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ON THE CONTROL OF TIME DELAY POWER SYSTEMS

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Abstract. *This paper deals with the control of a power system with time delay in the states. The power system is a seventh order synchronous machine infinite bus system. The linearized model of the system belongs to a class of uncertain linear systems with states delays. Two control schemes are proposed for the system. The first controller uses only the instantaneous states for feedback; the second controller combines the effects of the instantaneous as well as the delayed states. Using Lyapunov theory, it is proven that both control schemes guarantee the exponential stabilization of the power system. Detailed simulation results clearly indicate that the proposed control schemes work well.*
Keywords: Power system, Delays in the state, Control

1. **Introduction.** Time delays are present in many physical systems [1]. These time delays influence the stability of the systems and might degrade the performance of the system; hence they should be properly considered in the controller design [2,3].

Although time delay exists in the power system measurement and control loops, traditional power system controllers were generally designed based on local information and time delays were usually ignored. With the introduction of Wide-Area Measurement System (WAMS) technology, synchronized real-time measurements are provided in the form of phasor measurement unit (PMU) which can be used for stability studies of power systems. This leads to a more efficient controller design. However, time delays are present significantly in these measurements due to transmission channels [3].

Several researchers worked on the control of power systems while considering the time delay [2-41]. For example, in [4-8] the impact of time delay in the design of power system stabilizers was discussed. An effective method to eliminate the oscillations introduced by time-delayed feedback control was proposed in [9,10]. The work in [13] studied the effects of inclusion of delays on the small signal stability of power systems. The researchers in [14] designed a control scheme using phasor measurements considering delays for small signal stability of power systems. The authors in [15] presented a wide-area control system for damping generator oscillations. Additional studies on the influence of time delay on power system stability can be found in [11-41] and the references cited therein.

This paper contributes to the development of feedback controllers for power systems with states delays. Motivated by the work in [42], two control schemes are proposed for the system. The first controller uses only the instantaneous states for feedback; the second controller combines the effects of the instantaneous as well as the delayed states.

Using Lyapunov theory, it is proven that both control schemes guarantee the exponential stabilization of the power system.

The paper is organized as follows. The next section presents the dynamic model of the power system. The control problem is formulated in Section 3. Section 4 presents a robust feedback controller for uncertain linear systems with time delays in the states. Section 5 proposes a three term robust controller for the system. Detailed simulation results of the controlled power system are presented and discussed in Section 6. Finally, some concluding remarks are given in Section 7.

Notations. We use W^t, W^{-1}, $\lambda(W)$ to denote respectively, the transpose of, the inverse of, and the eignvalues of any square matrix W. The vector norm is taken to be the Euclidean norm and the matrix norm is the corresponding induced one; that is $\|W\| = \lambda_M^{1/2}(W^tW)$, where $\lambda_{M(m)}(W)$ stands for the operation of taking the maximum (minimum) eignvalue of W. We use $W > 0$ ($W < 0$) to denote a positive- (negative-) definite matrix W. We use x_t to represent a segment of $x(\tau)$ on $[t - \eta(t), t]$, that is $x_t : [-\eta(t), 0] \to \Re^n$ with $\|x_t\|_* = \sup \|x(\tau)\|$ with $t - \eta^* \leq \tau \leq t$. Let C_o^- denote the proper left-half of the complex plane. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2. **Dynamic Model of the Power System.**

A synchronous machine infinite bus power system is depicted in Figure 1. The dynamic model of the power system is derived in this section. The symbols used in the description of the model are listed below.

δ: the rotor angle with respect to the infinite bus system voltage,
ω_o: the synchronous angular speed,
$\Delta\omega$: the deviation of the rotor angular speed from the synchronous angular speed ω_o,
M: the effective inertia constant,
K_d: the equivalent damping factor,
E: the infinite bus voltage,
E_q: the direct q-axis voltage,
E_q': the transient q-axis voltage,
E_{fd}: the direct excitation voltage,
E_{fd}': the transient excitation voltage,
i_d: the d-axis current,
T_{do}': the equivalent transient rotor time constant,
x_d: the d-axis reactance,
x_d': the d-axis transient reactance,
x_q: the q-axis reactance,
x_e: the reactance of the transmission lines,
P_M: the mechanical power,
P_e: the electrical power,
V_t: the terminal bus voltage,
V_{ref}: the reference voltage,
ΔV: the difference between the reference voltage and the terminal bus voltage,
U_{PSS}: the power system stabilizer (PSS) control signal,
K_A: the amplifier gain constant,
T_A: the amplifier time constant,
y: the state of the governor system,
G: the governor control signal.

The dynamic model of the generator is described in terms of the following three first order differential equations:

$$\dot{\delta} = \omega_o \Delta\omega \tag{1}$$

$$\Delta\dot{\omega} = \frac{1}{M}\left(P_M + G + K_d \Delta\omega - P_e\right) \tag{2}$$

$$\dot{E}'_q = \frac{1}{T'_{do}}\left(E_{fd} - (x_d - x'_d)i_d - E'_q\right) \tag{3}$$

where

$$i_d = \frac{E_q - E\cos\delta}{x_e + x_q} \tag{4}$$

$$E_q = E'_q + (x_d - x'_d)i_d \tag{5}$$

$$P_e = \frac{E'_q E \sin\delta}{x_e + x'_d} \tag{6}$$

Combining (4) and (5), one obtains,

$$i_d = \frac{E'_q - E\cos\delta}{x_e + x_q - x_d + x'_d} \tag{7}$$

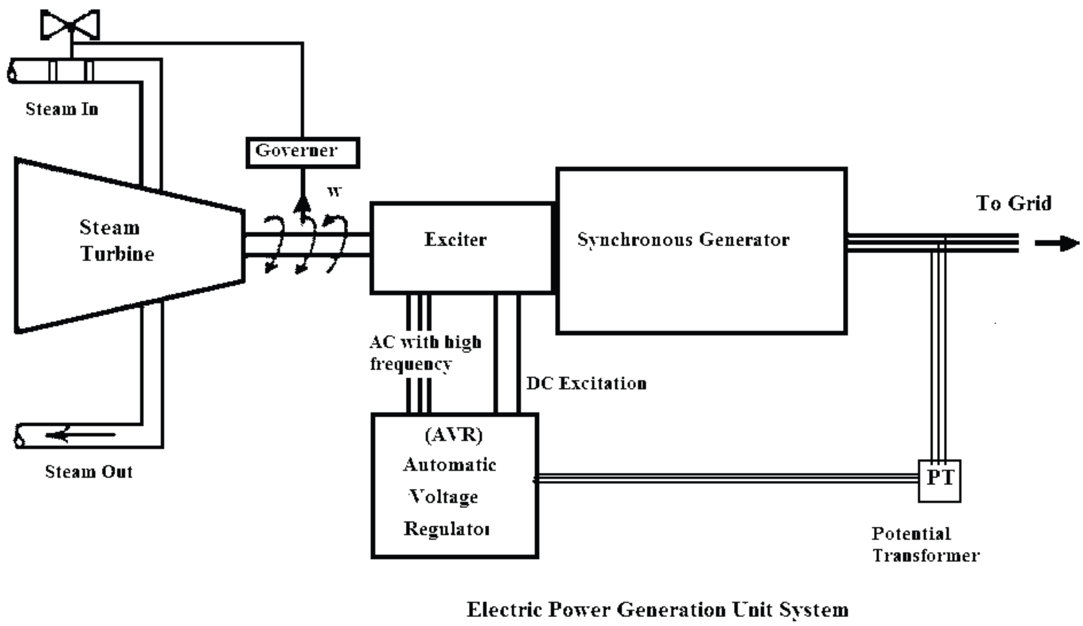

FIGURE 1. Block diagram of the electric power generation unit system

The following dynamic model for the Automatic Voltage Regulator (AVR) and the exciter is adopted:

