engineering
National Taipei University of Technology
No. 1, Section 3, Chung-Hsiao E. Rd., Taipei 10608, Taiwan
ltyao@ntut.edu.tw

Received November 2011; revised March 2012

Abstract. *An observer based adaptive PID controller with micro genetic algorithm (MGA) tuning its parameters is proposed in this paper. The task of the controller is to track the desired trajectory of a nonlinear system as best as it could. The proposed adaptive PID controller consists of 3 components including the PID controller, the control compensating for the external disturbance and the modeling error, and the supervisory control. The parameters of PID controller are learned using MGA. The Lyapunov stability theorem is utilized to design the fitness function for MGA so that the parameters under learning by MGA tend to stabilize the nonlinear system. To cope with the situation that the states are not always observable, an observer is designed and integrated with the adaptive PID controller.*

Keywords: Observer, PID controller, Adaptive control, Genetic algorithm, Nonlinear system, Lyapunov function

1. Introduction.

PID controllers have been well developed and widely used in the industry. In the past half century, PID controller has played its main role in the industrial automation and process control due to its simplicity of operation, design and maintenance. However, in some applications such as nonlinear system, system with unstructured dynamics, high order linear system, time-delayed system, or system with too much disturbance, conventional PID controllers are functionally limited. Various approaches have been proposed to overcome the afore-mentioned application limitations. Due to great technology advance and low cost of high performance micro-controller and embedded system, complex tuning schemes rather than conventional tuning method [1] for PID controllers are allowed. The auto-tuning adaptive PID controllers were widely investigated [2-6]. Mathematical tuning approaches are usually developed for these auto-tuning adaptive PID controllers. Ever since fuzzy theories are proposed in [7], fuzzy logic has been successfully adopted as one of major approaches for PID controller design. The fuzzy PID controllers [8-12] features that they are essentially still auto-tuning adaptive PID controllers and yet the tuning schemes are designed based on fuzzy inference.

For a system controlled by either gain-scheduling type or direct action type fuzzy PID controller, the fuzzy inference system acts like an efficient black box. The parameters in the fuzzy inference system within the fuzzy PID controller are generally too complicated to analyze the relationships between every parameter of fuzzy inference system and the fuzzy PID controller output. In spite of significant performance of fuzzy PID controllers over their conventional counterparts, the parameters of fuzzy inference systems for either gain scheduling or direct action type of controllers are generally tuned manually. In order to achieve optimal performance, genetic algorithm (GA) [13] is one of popular approaches utilized to learn the parameters of fuzzy inference system. In [14,15], a simple GA and a multi-objective GA were respectively applied to find the optimal tuning of parameters in a hybrid fuzzy PI+D controller. Similarly, the parameters of fuzzy inference system in a hybrid fuzzy PD+I controller were learned by a multi-GA [16]. In [17], a gain scheduled fuzzy PID controller consisting of both PI-like and PD-like fuzzy controllers was proposed. Both of fuzzy PI-like and PD-like controllers were weighted through adaptive gain scheduling. A modified GA called accumulated GA was proposed to learn the parameters of fuzzy inference system in the gain scheduled fuzzy PID controller.

Although GA can be applied to learn the parameters of fuzzy PID controllers, the calculation is conducted off-line since GA requires a great amount of computational effort. Therefore, the fuzzy PID controllers depending on GA to learn the controller parameters are usually restricted to the off-line applications. However, some applications require the adaptive design of PID controller so that the controller is able to cope with the deviation of system characteristics from the nominal values, suppression of time-varying disturbance, or unstructured system modeling, etc. On-line adaptive tuning for PID controller is thus required in some applications [2]. If the system allows the on-line calculation not so fast, i.e., the on-line calculation is allowed not as fast as gradient approach and is tolerant with the on-line calculation as long as, for instance, 1 second long, the adaptive PID controller utilizing fast type of GA can be applied to the on-line application. In [19-21], a micro-GA (MGA) with small size of gene pool, simple cross-over operation, and effective supplement scheme if no learning improvement achieved in one generation, was proposed to learn the parameters in a direct adaptive fuzzy-neural controller for uncertain nonlinear systems. The MGA is utilized in this paper to learn the proposed adaptive PID controller. A similar approach was proposed in [18] and given a different name called reduced-form genetic algorithm (RGA). To improve the learning efficiency of MGA, the Lyapunov stability theorem is utilized to help design an appropriate fitness function for the MGA so that the PID controller under learning tends to stabilize the nonlinear system to be controlled. In this paper, a direct adaptive PID controller is to be designed to stabilize a nonlinear system. It has been theoretically justified in [22] that a direct adaptive fuzzy controller is able to stabilize a nonlinear system with state feedback. However, the state feedback is not always available. An observer for output feedback is designed associated with the proposed PID controller to cope with the difficulty of state measurements in some applications.

The organization of this paper is as follows. The problem statement and the observer based adaptive PID controller are introduced in Section 2. The design of direct type adaptive PID controller is described in Section 3. Search for optimal parameters of PID controller using MGA is described in Section 4. Computer simulation showing the effectiveness and efficiency of the proposed adaptive PID controller is presented in Section 5. Concluding remarks are given in Section 6.