$$\dot{E}_{fd} = \frac{K_A}{T_A}(\Delta V + U_{PSS}) - \frac{E_{fd}}{T_A} \tag{8}$$

where

$$\Delta V = V_{ref} - V_t. \tag{9}$$

The following dynamic model is considered for the governor:

$$\dot{y} = \frac{1}{T_g}\left(b\Delta\omega(t-\tau) - y\right) \tag{10}$$

$$G = a\Delta\omega(t-\tau) + by. \tag{11}$$

The dynamic model for the conventional power system stabilizer (PSS) is as follows:

$$\dot{z}_1 = z_2 \tag{12}$$

$$\dot{z}_2 = -\frac{T_2}{T_Q}\left(z_1 + (T_2 + T_Q)z_2 - \Delta\omega(t-\tau)\right) \tag{13}$$

and

$$U_{PSS} = -\frac{K_J}{K_A}\left(-\frac{T_1}{T_2}z_1 + \frac{(T_Q - T_1T_2 - T_1T_Q)}{T_2}z_2 + \frac{T_1}{T_2}\Delta\omega(t-\tau)\right). \tag{14}$$

The overall model of the power system is given by the above equations. From the above equations, it can be seen that the model of the power system is of order 7 and it is nonlinear with state delays. To be able to analyze and control the power system, the model is linearized around the operating point $(\delta_o,\ \omega_o,\ E'_{q_o},\ E_{fd_o},\ y_o,\ z_{1o},\ z_{2o})$.

Let $\Delta\delta = \delta - \delta_o$, $\Delta\omega = \omega - \omega_o$, $\Delta E'_q = E'_q - E'_{q_o}$, $\Delta E_{fd} = E_{fd} - E_{fd_o}$, $\Delta y = y - y_o$, $\Delta z_1 = z_1 - z_{1o}$ and $\Delta z_2 = z_2 - z_{2o}$. Therefore, the linearized model of the power system can be written as:

$$\begin{aligned}
\Delta\dot{\delta} &= \omega_o\Delta\omega \\
\Delta\dot{\omega} &= \frac{1}{M}\left(\Delta P_M + \Delta G + K_d\Delta\omega - \Delta P_e\right) \\
\Delta\dot{E}'_q &= \frac{1}{T'_{do}}\left(\Delta E_{fd} - (\Delta E'_q + E\sin\delta_o\Delta\delta)\frac{x_d - x'_d}{x_e + x_q - x_d + x'_d} - \Delta E'_q\right) \\
\Delta\dot{E}_{fd} &= \frac{K_A}{T_A}\left(\Delta V + \Delta U_{PSS}\right) - \frac{\Delta E_{fd}}{T_A} \\
\Delta\dot{y} &= \frac{1}{T_g}\left(b\Delta\omega(t-\tau) - \Delta y\right) \\
\Delta\dot{z}_1 &= \Delta z_2 \\
\Delta\dot{z}_2 &= -\frac{1}{T_2T_Q}\left(\Delta z_1 + (T_2 + T_Q)\Delta z_2 - \Delta\omega(t-\tau)\right)
\end{aligned} \tag{15}$$

Further manipulations of the above model yield:

$$\begin{aligned}
\Delta\dot{\delta} &= \omega_o\Delta\omega \\
\Delta\dot{\omega} &= \frac{1}{M}\left(\Delta P_M + a\Delta\omega(t-\tau) + b\Delta y + K_d\Delta\omega - \frac{E'_{qo}E\sin\delta_o}{x_e + x'_d}\Delta E'_q \right.\\
&\qquad \left. - \frac{E'_{qo}E\cos\delta_o}{x_e + x'_d}\Delta\delta\right) \\
\Delta\dot{E}'_q &= \frac{1}{T'_{do}}\left(\Delta E_{fd} - \frac{x_e + x_q}{x_e + x_q - x_d + x'_d}\Delta E'_q - E\sin\delta_o\frac{x_d - x'_d}{x_e + x_q - x_d + x'_d}\Delta\delta\right) \\
\Delta\dot{E}_{fd} &= \frac{K_A}{T_A}\left(\Delta V + \frac{K_JT_1}{K_AT_2}\Delta z_1 - \frac{K_J(T_Q - T_1T_2 - T_1T_Q)}{K_AT_2}\Delta z_2 \right.\\
&\qquad \left. - \frac{K_JT_1}{K_AT_2}\Delta\omega(t-\tau)\right) - \frac{\Delta E_{fd}}{T_A} \\
\Delta\dot{y} &= \frac{1}{T_g}\left(b\Delta\omega(t-\tau) - \Delta y\right) \\
\Delta\dot{z}_1 &= \Delta z_2 \\
\Delta\dot{z}_2 &= -\frac{1}{T_2T_Q}\left(\Delta z_1 + (T_2 + T_Q)\Delta z_2 - \Delta\omega(t-\tau)\right)
\end{aligned} \tag{16}$$

The outputs of the system are taken to be the deviation of the rotor angle and the deviation of the rotor angular speed.

Define the states and the inputs of the power system such that:

$$x = \begin{pmatrix} \Delta\delta & \Delta\omega & \Delta E'_q & \Delta E_{fd} & \Delta y & \Delta z_1 & \Delta z_2 \end{pmatrix}^t \text{ and } u = (\Delta P_M \;\; \Delta V)^t$$

Hence, the linearized model of the power system can be written in a compact form as,

$$\begin{aligned} \dot{x}(t) &= A_o x(t) + B_o u(t) + Dx(t-\tau) + \zeta(x,t) \\ z(t) &= E_o x(t) \end{aligned} \tag{17}$$

where

$$A_o = \begin{bmatrix} 0 & \omega_o & 0 & 0 & 0 & 0 & 0 \\ K_1 & K_d/M & K_2 & 0 & b/M & 0 & 0 \\ K_3 & 0 & K_4 & \frac{1}{T'_{do}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_A} & 0 & K_5 & K_6 \\ 0 & 0 & 0 & 0 & \frac{-1}{T_g} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & K_7 & K_8 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a/M & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_9 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_{11} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_o = \begin{bmatrix} 0 & 0 \\ \frac{1}{M} & 0 \\ 0 & 0 \\ 0 & \frac{K_A}{T_A} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad E_o = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and, the parameters K_1-K_{11} are defined such that:

$$\begin{aligned}
K_1 &= -\frac{E'_{qo} E \cos\delta_o}{M(x_e + x'_d)} \\
K_2 &= -\frac{E'_{qo} E \sin\delta_o}{M(x_e + x'_d)} \\
K_3 &= -E \sin\delta_o \frac{x_d - x'_d}{T'_{do}(x_e + x_q - x_d + x'_d)} \\
K_4 &= -\frac{x_e + x_q}{T'_{do}(x_e + x_q - x_d + x'_d)} \\
K_5 &= \frac{K_J \, T_1}{T_A \, T_2} \\
K_6 &= \frac{-K_J(T_Q - T_1 T_2 - T_1 T_Q)}{T_A \, T_2} \\
K_7 &= \frac{-1}{T_2 \, T_Q} \\
K_8 &= -\frac{T_2 + T_Q}{T_2 \, T_Q} \\
K_9 &= -\frac{K_J \, T_1}{T_A \, T_2}
\end{aligned}$$

$$K_{10} = \frac{b}{T_g}$$
$$K_{11} = \frac{1}{T_2\, T_Q}$$

Note that the term $\zeta(x,t)$ is added in (17) to represent the external disturbances acting on the system. System (17) belongs to the general class of uncertain linear systems with state delays. The next three sections deal with the formulation and the design of control schemes for this class of systems.

3. **Formulation of the Control Problem.** Consider a class of dynamical systems with time-varying state-delay of the form:

$$\dot{x}(t) = A_o x(t) + B_o u(t) + D x(t - \eta(t)) + \zeta(x,t) \tag{18}$$
$$0 \le \eta(t) \le \eta^* \text{ and } \dot{\eta}(t) \le \eta^+ < 1$$
$$x(t) = \varrho_d(t) \text{ for } t \in [-\eta^*, 0]$$

where $t \in \Re$ is the time, $x \in \Re^n$ is the instantaneous state; $u \in \Re^m$ is the control input. The variable $\eta(t)$ represents the delay of the system with bounds η^* and η^+ which are assumed to be known. The matrices $A_o \in \Re^{n\times n}$, $B_o \in \Re^{n\times m}$, represent the nominal system. The uncertainties within the system are represented by a delay factor multiplied by the matrix D and a current factor contained in the nonlinear vector $\zeta(x,t)$.

Remark 3.1. *The assumption that $\dot{\eta}(t) \le \eta^+ < 1$ stems from the need to bound the growth variations in the delay factor as a time-function. This assumption can be considered quite realistic and holds for a wide class of uncertain dynamical systems. Thus our design results are applicable for the class of linear time-delay systems with bounded state-delay in the manner of (18).*

Definition 3.1. *The uncertain state-delay system (18) is said to be* **robustly stable** *if the solution $x(t) = 0$ of system (18) with $u(t) = 0$ is uniformly asymptotically stable for all admissible realizations of the uncertainties D and $\zeta(.,.)$.*

Definition 3.2. *The uncertain state-delay system (18) is said to be* **robustly stabilizable** *with a degree of stability $\gamma > 0$ if there exists a feedback control $u[x(t)]$ such that the resulting closed-loop system is robustly stable and the solution of the controlled system satisfies:*

$$\|x(\tau_a)\| \le \|x(\tau_b)\| \exp\left(-\gamma(\tau_a - \tau_b)\right) \quad \forall \tau_a \ge \tau_b \in \Re^+. \tag{19}$$

Assumptions:

The following structural and growth constraints on $\zeta(.,.)$ and D are considered.

A1. There exist sufficiently smooth functions $h(.) : \Re^n \times \Re \to \Re^m$ and $g(.) : \Re^n \times \Re \to \Re^n$, such that:

$$\zeta(x,t) = B_o h(x,t) + g(x,t) \quad \forall (x,t) \in \Re^n \times \Re \tag{20}$$

A2. There exist positive constants α and β such that:

$$\begin{aligned} \|h(x,t)\| &\le \alpha \|x(t)\| \\ \|g(x,t)\| &\le \beta \|x(t)\| \end{aligned} \tag{21}$$

A3. There exist functions $G \in \Re^{m\times n}$ and $L \in \Re^{n\times n}$ such that:

$$D = B_o G + L \tag{22}$$

A4. Suppose that $\lambda(A_o) \in C_o^-$. It follows that there exist matrices $0 < P = P^t \in \Re^{n\times n}$ and $0 < Q = Q^t \in \Re^{n\times n}$ such that for some $\kappa > 0$,

$$P(A_o + \kappa I) + (A_o + \kappa I)^t P = -Q \tag{23}$$

For convenience, let

$$\sigma(P,Q) := \lambda_m(Q) - 2\beta\lambda_M(P) - 2\|L^tP\|^2 \text{ and } \omega = \lambda_M(GG^t) \tag{24}$$

Remark 3.2. *One interpretation of* **A1** *and* **A2** *is that the nonlinear uncertainty function* $\zeta(.,.)$ *is mismatched and cone-bounded. The component* $h(.,.)$ *represents the matched part while the component* $g(.,.)$ *stands for the amount of mismatch.*

Remark 3.3. *If* A_o *is not a stable matrix, and the pair* (A_o, B_o) *is controllable, then one can easily design* $u(t) = -K_s x(t) + v(t)$*, where* K_s *is chosen such that* $A_o - B_oK_s$ *is a stable matrix. In this case,* $v(t)$ *is the controller to be designed in the next sections.*

The following fact will be used in the analysis to follow.

Fact 1. Given any real matrices M_1, M_2, M_3 with appropriate dimensions such that $0 < M_3 = M_3^t$. Then the following inequality holds:

$$M_1^tM_2 + M_2^tM_1 \le M_1^tM_3M_1 + M_2^tM_3^{-1}M_2 \tag{25}$$

4. **Design of a Robust Feedback Controller.** The design approach is based on Lyapunov stability theory. For this purpose and in view of **A4**, we define a quadratic Lyapunov function candidate V_1 such that,

$$V_1(x) = e^{2\kappa t}x^t(t)P_ox(t) + \rho\int_{t-\eta(t)}^{t} e^{2\kappa\tau}x_t^t(\tau)x_t(\tau)d\tau; \quad \kappa > 0 \tag{26}$$

where ρ is a weighting factor; x_t represents a segment of $x(\tau)$ on $[t-\eta(t), t]$. Note that

$$\lambda_m(P_o)\|x\|^2e^{2\kappa t} \le V_1(x) \le (\lambda_M(P_o) + \rho\eta^*)\,\|x_t\|_*^2e^{2\kappa t}. \tag{27}$$

In the following analysis, the weighting factor ρ is taken to be:

$$\rho = 2(1-\eta^+)^{-1}e^{2\kappa\eta^*}(1+\omega). \tag{28}$$

We will choose the matrix $0 < Q_o = Q_o^t \in \Re^{n\times n}$ such that $0 < P_o = P_o^t \in \Re^{n\times n}$ is the solution of (23) for $Q = Q_o + \rho I$. Also let,

$$\sigma_o = \sigma(P_o, Q_o) = \lambda_m(Q_o) - 2\beta\lambda_M(P_o) - 2\|L^tP_o\|^2. \tag{29}$$

Define the $Sgn(.)$ function as the vector signum function defined such that,

$$Sgn(\sigma) = \left\{\begin{array}{c} sgn(\sigma_1)\\ sgn(\sigma_2)\\ \vdots\\ sgn(\sigma_m)\end{array}\right. \qquad \forall\ \sigma = \begin{pmatrix}\sigma_1\\ \sigma_2\\ \vdots\\ \sigma_m\end{pmatrix}$$

where $sgn(\sigma_i) = 1$ if $\sigma_i > 0$, $sgn(\sigma_i) = -1$ if $\sigma_i < 0$ and $sgn(\sigma_i) = 0$ if $\sigma_i = 0$.

Let W_1 be a positive scalar. The following result is established.