2. Problem Statement and the Observer Based Adaptive PID Controller.

Consider an nth order nonlinear system described by the following nonlinear differential equation:

$$y^{(n)} = f\left(y, \dot{y}, \cdots, y^{(n-1)}\right) + g\left(y, \dot{y}, \cdots, y^{(n-1)}\right) u + d \tag{1}$$

where d is the external bounded disturbance, $u \in \Re$ is the system input, and $y \in \Re$ is the system output. Both $f(\cdot)$ and $g(\cdot)$ are uncertain nonlinear functions. The function $g(\cdot)$ is assumed to be strictly positive with upper and lower boundaries. In addition, only the system output y is assumed to be measurable. The control objective is to design an output-feedback adaptive PID controller u such that the system output follows a given bounded reference signal y_m. Letting $x_1 = x, x_2 = \dot{x}_1 = \dot{x}, \ldots,$ and $x_n = \dot{x}_{n-1} = x^{(n-1)}$, (1) can be rewritten as a nonlinear dynamic state equation

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}(f(\boldsymbol{x}) + g(\boldsymbol{x})u + d) \tag{2}$$

$$y = \boldsymbol{C}^T\boldsymbol{x} \tag{3}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \tag{4}$$

and $\boldsymbol{x} = [x, \dot{x}, \cdots, x^{(n-1)}]^T = [x_1, x_2, \cdots, x_n]^T \in \Re^n$ is a vector of states. The tracking performance due to the adaptive PID controller to be designed is analyzed by the tracking error $e \equiv y_m - y$. Denote the output and reference output vector $\boldsymbol{y}$ and $\boldsymbol{y_m}$ as $\boldsymbol{y} \equiv [y, \dot{y}, \ldots, y^{(n-1)}]^T$ and $\boldsymbol{y_m} \equiv [y_m, \dot{y}_m, \ldots, y_m^{(n-1)}]^T$, respectively, then the tracking error vector $\boldsymbol{e}$ is defined as $\boldsymbol{e} \equiv [e, \dot{e}, \cdots, e^{(n-1)}]^T = \boldsymbol{y_m} - \boldsymbol{y}$. Since only the system output y is assumed to be measurable, an observer is to be designed along with the proposed adaptive PID controller. Denote the estimate of $\boldsymbol{x}$ due to the observer as $\hat{\boldsymbol{x}}$. If the nonlinear functions $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are known and the system have no external disturbance [22] shows that the optimal control u^* is given as:

$$u^* = \frac{1}{g(\boldsymbol{x})}\left[-f(\boldsymbol{x}) + y_m^{(n)} + \boldsymbol{\Gamma_c}^T\hat{\boldsymbol{e}}\right] \tag{5}$$

where $\hat{\boldsymbol{e}} = \boldsymbol{y_m} - \hat{\boldsymbol{x}}$. The feedback gain vector $\boldsymbol{\Gamma_c} \equiv [\gamma_1^c, \gamma_2^c, \ldots, \gamma_n^c]^T \in \Re^n$ is designed such that the characteristic polynomial $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{\Gamma_c}^T$ is Hurwitz given the fact that $(\boldsymbol{A}, \boldsymbol{B})$ in (4) is controllable. However, both $f(\cdot)$ and $g(\cdot)$ are unknown, the optimal control law in (5) cannot be implemented. To overcome this difficulty, an adaptive PID controller along with a compensation scheme is designed. Define the output of PID controller as:

$$u_{PID} = k_p\hat{e} + k_i\int_0^t \hat{e}(\tau)d\tau + k_d\dot{\hat{e}} \tag{6}$$

Differentiating both sides of (6) yields

$$\dot{u}_{PID} = k_p\dot{\hat{e}} + k_i\hat{e} + k_d\ddot{\hat{e}} \tag{7}$$

Referring to (7), the PID controller can be implemented as a velocity type PID controller with the increment of PID controller output

$$\Delta u_{PID}(\boldsymbol{K_{PID}}) = k_p\dot{\hat{e}} + k_i\hat{e} + k_d\ddot{\hat{e}} = \boldsymbol{K_{PID}}^T\hat{\boldsymbol{e}}_{\boldsymbol{PID}} \tag{8}$$

where $\boldsymbol{K_{PID}} = [k_p, k_i, k_d]^T$ and $\hat{\boldsymbol{e}}_{\boldsymbol{PID}} = [\dot{\hat{e}}, \hat{e}, \ddot{\hat{e}}]^T$. Denote u_{PID}^l as the PID controller output at the previous sampling interval. The velocity type of PID controller output is defined as

$$u_{PID} = u_{PID}^l + \Delta u_{PID}(\boldsymbol{K_{PID}}) \tag{9}$$

where $\Delta u_{PID}(\boldsymbol{K_{PID}})$ is defined as in (8). The optimal control in (5) is to be approximated by a controller as following:

$$u = u_{PID} + u_d + u_s \tag{10}$$

where u_d is employed to compensate for the external disturbance and the modeling error and u_s is a supervisory control with the function of improving system stability. Substituting (10) into (2) yields

$$x^{(n)} = f(\boldsymbol{x}) + g(\boldsymbol{x})(u_{PID} + u_d + u_s) + d \tag{11}$$

Subtracting $y_m^{(n)}$ from $x^{(n)}$ in (11) yields

$$e^{(n)} = -\boldsymbol{\Gamma}_{\boldsymbol{c}}^T\hat{\boldsymbol{e}} + \frac{g(\boldsymbol{x})}{g(\boldsymbol{x})}(-f(\boldsymbol{x}) + y_m^{(n)} + \boldsymbol{\Gamma}_{\boldsymbol{c}}^T\hat{\boldsymbol{e}}) - g(\boldsymbol{x})(u_{PID} + u_d + u_s) - d \tag{12}$$

Substituting u^* in (5) into (12) yields

$$e^{(n)} = -\boldsymbol{\Gamma}_{\boldsymbol{c}}^T\hat{\boldsymbol{e}} + g(\boldsymbol{x})(u^* - u_{PID} - u_d - u_s) - d \tag{13}$$

The dynamic equation corresponding to (13) can be written as:

$$\dot{\boldsymbol{e}} = \boldsymbol{A}\boldsymbol{e} - \boldsymbol{B}\boldsymbol{\Gamma}_{\boldsymbol{c}}^T\hat{\boldsymbol{e}} + \boldsymbol{B}(g(\boldsymbol{x})(u^* - u_{PID} - u_d - u_s) - d) \tag{14}$$

$$e_1 = \boldsymbol{C}^T\boldsymbol{e} \tag{15}$$

Note that $e_1 = y_m - y$. Referring to (14) and (15), the proposed track following problem is transformed to be a regulation problem.

3. **Design of Observer Based Adaptive PID Controller.** In some situations, only system output is measurable. The states and therefore the state errors in (14) are not measurable. Let $\hat{\boldsymbol{e}}$ be the estimate of $\boldsymbol{e}$. Design an observer with the following dynamic equation for the error estimation.

$$\dot{\hat{\boldsymbol{e}}} = \boldsymbol{A}\hat{\boldsymbol{e}} - \boldsymbol{B}\boldsymbol{\Gamma}_{\boldsymbol{c}}^T\hat{\boldsymbol{e}} + \boldsymbol{\Gamma}_{\boldsymbol{o}}(e_1 - \hat{e}_1) \tag{16}$$

where $\boldsymbol{\Gamma}_{\boldsymbol{o}} \equiv [\gamma_1^o, \gamma_2^o, \ldots, \gamma_n^o]^T \in \Re^n$ is a vector of observer gains. Denote the observation error for the observer as $\tilde{\boldsymbol{e}} = \boldsymbol{e} - \hat{\boldsymbol{e}}$. Subtracting (16) from (14) yields

$$\dot{\tilde{\boldsymbol{e}}} = \boldsymbol{A}_{\boldsymbol{o}}\tilde{\boldsymbol{e}} + \boldsymbol{B}(g(\boldsymbol{x})(u^* - u_{PID} - u_d - u_s) - d) \tag{17}$$

$$\tilde{e}_1 = \boldsymbol{C}^T\tilde{\boldsymbol{e}} \tag{18}$$

where

$$\boldsymbol{A}_{\boldsymbol{o}} = \boldsymbol{A} - \boldsymbol{\Gamma}_{\boldsymbol{o}}\boldsymbol{C}^T \tag{19}$$

Since $(\boldsymbol{C}, \boldsymbol{A})$ is observable, the observer gain vector $\boldsymbol{\Gamma}_{\boldsymbol{o}}$ can be designed such that $\boldsymbol{A} - \boldsymbol{B}\boldsymbol{\Gamma}_{\boldsymbol{o}}^T$ is strictly Hurwitz. There exists a symmetric and positive definite matrix $\boldsymbol{P}$ and a positive definite matrix $\boldsymbol{Q}_{\boldsymbol{o}}$ such that

$$\boldsymbol{A}_{\boldsymbol{o}}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_{\boldsymbol{o}} = -\boldsymbol{Q} \tag{20}$$

Denote $\mathrm{L}(\cdot)$ as the Laplace Transform. The transfer function of the dynamic equation in (17) and (18) can be written as

$$\mathrm{L}(\tilde{e}_1) = N(s)\mathrm{L}(g(\boldsymbol{x})(u^* - u_{PID} - u_d - u_s) - d) \tag{21}$$

where $N(s) = \boldsymbol{C}^T(s\boldsymbol{I} - \boldsymbol{A}_{\boldsymbol{o}})^{-1}\boldsymbol{B}$. The SPR-Lyapunov design approach [23] is utilized in the adaptive PID controller design. Choose an $M(s) = s^m + b_1 s^{m-1} + \ldots + b_m$, $m < n$, so that $M^{-1}(s)$ is a proper stable transfer function and $N(s)M(s)$ is a proper SPR transfer function. $\mathrm{L}(\tilde{e}_1)$ in (21) can be further written as

$$\begin{aligned} \mathrm{L}(\tilde{e}_1) \;=\; & N(s)M(s)M(s)^{-1}\mathrm{L}(g(\boldsymbol{x})(u^* - u_{PID}) - d) \\ & -N(s)M(s)M(s)^{-1}\mathrm{L}(g(\boldsymbol{x})u_d) - N(s)M(s)M(s)^{-1}\mathrm{L}(g(\boldsymbol{x})u_s) \\ & +N(s)M(s)\mathrm{L}(u^* - u_{PID}) - N(s)M(s)\mathrm{L}(u^* - u_{PID}) \end{aligned} \tag{22}$$

Denote the function φ so that $\mathrm{L}(\varphi) = M(s)^{-1}\mathrm{L}(g(\boldsymbol{x})(u^* - u_{PID}) - d) - \mathrm{L}(u^* - u_{PID})$, $\mathrm{L}(\bar{u}_d) = M(s)^{-1}\mathrm{L}(g(\boldsymbol{x})u_d)$, and $\mathrm{L}(\bar{u}_s) = M(s)^{-1}\mathrm{L}(g(\boldsymbol{x})u_s)$, then (22) is simplified as

$$\mathrm{L}(e_1) = N(s)M(s)\mathrm{L}(u^* - u_{PID} + \varphi - \bar{u}_d - \bar{u}_s) \tag{23}$$

The dynamic equation corresponding to (23) is defined as

$$\dot{\tilde{\boldsymbol{e}}} = \boldsymbol{A}_{\boldsymbol{o}}\tilde{\boldsymbol{e}} + \boldsymbol{B}_{\boldsymbol{m}}(u^* - u_{PID} + \varphi - \bar{u}_d - \bar{u}_s) \tag{24}$$

$$\tilde{e}_1 = \boldsymbol{C}_{\boldsymbol{m}}^T\tilde{\boldsymbol{e}} \tag{25}$$

where $\boldsymbol{B_m} = [0, 0, \ldots, b_1, b_2, \ldots, b_m]^T \in \Re^n$, $\boldsymbol{C_m} = [1, 0, \ldots, 0]^T \in \Re^n$. To further analyze the system stability due to the proposed PID controller, the following 2 assumptions [18] are required.

Assumption 3.1. *The uncertain nonlinear function $f(\boldsymbol{x})$ for the states is bounded by a upper bound function $f^u(\boldsymbol{x})$, i.e., $f(\boldsymbol{x}) \leq f^u(\boldsymbol{x})$. The uncertain nonlinear function $g(\boldsymbol{x})$ associated with the input is bounded by*

$$g_l \leq \|g(\boldsymbol{x})\| \leq g^u \tag{26}$$

where both upper and lower bound g^u and g_l are positive constants.