Theorem 4.1. *Suppose that* **A1-A4** *are satisfied. Then the memoryless state-feedback control law:*

$$u(t) = -\mu B_o^tP_ox(t) - W_1Sgn(B_o^tP_ox(t)) \tag{30}$$

renders the uncertain state-delay system (18) **robustly stabilizable** *with degree* κ *provided that,*

$$\sigma_o > 0 \text{ and } \mu > \mu_o = 1 + \frac{\alpha^2\sigma_o^{-1}}{2}. \tag{31}$$

Proof: The time derivative of the Lyapunov function candidate $V_1(.)$ in (26) evaluated along the trajectories of (18) is given by,

$$\begin{aligned} \dot{V}_1(x) &= e^{2\kappa t}\left[x^t(t)P_o\dot{x}(t) + \dot{x}^t(t)P_o x(t) + 2\kappa x^t(t)P_o x(t)\right. \\ &\quad \left.+\rho x^t(t)x(t) - \rho e^{-2\kappa\eta}(1-\dot{\eta})x^t(t-\eta)x(t-\eta)\right] \\ &= e^{2\kappa t}\left[x^t(t)\left(P_o(A_o+\kappa I) + (A_o+\kappa I)^t P_o + \rho I\right)x(t) + 2x^t(t)P_oB_ou(t)\right. \\ &\quad \left.+2x^t(t)P_o\zeta(x,t) + 2x^t(t)P_oDx(t-\eta) - \rho e^{-2\kappa\eta}(1-\dot{\eta})x^t(t-\eta)x(t-\eta)\right] \end{aligned} \tag{32}$$

By substituting (20), (22), (23) and (30) into (32), we get

$$\begin{aligned} \dot{V}_1(x) &= e^{2\kappa t}\Big[-x^t(t)Q_ox(t) - 2\mu x^t(t)P_oB_oB_o^tP_ox(t) - 2W_1x^t(t)P_oB_oSgn(B_o^tP_ox(t)) \\ &\quad +2x^t(t)P_o\zeta(x,t) + 2x^t(t)P_oDx(t-\eta) - \rho e^{-2\kappa\eta}(1-\dot{\eta})x^t(t-\eta)x(t-\eta)\Big] \\ &= e^{2\kappa t}\Big[-x^t(t)Q_ox(t) - 2\mu x^t(t)P_oB_oB_o^tP_ox(t) - 2W_1x^t(t)P_oB_oSgn(B_o^tP_ox(t)) \\ &\quad + 2x^t(t)P_o\left(B_oh+g\right) + 2x^t(t)P_o\left(B_oG+L\right)x(t-\eta) \\ &\quad -\rho e^{-2\kappa\eta}(1-\dot{\eta})x^t(t-\eta)x(t-\eta)\Big] \end{aligned} \tag{33}$$

Algebraic manipulation of (33) yields:

$$\begin{aligned} \dot{V}_1(x) \leq e^{2\kappa t}&\Big[-\lambda_m(Q_o)\|x(t)\|^2 - 2\mu\|B_o^tP_ox\|^2 + 2x^t(t)P_oB_oh + 2x^t(t)P_og \\ &+ 2x^t(t)P_oB_oGx(t-\eta) + 2x^t(t)P_oLx(t-\eta) \\ &-\rho e^{-2\kappa\eta^*}(1-\eta^+)x^t(t-\eta)x(t-\eta)\Big] \end{aligned} \tag{34}$$

Application of **Fact 1** to (34), using (21) and rearranging terms gives:

$$\begin{aligned} \dot{V}_1(x) \leq\, & e^{2\kappa t}\Big[-\lambda_m(Q_o)\|x(t)\|^2 - 2\mu\|B_o^tP_ox\|^2 + 2\alpha\|B_o^tP_ox\|, \|x\| + 2\beta\lambda_M(P_o)\|x\|^2 \\ &+ 2x^t(t)P_oB_oB_o^tP_ox(t) + 2x^t(t-\eta)G^tGx(t-\eta) + 2x^t(t)P_oLL^tP_ox(t) \\ &+ 2x^t(t-\eta)x(t-\eta) - \rho e^{-2\kappa\eta^*}(1-\eta^+)x^t(t-\eta)x(t-\eta)\Big] \\ \leq\, & e^{2\kappa t}\Big[-\left\{\lambda_m(Q_o) - 2\beta\lambda_M(P_o) - 2\|L^tP_o\|^2\right\}\|x\|^2 - 2\left(\mu-1\right)\|B_o^tP_ox\|^2 \\ &+ 2\,\alpha\|B_o^tP_ox\|\,\|x\| + \left(2+2\omega-\rho e^{-2\kappa\eta^*}(1-\eta^+)\right)\|x(t-\eta)\|^2\Big] \end{aligned} \tag{35}$$

Using (28), we get from (35) the inequality:

$$\dot{V}_1(x) \leq -e^{2\kappa t}\begin{pmatrix}\|x\| & \|B_o^tP_ox\|\end{pmatrix}\Omega_1\begin{pmatrix}\|x\| \\ \|B_o^tP_ox\|\end{pmatrix} \tag{36}$$

$$\Omega_1 = \begin{bmatrix}\sigma_o & -\alpha \\ -\alpha & 2(\mu-1)\end{bmatrix}$$

It is readily seen that the positive-definiteness of Ω_1 is satisfied when:

$$\sigma_o > 0 \text{ and } 2\left(\mu-1\right)\sigma_o > \alpha^2. \tag{37}$$

Simple rearrangement of (37) entails conditions (31). Therefore, we have for all realizations of uncertainties and $\forall t \in \Re$, $x \in \Re^n$, the Lyapunov stability condition:

$$\dot{V}_1(x) \leq -\epsilon e^{2\kappa t}\|x\|^2 \leq 0 \text{ for } \epsilon > 0. \tag{38}$$

By (20)-(22), the solution of the controlled uncertain system (18) is given by:

$$\begin{aligned} \dot{x}(t) = &\left(A_o - \mu B_oB_o^tP_o\right)x(t) + B_o\left(h(x,t) + Gx(t-\eta)\right) \\ &+ g(x,t) + Lx(t-\eta) - W_1B_oSgn(B_o^tP_ox(t)). \end{aligned} \tag{39}$$

From the previous analysis, it can be concluded that the solution of (39) satisfies the state-norm inequality

$$\|x(\tau)\| \leq \sqrt{\frac{\lambda_M(P_o) + \rho\eta^*}{\lambda_m(P_o)}}\, \|x(\xi)\|\, e^{-\kappa(\tau-\xi)}, \quad \forall \tau \geq \xi. \tag{40}$$

In turn, this implies that,

$$\|x(\tau)\|_* \leq \sqrt{\frac{\lambda_M(P_o) + \rho\eta^*}{\lambda_m(P_o)}}\, \|x(\xi)\|_*\, e^{-\kappa(\tau-\xi)}\ , \ \forall \tau \geq \xi. \tag{41}$$

Therefore, we conclude that system (18) is robustly stabilizable by controller (30) and has a degree of stability $\kappa > 0$.

Remark 4.1. *The term $-W_1 Sgn(B_o^t P_o x(t))$ is added to the control law in (30) because it is expected that this term will enhance the robustness of the closed loop system.*

Remark 4.2. *The case of constant delay where $\eta(t) = \eta^* = d$ and hence $\dot{\eta} = 0$ can be easily obtained from the previous result. In this case, the dynamic model is given by:*

$$\dot{x}(t) = A_o x(t) + B_o u(t) + Dx(t-d) + \zeta(x,t) \tag{42}$$

and we use the proposed controller for robust feedback synthesis based on the Lyapunov function (26). In this case, set

$$\rho_* = 2e^{2\kappa d}(1+\omega) \tag{43}$$

and choose a matrix $0 < Q_* = Q_*^t \in \Re^{n\times n}$ such that $0 < P_* = P_*^t \in \Re^{n\times n}$ is the solution of (23) for $Q = Q_* + \rho_* I$. Define

$$\sigma_* = \sigma(P_*, Q_*) := \lambda_m(Q_*) - 2\beta\lambda_M(P_*) - 2\|L^t P_*\|^2 \tag{44}$$

Therefore, we obtain the following corollary for $W_1 > 0$.