Assumption 3.2. *The function φ is bounded by*

$$\|\varphi\| \leq \varepsilon \tag{27}$$

where ε is a positive constant.

To analyze the system stability, define the Lyapunov function

$$V = \frac{1}{2}\tilde{\boldsymbol{e}}^T \boldsymbol{P} \tilde{\boldsymbol{e}} \tag{28}$$

where the matrix $\boldsymbol{P} = \boldsymbol{P}^T > 0$. Differentiating V with respect to t and substituting (24) into the result yields

$$\begin{aligned} \dot{V} \;=\; & \tfrac{1}{2}((\tilde{\boldsymbol{e}}^T \boldsymbol{A_o}^T \boldsymbol{P} \tilde{\boldsymbol{e}} + \tilde{\boldsymbol{e}}^T \boldsymbol{P} \boldsymbol{A_o} \tilde{\boldsymbol{e}}) + (u^* - u_{PID} + \varphi - \bar{u}_d - \bar{u}_s)^T \boldsymbol{B_m}^T \boldsymbol{P} \tilde{\boldsymbol{e}} \\ & + \tilde{\boldsymbol{e}}^T \boldsymbol{P} \boldsymbol{B_m} (u^* - u_{PID} + \varphi - \bar{u}_d - \bar{u}_s)). \end{aligned} \tag{29}$$

Since $N(s)M(s)$ is SPR, there exists a $\boldsymbol{P} = \boldsymbol{P}^T > 0$ such that

$$\boldsymbol{A_o}^T \boldsymbol{P} + \boldsymbol{P} \boldsymbol{A_o} = -\boldsymbol{Q} \tag{30}$$

$$\boldsymbol{P} \boldsymbol{B_m} = \boldsymbol{C_m} \tag{31}$$

where $\boldsymbol{Q} = \boldsymbol{Q}^T > 0$. Substituting (30), (31) and (25) into (29) yields

$$\dot{V} = -\frac{1}{2}\tilde{\boldsymbol{e}}^T \boldsymbol{Q} \tilde{\boldsymbol{e}} + \tilde{e}_1(u^* - u_{PID} + \varphi - \bar{u}_d) - \tilde{e}_1 \bar{u}_s \tag{32}$$

Referring to (32),

$$\dot{V} \leq -\frac{1}{2}\left|\tilde{\boldsymbol{e}}^T \boldsymbol{Q} \tilde{\boldsymbol{e}}\right| + |e_1|\,|u^* - u_{PID}| + \tilde{e}_1(\varphi - \bar{u}_d) - \tilde{e}_1 \bar{u}_s \tag{33}$$

The compensation of the input signal u_d is designed so that $\tilde{e}_1(\varphi - \bar{u}_d) \leq 0$. Based on Assumption 3.2, u_d can be designed as following.

$$u_d = \begin{cases} \varepsilon + \kappa, & \text{if } \tilde{e}_1 \geq 0 \text{ and } \varphi > 0; \\ 0, & \text{if } \tilde{e}_1 \geq 0 \text{ and } \varphi < 0; \\ 0, & \text{if } \tilde{e}_1 < 0 \text{ and } \varphi > 0; \\ -(\varepsilon + \kappa), & \text{if } \tilde{e}_1 < 0 \text{ and } \varphi < 0; \end{cases} \tag{34}$$

where κ is a positive constant. With u_d designed in (34), $\dot{V}$ in (33) can be written as

$$\begin{aligned} \dot{V} &\leq -\tfrac{1}{2}\left|\tilde{\boldsymbol{e}}^T \boldsymbol{Q} \tilde{\boldsymbol{e}}\right| + |\tilde{e}_1|\,|u^* - u_{PID}| - \tilde{e}_1 \bar{u}_s \\ &\leq -\tfrac{1}{2}\lambda_{\min}(\boldsymbol{Q})\,|\tilde{e}_1|^2 + |\tilde{e}_1|\,|u^* - u_{PID}| - \tilde{e}_1 \bar{u}_s \end{aligned} \tag{35}$$

where $\lambda_{\min}(\boldsymbol{Q})$ denotes the minimum eigenvector of $\boldsymbol{Q}$. Substituting (5) into (35) replacing the state vector $\boldsymbol{x}$ with its estimate $\hat{x}$ due to the observer and employing the bounds for $f(\cdot)$ and $g(\cdot)$ in Assumption 3.1 yields

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(\boldsymbol{Q})\,|\tilde{e}_1|^2 + |\tilde{e}_1| \left(\frac{1}{|g_l|}(|f^u(\hat{\boldsymbol{x}})| + \left|y_m^{(n)}\right| + \left|\boldsymbol{\Gamma_c}^T \hat{\boldsymbol{e}}\right|) + |u_{PID}| \right) - \tilde{e}_1 u_s \tag{36}$$

MGA is utilized in this paper to design the PID controller so that (36) is satisfied. Recall that u_s is a supervisory control signal. Let

$$V_d(\boldsymbol{K_{PID}}) = -\frac{1}{2}\lambda_{\min}(\boldsymbol{Q})\,|\tilde{e}_1|^2 + |\tilde{e}_1|\left(\frac{1}{|g_l|}(|f^u(\hat{\boldsymbol{x}})| + \left|y_m^{(n)}\right| + \left|\boldsymbol{\Gamma}_{\boldsymbol{c}}^T\hat{\boldsymbol{e}}\right|) + |u_{PID}|\right) \tag{37}$$