Corollary 4.1. *Suppose that* **A1-A4** *are satisfied. Then the control law:*

$$u(t) = -\mu B_o^t P_* \mathbf{x}(t) - W_1 Sgn(B_o^t P_* x(t))$$

renders the uncertain system (42) **robustly stabilizable** *with a degree κ provided that*

$$\sigma_* > 0, \quad \mu > \mu_* = 1 + \frac{\alpha^2\sigma_*^{-1}}{2} \tag{45}$$

The proof follows the procedure of Theorem 4.1.

5. **Design of a Robust Feedback Controller with a Delay Term.** Now, we consider the case of constant delay where $\eta(t) = \eta^* = d$ and hence $\dot{\eta} = 0$. In this case the dynamic model of the system is given by,

$$\dot{x}(t) = A_o x(t) + B_o u(t) + Dx(t-d) + \zeta(x,t). \tag{46}$$

Let W_2 be a positive scalar. We now propose the controller,

$$u(t) = -\mu B_o^t \bar{P} x(t) + \bar{K} x(t-d) - W_2 Sgn(B_o^t \bar{P} x(t)) \tag{47}$$

which combines the effect of the instantaneous as well as the delay states. This controller provides three degrees of freedom: one by the proportional term $-\mu B_o^t \bar{P} x(t)$ and the other through the delay term $\bar{K} x(t-d)$ and the third through $-W_2 Sgn(B_o^t \bar{P} x(t))$ term. Note that the third term is used to enhance the robustness of the closed loop system.

To study the stability behavior in this case, we define a quadratic Lyapunov function candidate as

$$V_2(x) = e^{2\kappa t} x^t(t)\bar{P} x(t) + \int_{t-d}^{t} e^{2\kappa\tau} x_t^t(\tau) x_t(\tau) d\tau; \quad \kappa > 0 \tag{48}$$

Choose a matrix $0 < \bar{Q} = \bar{Q}^t \in \Re^{n\times n}$ such that $0 < \bar{P} = \bar{P}^t \in \Re^{n\times n}$ is the solution of (23) for $\bar{Q} + I$. Let

$$\bar{\sigma} = \sigma(\bar{P}, \bar{Q}) := \lambda_m(\bar{Q}) - 2\beta\lambda_M(\bar{P}) - 2\|L^t\bar{P}\|^2 \tag{49}$$

$$\zeta = 2(\mu - 2) \ \text{ and } \ \upsilon = -2(1+\omega) + e^{-2\kappa d} \tag{50}$$

The stability behavior is established by the following theorem.

Theorem 5.1. *Suppose that* **A1-A4** *are satisfied. Then the proportional-delay feedback control (47) renders the uncertain system (46) robustly stabilizable with degree* $\kappa > 0$ *provided that*

$$\lambda_m(\bar{Q}) > 2\beta\lambda_M(\bar{P}) + 2\|L^t\bar{P}\|^2 \tag{51}$$

$$\mu > \bar{\mu} = 2 + \frac{\alpha^2\bar{\sigma}^{-1}}{2}, \tag{52}$$

$$\|\bar{K}\| < \bar{k} = \sqrt{\upsilon/2}. \tag{53}$$

Proof: The time derivative of the Lyapunov function candidate $\dot{V}_2(.)$ evaluated along the solutions of system (46) while using the controller (47) is given by,

$$\begin{aligned}\dot{V}_2(x) =& e^{2\kappa t}\big[x^t(t)\bar{P}\dot{x}(t) + \dot{x}^t(t)\bar{P}x(t) + 2\kappa x^t(t)\bar{P}x(t) + x^t(t)x(t)\\ &-e^{-2\kappa d}x^t(t-d)x(t-d)\big]\\ =& e^{2\kappa t}\big[x^t(t)\big(\bar{P}(A_o+\kappa I) + (A_o+\kappa I)^t\bar{P} + I\big)x(t) - 2\mu x^t(t)\bar{P}B_oB_o^t\bar{P}x(t)\\ &+2x^t(t)\bar{P}B_o\bar{K}x(t-d) - 2W_2x^t(t)\bar{P}B_oSgn(B_o^t\bar{P}x(t)) + 2x^t(t)\bar{P}\zeta(x,t)\\ &+2x^t(t)\bar{P}Dx(t-d) - e^{-2\kappa d}x^t(t-d)x(t-d)\big]\end{aligned} \tag{54}$$

By substituting (20), (22) and (23) into (54), we get,

$$\begin{aligned}\dot{V}_2(x) \le& e^{2\kappa t}\big[-\lambda_m(\bar{Q})\|x(t)\|^2 - 2\mu\|B_o^t\bar{P}x\|^2 + 2x^t(t)\bar{P}B_o\bar{K}x(t-d)\\ &+2x^t(t)\bar{P}B_oh + 2x^t(t)\bar{P}g + 2x^t(t)\bar{P}B_oGx(t-d) + 2x^t(t)\bar{P}Lx(t-d)\\ &-e^{-2\kappa d}x^t(t-d)x(t-d)\big]\end{aligned} \tag{55}$$

Application of **Fact 1** to (55) and rearranging terms gives:

$$\begin{aligned}\dot{V}_2(x) \le\ & e^{2\kappa t}\big[-\lambda_m(\bar{Q})\|x(t)\|^2 - 2\mu\|B_o^t\bar{P}x\|^2 + 2x^t(t)\bar{P}B_oB_o^t\bar{P}x(t)\\ &+2x^t(t-d)\bar{K}^t\bar{K}x(t-d) + 2\alpha\|B_o^t\bar{P}x\|\,\|x\| + 2\beta\lambda_M(\bar{P})\|x\|^2\\ &+2x^t(t)\bar{P}B_oB_o^t\bar{P}x(t) + 2x^t(t-d)G^tGx(t-d)\\ &+2x^t(t)\bar{P}LL^t\bar{P}x(t) + 2x^t(t-d)x(t-d) - e^{-2\kappa d}x^t(t-d)x(t-d)\big]\\ \le\ & e^{2\kappa t}\big[-\big\{\lambda_m(\bar{Q}) - 2\beta\lambda_M(\bar{P}) - 2\|L^t\bar{P}\|^2\big\}\,\|x\|^2 - 2\,(\mu-2)\,\|B_o^t\bar{P}x\|^2\\ &+2\alpha\|B_o^t\bar{P}x\|\,\|x\| + \big(2+2\omega+2\|\bar{K}\|^2 - e^{-2\kappa d}\big)\,\|x(t-d)\|^2\big]\\ \le\ & e^{2\kappa t}\big[-\bar{\sigma}\,\|x\|^2 - \zeta\,\|B_o^t\bar{P}x\|^2 + 2\alpha\|B_o^t\bar{P}x\|\,\|x\|\\ &-\big(\upsilon - 2\|\bar{K}\|^2\big)\,\|x(t-d)\|^2\big]\end{aligned} \tag{56}$$

Upon using (49) we get the inequality:

$$\dot{V}_2(x) \le -e^{2\kappa t}\begin{pmatrix}\|x\| & \|B_o^t\bar{P}x\| & \|x(t-d)\|\end{pmatrix}\Omega_2\begin{bmatrix}\|x\|\\ \|B_o^t\bar{P}x\|\\ \|x(t-d)\|\end{bmatrix} \tag{57}$$

where,

$$\Omega_2 = \begin{bmatrix}\bar{\sigma} & -\alpha & 0\\ -\alpha & \zeta & 0\\ 0 & 0 & \upsilon - 2\|\bar{K}\|^2\end{bmatrix}.$$

The positive definiteness of Ω_2 is satisfied when

$$\bar{\sigma} > 0, \quad \bar{\sigma}\zeta - \alpha^2 > 0 \text{ and } \upsilon - 2\|\bar{K}\|^2 > 0 \tag{58}$$

Algebraic manipulation of (58) in view of (49) and (50) gives immediately conditions (51)-(53). Therefore, we have for all realizations of uncertainties and $\forall t \in \Re$, $\forall x \in \Re^n$ the Lyapunov stability condition:

$$\dot{V}_2(x) \leq -\epsilon e^{2\kappa t}\|x\|^2 \leq 0 \tag{59}$$

is satisfied for some $\epsilon > 0$. The remaining part is obtained in a similar manner to the proof of Theorem 4.1.