If $\boldsymbol{K_{PID}}$ estimated by MGA results in $V_d(\boldsymbol{K_{PID}}) < 0$, $\dot{V} < 0$ is satisfied given that u_s is not applied in the input in (10). Conversely, if $V_d(\boldsymbol{K_{PID}}) > 0$, u_s should be applied leading to the condition $\dot{V} < 0$. In order to smooth out the effect caused by u_s being either included or excluded in the PID controller, define a gate function

$$\Theta(V_d) = \begin{cases} 0, & \text{if } V_d < 0; \\ \frac{V_d}{V_m}, & \text{if } 0 \le V_d < V_m; \\ 1, & \text{if } V_d \ge V_m. \end{cases} \tag{38}$$

where V_m is a positive constant. With the gate function, the supervisory control

$$u_s = \Theta(V_d)\mathrm{sgn}(\tilde{e}_1)\left(\frac{1}{|g_l|}(|f^u(\hat{\boldsymbol{x}})| + \left|y_m^{(n)}\right| + \left|\boldsymbol{\Gamma_c}^T\hat{\boldsymbol{e}}\right|) + |u_{PID}|)\right). \tag{39}$$

4. **Search for Optimal Parameters of PID Controller Using Micro-GA.** The nonlinear system to be controlled by the proposed PID controller is assumed to be not so fast and allows a simple type GA such as MGA to search for the best controller on-line. MGA features a small number of chromosomes and simple crossover operation in every generation. Denote G as the number of chromosomes in every generation. G is usually set to be 4 or 6 in MGA. The chromosome consists of cascaded real representation of three controller parameters k_p, k_i and k_d in $\boldsymbol{K_{PID}}$ and $k_p \in [\omega^p_{\min}, \omega^p_{\max}]$, $k_i \in [\omega^i_{\min}, \omega^i_{\max}]$, $k_d \in [\omega^d_{\min}, \omega^d_{\max}]$. Only 3 parameters encoded by real numbers are in the chromosome resulting in simple crossover operation. The PID controller u_{PID} is a function of $\boldsymbol{K_{PID}}$ as shown in (9). MGA is employed to search for optimal $\boldsymbol{K_{PID}}$ aiming to make $V_d(\boldsymbol{K_{PID}}) < 0$ in (37). If the condition $V_d(\boldsymbol{K_{PID}}) < 0$ is violated, the fitness function of the MGA is set to be penalized by adding a large positive value putting the associated $\boldsymbol{K_{PID}}$ in the lower priority for being selected for reproduction in the next generation. Let $\boldsymbol{K^j_{PID}}(g) \equiv [k^j_p(g), k^j_i(g), k^j_d(g)]$ be the PID controller parameters associated with the jth chromosome in the gth generation. Denote $\Omega(\boldsymbol{K^j_{PID}}(g))$as the fitness value corresponding to $\boldsymbol{K^j_{PID}}(g)$. Then,

$$\Omega(\boldsymbol{K^j_{PID}}(g)) = V_d(\boldsymbol{K^j_{PID}}(g)) + \upsilon \tag{40}$$

where υ is defined as

$$\upsilon = \begin{cases} 0, & \text{if } V_d(\boldsymbol{K^j_{PID}}(g)) < 0; \\ \varsigma \gg 0, & \text{otherwise.} \end{cases} \tag{41}$$

Note that ς in (41) is a large positive constant. Let $\boldsymbol{K^*_{PID}}(g) \equiv [k^*_p(g), k^*_i(g), k^*_d(g)]$ be the best estimate of $\boldsymbol{K_{PID}}$ searched in the gth generation of MGA and $M(\boldsymbol{K_{PID}})$ be the space of all possible solutions of $\boldsymbol{K_{PID}}$. Then, $\boldsymbol{K^*_{PID}}(g)$ is the one minimizing the fitness value in (40) and (41), i.e.,

$$\boldsymbol{K^*_{PID}}(g) = \underset{\boldsymbol{K^j_{PID}}(g) \in M(\boldsymbol{K_{PID}}),\ j=1,\ldots,G}{Arg\min} (\Omega(\boldsymbol{K^j_{PID}}(g)) \tag{42}$$

The crossover operation is conducted at every gene, i.e., at every controller parameter. Let $k^j_q(g)$ be the parameter estimated at the jth chromosome in the gth generation,

$q \in \{p, i, d\}$. Then, the parameter generated in the next generation through crossover operation is defined as

$$k_q^j(g+1) = (1-\delta)k_q^j(g) + \delta k_q^{j+G/2}(g), \tag{43}$$

$$k_q^{j+G/2}(g+1) = (1-\delta)k_q^{j+G/2}(g) + \delta k_q^j(g), \tag{44}$$

where $\delta \in [0,1]$ is a random number, $j = 1, \ldots, (G/2)$ and $q \in \{p, i, d\}$. The mutation operation is conducted along with the crossover operation with probability p_u. Define $\Lambda(\cdot)$ as a function regulating the effect of mutation so that it reaches the maximum at the beginning of generations and it decreases generation by generation, i.e.,

$$\Lambda(g, z) = z \times \gamma \times \left(1 - \frac{g}{G_{\max}}\right)^{\gamma} \tag{45}$$

where $G_{\max}$ is the maximal number of generation set before running MGA and $\gamma \in [0,1]$ is a random number determining the dependency of mutation operator on the number of generations. Let $\mu \in [0,1]$ be a random number and $\underline{k}_q^j(\cdot)$ be the mutated parameter of $k_q^j(\cdot)$.

$$\underline{k}_q^j(g) = \begin{cases} k_q^j(g) + \Lambda(g, \omega_q^{\max} - k_q^1(g-1)), & \text{if } \mu > 0.5; \\ k_q^j(g) + \Lambda(g, k_q^1(g-1) - \omega_q^{\min}), & \text{if } \mu \leq 0.5; \end{cases} \tag{46}$$

where $q \in \{p, i, d\}$. Referring to (45) and (46), the mutation operator allows MGA to explore wider parameter space at the early stage of MGA and restricts the search space as MGA gradually converges to the optimal solution after running certain number of generations. Note that the chromosomes are re-ordered in an ascending order every generation in the gene pool based on the associated fitness value. In other words, the chromosome corresponding to $\boldsymbol{K}^*_{\boldsymbol{PID}}(g)$ is placed at the first chromosome of the gene pool.

5. **Computer Simulations.** Consider the Duffing forced oscillation system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12\cos t + u + d \\ y &= x_1 \end{aligned} \tag{47}$$

the external disturbance d is a square wave with amplitude 1 and a period 2π. The control objective is to design an adaptive PID controller u in (47) so that the output y tracks the reference trajectory y_m, under the condition that only the system output y is measureable. The reference trajectory y_m is set as $y_m = \cos(t)$. The parameter V_m in the gate function $\Theta(V_d)$ in (38) is set to be 0.015. The vector of controller gains $\boldsymbol{\Gamma_c}$ in (13) and the vector of observer gains $\boldsymbol{\Gamma_o}$ in (16) are designed as $\boldsymbol{\Gamma_c} = [144\ 24]^T$ and $\boldsymbol{\Gamma_o} = [60\ 900]^T$, respectively. The filter $M(s)$ in (23) is set as $M(s) = 1/(s+2)$ and the matrix $\boldsymbol{Q}$ in (30) is set as $\boldsymbol{Q} = \text{diag}(500\ 500)$, where diag$(\cdot)$ denotes a diagonal function. For the real number representation of the PID controller parameters in MGA, the PID controller parameters are assumed to vary in the range of -20 to 20, i.e., $\omega_q^{\min} = -20$ and $\omega_q^{\max} = 20$, $q \in \{p, i, d\}$. The number of chromosomes, G, in the gene pool of MGA is set as $G = 4$. The maximal number of generations $G_{\max}$ in (45) is set as $G_{\max} = 200$, and $\varsigma = 10000$ in (41). Denote g as the number of generation for MGA, the probability of mutation $p_u = (1 - g/G_{\max}) \times 0.05 + 0.05$. The initial conditions are set as $x_1(0) = 3$, $x_2(0) = 3$, $\hat{x}_1(0) = -1$, $\hat{x}_2(0) = -1$.

With the proposed adaptive PID controller, the output y of the Duffing forced oscillation system in (47) and the reference trajectory y_m are compared in Figure 1. It is shown in Figure 1 that the trajectory following performs well for the adaptive PID controller under the condition that the external disturbance d is constantly applied. The output y starts tracking y_m within less than 0.8 second showing that the adaptive PID controller is capable of overcoming the initial conditions and the constant external disturbance within a very short time. Referring to (10), the total input signal u consists of 3 components including the output of PID controller u_{PID} learned by MGA, the signal u_d compensating for the external disturbance and the modeling error and the supervisory control signal u_s. The total input signal u combining u_{PID}, u_d and u_s is shown in Figure 2, where as u_{PID}, u_d and u_s are shown in Figures 3-5, respectively.