6. **Simulation Results.** The controllers proposed in the previous two sections are applied to the power system presented in Section 2. The performances of the system are simulated using the MATLAB software. The numerical parameters of the power system are given as follows.

The parameters of the synchronous machine are:

$$\begin{array}{lll} \omega_o = 314.16\text{rad./sec.} & M = 6.92 & K_d = -0.027 \\ x_d = 1.24\text{p.u.} & x_q = 0.743\text{p.u.} & x'_d = 0.022\text{p.u.} \\ x_e = 0.8\text{p.u.} & P_M = 0.437\text{p.u.} & E = 1.0\text{p.u.} \\ T'_{d0} = 5.0\text{sec.} & \delta_o = 20^o & E'_{qo} = 1.05\text{p.u.} \end{array}$$

The parameters of the AVR and excitation system are:

$$K_A = 250 \quad T_A = 0.001\text{sec.} \quad V_{ref} = 1.0\text{p.u.}$$

The parameters of the governor system are:

$$a = -0.001238 \quad b = -0.17 \quad T_g = 0.25\text{sec.}$$

The parameters of the conventional PSS are:

$$K_J = 12 \quad T_Q = 2.5\text{sec.} \quad T_1 = 0.1\text{sec.} \quad T_2 = 0.03\text{sec.}$$

The initial condition of the system is taken as, $x_o = [0.7859 \;\; 0 \;\; 1.0291 \;\; 1.2075 \;\; 0 \;\; 0 \;\; 0]$. The time delay τ is 0.7. It should be noted that since the delay factor τ occurs in the dynamics of the governor and the PSS, it can be estimated using off-line computation. The disturbance used is $\zeta(t) = [\sin(2\pi ft) \quad \cos(2\pi ft) \quad \sin(2\pi ft) \quad \cos(2\pi ft) \quad \sin(2\pi ft) \quad \cos(2\pi ft) \quad \sin(2\pi ft)]$ with $f = 50$Hz.

To show the need for controlling the power system with delays in the states, the linearized model of the system given by (16) is simulated when the time delay $\tau = 0.7$. The simulation results are presented in Figures 2-4. Figure 2 shows the deviation of the rotor angle versus time; Figure 3 depicts the deviation of the rotor angular speed versus time. Figure 4 shows the deviation of the direct q-axis voltage versus time. It is clear from these figures that the uncontrolled system is unstable.

6.1. **Simulation results of the power system controlled using the first control scheme.** This subsection presents the simulation results of the power system when it is controlled using the control scheme given by Equation (30). The parameters of the controller are taken to be $\mu = 0.08$ and $W_1 = 0.8$.

Since the proposed controller assumes that the system matrix A_o is a stable matrix, the MATLAB command *eig* is used to compute the eigenvalues of A_o. The eigenvalues of A_o are found to be $[-0.048 + j7.377, -0.048 - j7.377, -0.857, -1000, -33.33, -0.4, -4.0]$ where $j^2 = -1$. Therefore, the matrix A_o is a stable matrix and the required condition is satisfied.

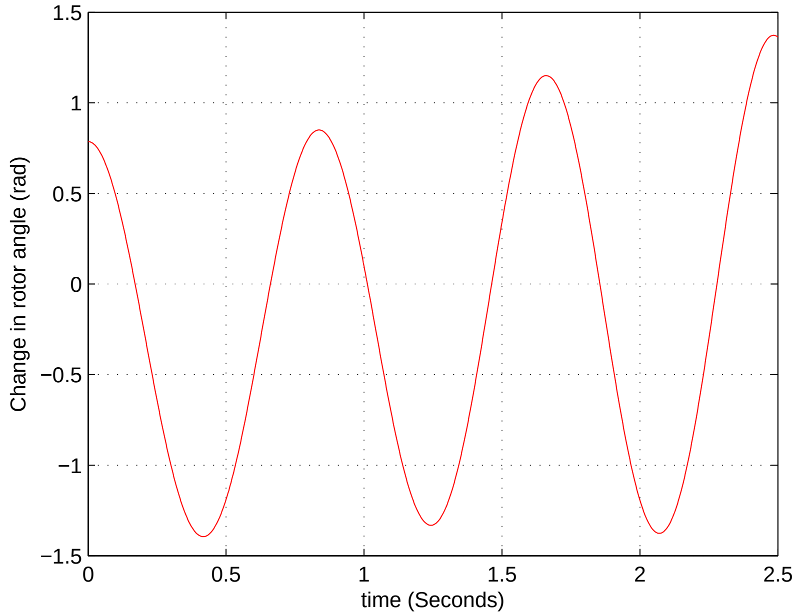

FIGURE 2. The deviation of the rotor angle $\delta(t)$ versus time (no control)

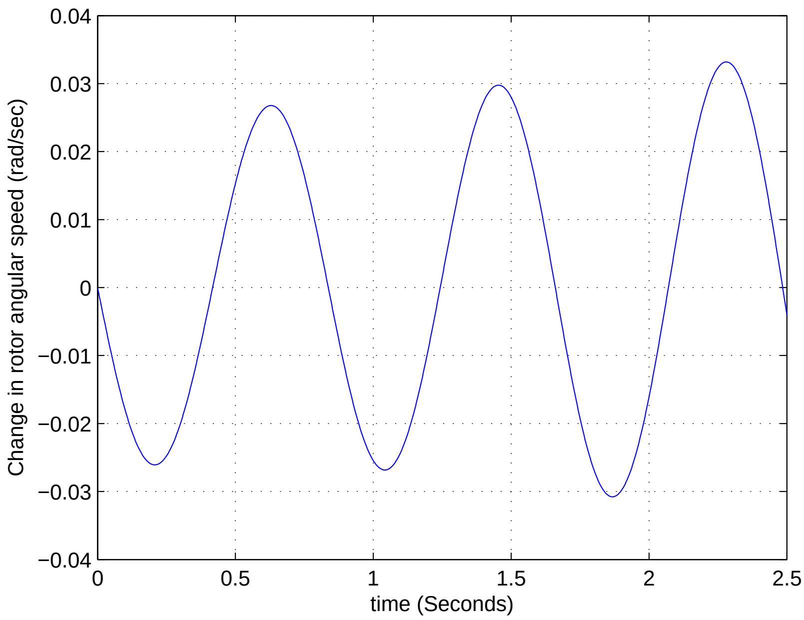

FIGURE 3. The deviation of the rotor angular speed $\omega(t)$ versus time (no control)

The following procedure is used to implement the control scheme given by Equation (30).

Step 1: Given the power system represented by (16), the parameters α and β are selected such that (21) is satisfied. Note that these parameters depend on the uncertainties present in the system.

Step 2: Find G and L such that (22) is satisfied. Note that the choice of G and L is not unique.

Step 3: Compute ω using the equation $\omega = \lambda_M(GG^t)$.