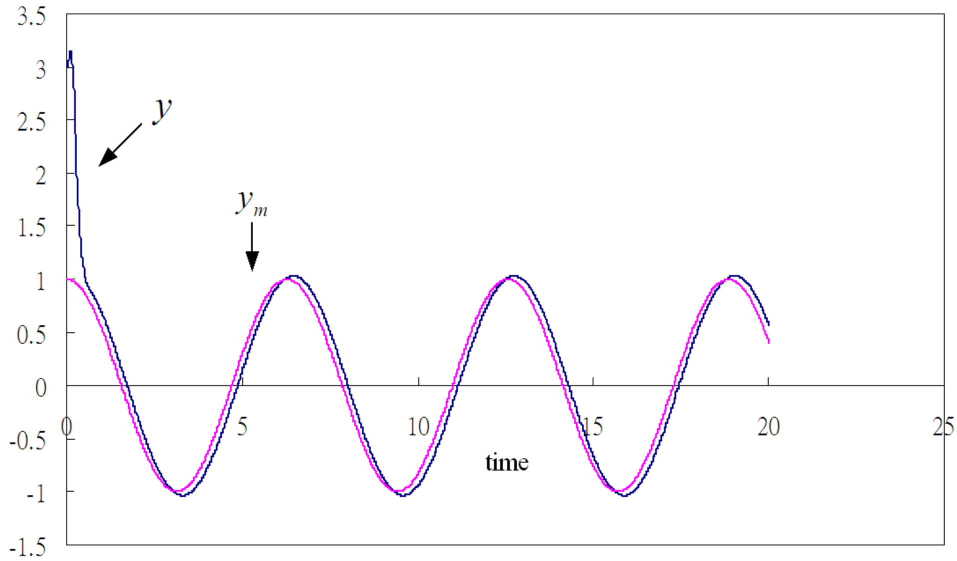

FIGURE 1. Comparison of output y and reference trajectory y_m

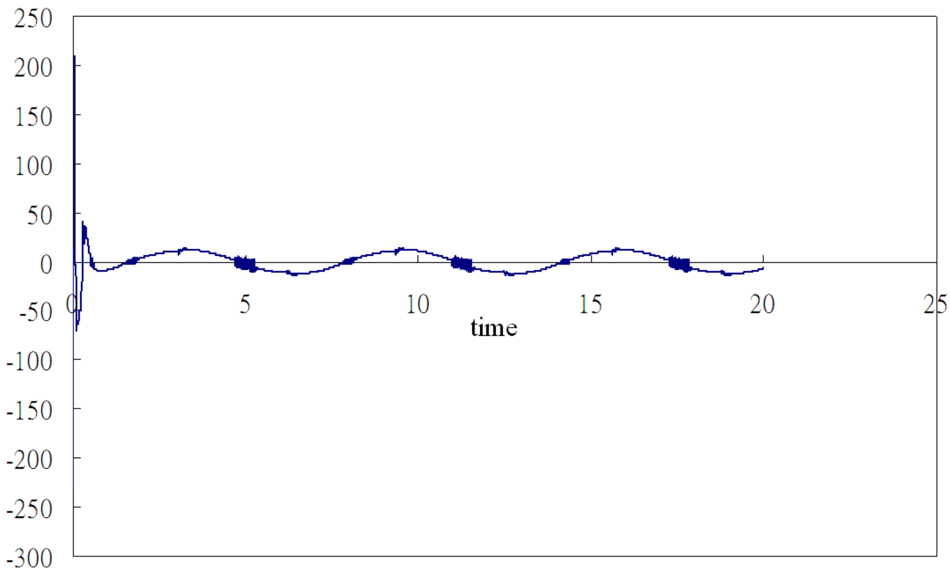

FIGURE 2. The total input signal u

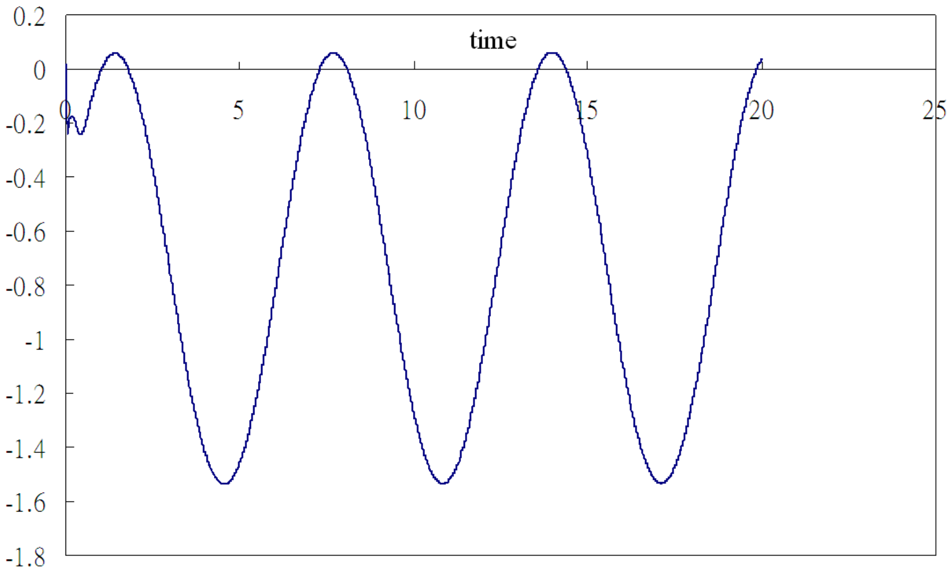

FIGURE 3. The output of PID controller u_{PID}

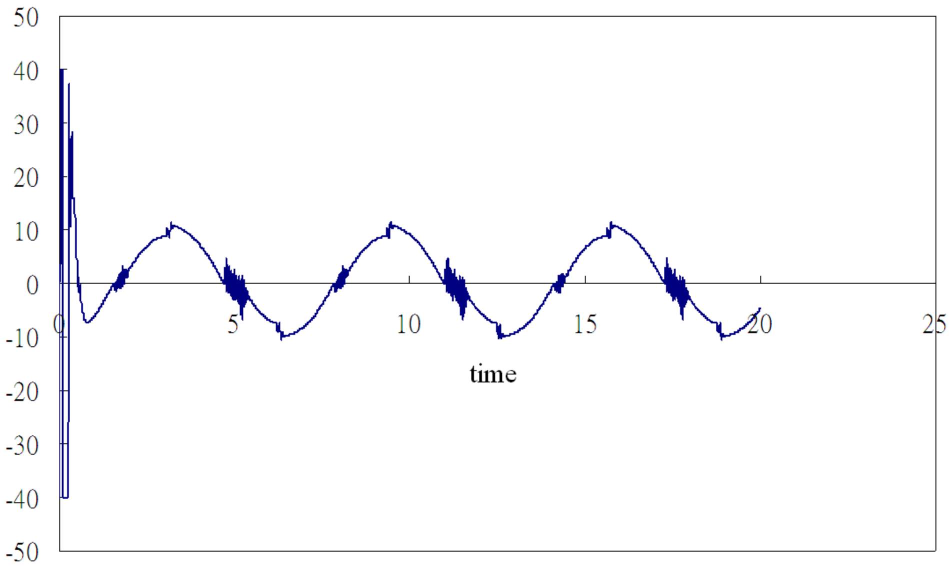

FIGURE 4. The signal u_d compensating for the external disturbance and the modeling error

6. **Conclusions.** The MGA has been successfully applied to the on-line learning of an adaptive PID controller. Since only a small number of chromosomes are used to learn the parameters of PID controller, the fitness function needs to be delicately designed so that the parameters under learning lead to stabilizing the nonlinear system. The SPR-Lyapunov design approach has been utilized to help define the range of variations for the PID controller parameters. Every parameter under learning is directed to stabilize the nonlinear system. To prevent from the situation that the parameters under learning

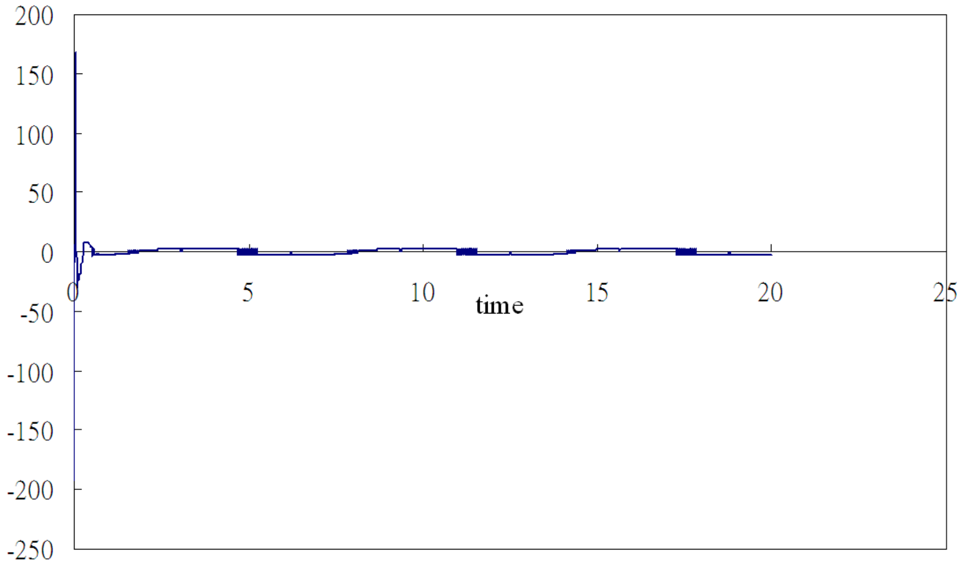

FIGURE 5. The signal u_s for supervisory control

cannot stabilize the nonlinear system at certain generation, a supervisory control mechanism has been designed along with the on-line learning of PID controller parameters using MGA to maintain the stability of nonlinear system. A control signal compensating for the external disturbance and modeling error is also designed within the adaptive PID controller. Both the supervisory control and compensation mechanism for disturbance and modeling error have been designed using SPR-Lyapunov design approach.