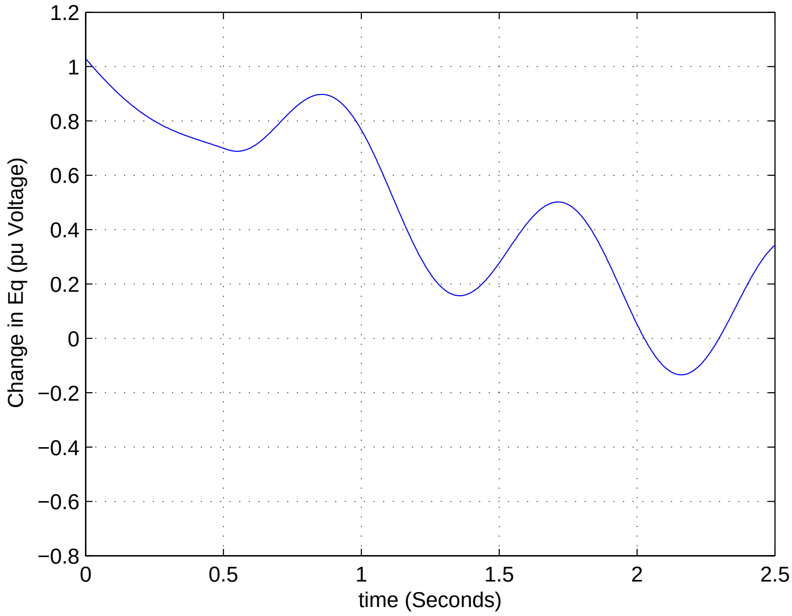

FIGURE 4. The deviation of the direct q-axis voltage $E_q(t)$ versus time (no control)

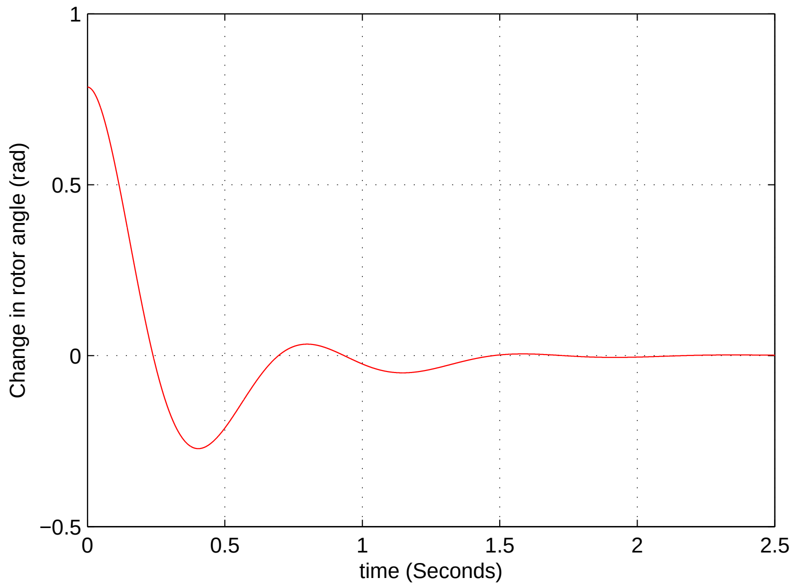

FIGURE 5. The deviation of the rotor angle $\delta(t)$ versus time (first controller)

Step 4: Select κ; one should start with a small value of κ and then increase this value incrementally. Note that κ represents some sort of stability margin.

Step 5: Using the bounds η^* and η^+, compute ρ from Equation (28). For constant time delay τ, $\eta^* = \eta = \tau$ and $\eta^+ = 0$

Step 6: Choose $Q_o > 0$ such that P_o is the solution of (23) with $Q = Q_o + \rho I$.

Step 7: Compute σ_o using Equation (29).

Step 8: Choose μ such that (31) is satisfied.

Step 9: Simulate and analyze the performance of the system when the memoryless state-feedback controller (30) is used.

At first, the system is simulated assuming that the system has no uncertainties. The simulation results are presented in Figures 5-7. Figure 5 shows the deviation of the rotor angle versus time; Figure 6 depicts the deviation of the rotor angular speed versus time. Figure 7 shows the deviation of the direct q-axis voltage versus time. The figures show that the deviations in the rotor angle and in the rotor angular speed converge to zero in less than 1.5 seconds. The deviation of the direct q-axis voltage converges to zero in about 2 seconds. Therefore, it can be concluded that the first control scheme works well when applied to the power system with state delays and without uncertainties.

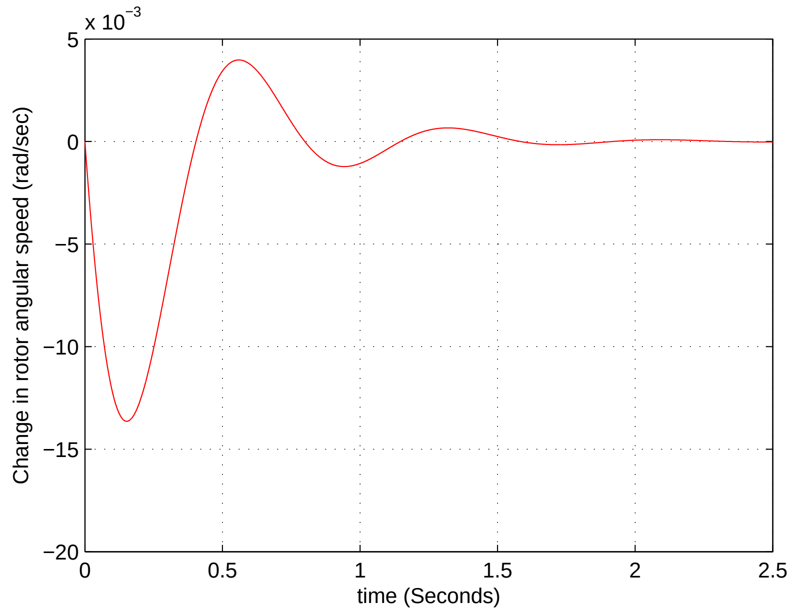

FIGURE 6. The deviation of the rotor angular speed $\omega(t)$ versus time (first controller)

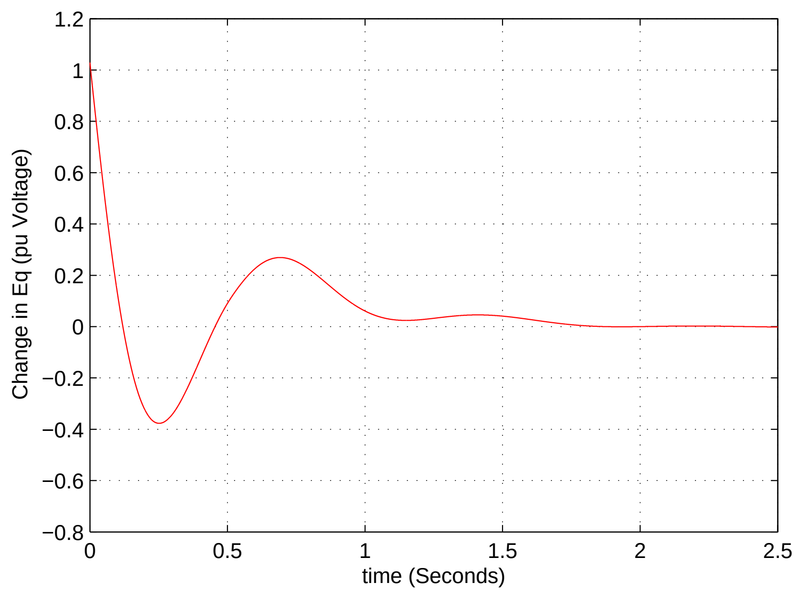

FIGURE 7. The deviation of the direct q-axis voltage $E_q(t)$ versus time (first controller)

Next, the performance of the uncertain linearized power system is simulated. The uncertainties are taken to be $\zeta(t)$. For these uncertainties, the parameters α, β, G and L are taken such that $\alpha = 0.002184$, $\beta = 0.003783$, G and L are such that

$$G = \begin{bmatrix} 0.02 & -0.841 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.00289 & 0.1214 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.0004 & 0 & -0.0001 & 0 & 0.005 & -0.09 \\ 0 & -0.68 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1333 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The simulation results are presented in Figures 8-10. Figure 8 shows the deviation of the rotor angle versus time; Figure 9 depicts the deviation of the rotor angular speed versus time. Figure 10 shows the deviation of the direct q-axis voltage versus time. The figures show that the deviations in the rotor angle and in the rotor angular speed converge to zero in less than 2.2 second. The deviation of the direct q-axis voltage converges to zero in about 2.0 seconds.