The proposed observer based adaptive PID controller has mainly designed for real-time application using MGA to learn the PID controller parameters. The restriction is that the nonlinear system to be applied allows the on-line learning of MGA. It takes at most 700 ms for the MGA to converge to a reasonable solution. Therefore, a suitable sampling interval for the proposed on-line tuning adaptive PID controller is 1 second. The slowly-time varying nonlinear systems that allow at least 1 second time interval updating the control signal are suitable for the proposed adaptive PID controller.

REFERENCES

[1] J. G. Ziegler and N. B. Nichols, Optimum settings for automatic controller, *ASME Trans.*, vol.64, pp.756-768, 1942.
[2] H.-P. Huang, M.-L. Roan and J.-C. Jeng, On-line adaptive tuning for PID controllers, *IEE Proc. of Control Theory Appl.*, vol.149, no.1, pp.60-67, 2002.
[3] F. Karray, W. Gueaieb and S. Al-Sharhan, The hierarchical expert tuning of PID controllers using tools of soft computing, *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol.32, no.1, pp.77-90, 2002.
[4] W. K. Ho, C. C. Hang and J. Zhou, Self tuning PID control of a plant with underdamped response with specification on gain and phase margin, *IEEE Trans. Contr. Syst., Technol.*, vol.5, pp.446-452, 1997.
[5] A. Leva and A. M. Colombo, Method for optimizing set-point weights in ISA-PID autotuners, *IEE Proc. of Control Theory Appl.*, vol.146, no.2, pp.137-146, 1999.
[6] W. K. Ho, Y. Hong, A. Hansson, H. Hjalmarsson and J. W. Deng, Relay auto-tuning of PID controllers using iterative feedback tuning, *Automatica,* vol.39, no.1, pp.1469-157, 2003.
[7] L. A. Zedeh, Fuzzy sets, *Inform. Contr.,* vol.8, pp.338-353, 1965.
[8] G. Chen, Conventional and fuzzy PID controllers: An overview, *Int. J. Intell. Control Syst.,* vol.1, pp.235-246, 1996.

[9] H. A. Malki, H. Li and G. Chen, New design and stability analysis of fuzzy proportional-derivative control systems, *IEEE Trans. Fuzzy Syst.,* vol.2, pp.245-254, 1994.

[10] H.-X. Li, A comparative design and tuning for conventional fuzzy control, *IEEE Trans. Syst., Man, Cybern. B,* vol.27, pp.884-889, 1997.

[11] J. Carvajal, G. Chen and H. Ogmen, Fuzzy PID controller: Design performance evaluation, and stability analysis, *Inform. Sci.,* vol.123, pp.249-270, 2000.

[12] H. A. Malki, D. Feigenspan, D. Misir and G. Chen, Fuzzy PID control of a flexible-joint robot arm with uncertainties from time-varying loads, *IEEE Trans. Contr. Syst. Tech.,* vol.5, pp.371-378, 1997.

[13] D. E. Goldberg, *Genetic Algorithm in Search, Optimization, and Machine Learning,* Addison-Wesley, 1989.

[14] M. J. Er and Y.-L. Sun, Hybrid fuzzy proportional-integral plus conventional derivative control of linear and nonlinear systems, *IEEE Trans. Ind. Elect.,* vol.48, no.6, pp.1109-1117, 2001.

[15] K. S. Tang, K. F. Man, G. Chen and S. Kowng, An optimal fuzzy PID controller, *IEEE Trans. Ind. Electrn.,* vol.48, no.4, pp.757-765, 2001.

[16] K. S. Tang, K. F. Man, G. Chen and S. Kwong, A GA-optimized fuzzy PD+I controller for nonlinear systems, *Proc. of IEEE IECON*, Denver, volorado, pp.718-723, 2001.

[17] L. Yao and C.-C. Lin, On a genetic algorithm based scheduled fuzzy PID controller, *Internatinal Journal of Innovative Computing, Information and Control*, vol.5, no.10(B), pp.3593-3603, 2009.

[18] W.-Y. Wang, C.-Y. Cheng and Y.-G. Leu, An online GA-based output-feedback direct adaptive fuzzy-neural controller for uncertain nonlinear systems, *IEEE Trans. Syst., Man, Cybern. B*, vol.34, no.1, pp.334-345, 2004.

[19] L. Cen and Y. Lian, A modified micro-genetic algorithm for the design of multiplierless digital FIR filters, *IEEE Conf. TENCON*, vol.A, pp.52-55, 2004.

[20] J. E. Mendoza, D. A. Morales, R. A. Lopez, E. A. Lopez, J. C. Vannier and C. C. A. Coello, Multiobjective location of automatic voltage regulators in a radial distribution network using a micro genetic algorithm, *IEEE Trans. Power Syst.*, vol.22, no.1, pp.404-412, 2007.

[21] S. Chakravarty and R. Mittra, Application of the micro-genetic algorithm to the design of spatial filters with frequency-selective surfaces embedded in dielectric media, *IEEE Trans. Antennas Propa.*, vol.44, no.2, pp.338-346, 2002.

[22] L.-X. Wang, *Adaptive Fuzzy Systems and Control*, Prentice-Hall, Englewood Cliffs, NJ, 1994.

[23] P. A. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, Englewood Cliffs, NJ, 1996.