Therefore, it can be concluded that the proposed first controller is robust with respect to uncertainties in D and $\zeta(.,.)$ satisfying assumptions **A1-A3**.

6.2. **Simulation results of the power system controlled using the second control scheme.** This subsection presents the simulation results of the power system when it is controlled using the control scheme given by Equation (47). The parameters of the controllers are taken to be $\mu = 0.08$, $W_1 = 0.8$ and $\bar{K}$:

$$\bar{K} = \begin{bmatrix} 0.16 & 0.0003 & 0 & 0 & 0 & 0 & 0 \\ 0.0003 & .01 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

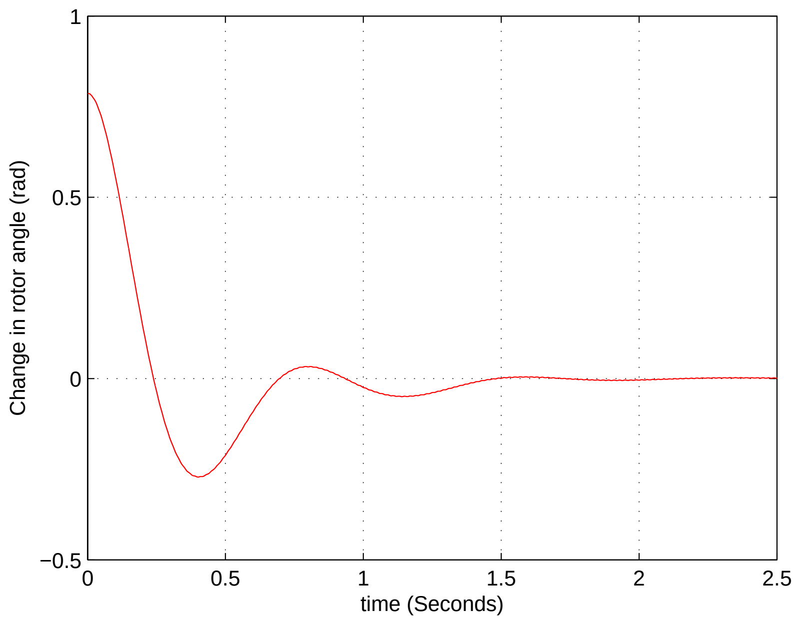

FIGURE 8. The deviation of the rotor angle $\delta(t)$ versus time (first controller + uncertainties)

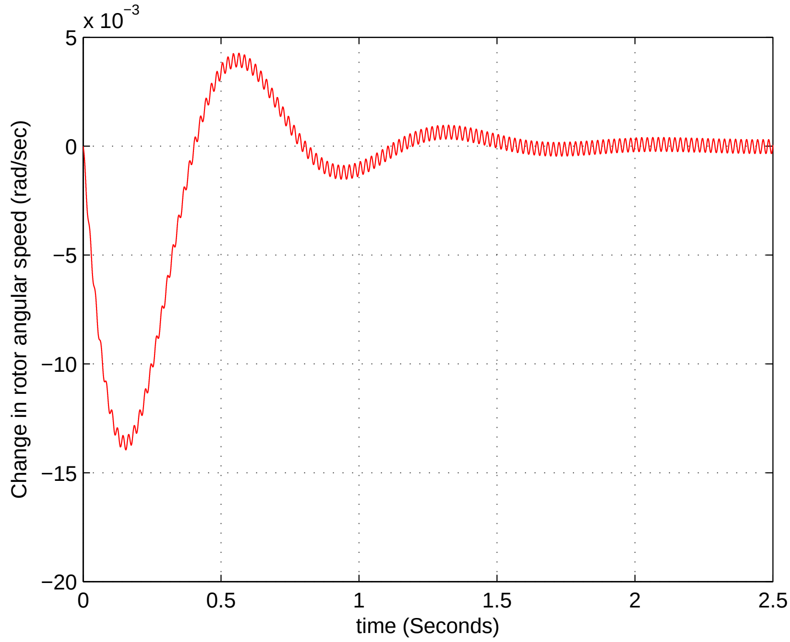

FIGURE 9. The deviation of the rotor angular speed $\omega(t)$ versus time (first controller + uncertainties)

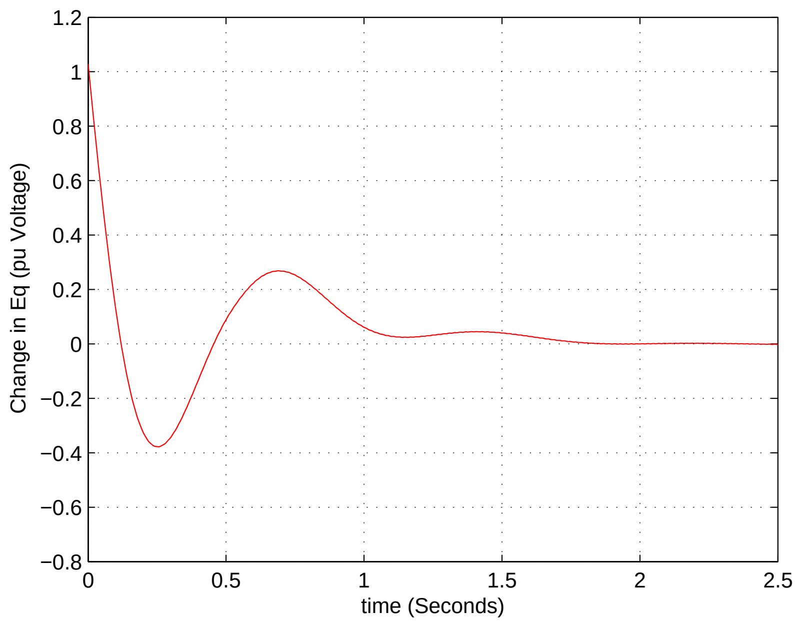

FIGURE 10. The deviation of the direct q-axis voltage $E_q(t)$ versus time (first controller + uncertainties)

A procedure similar to the procedure presented in the previous subsection is used to implement the control scheme given by Equation (47).

Again, the system is simulated assuming that the system has no uncertainties. The simulation results are presented in Figures 11-13. Figure 11 shows the deviation of the rotor angle versus time; Figure 12 depicts the deviation of the rotor angular speed versus time. Figure 13 shows the deviation of the direct q-axis voltage versus time. The figures show that the deviations in the rotor angle and in the rotor angular speed converge to zero in less than 1.7 seconds. The deviation of the direct q-axis voltage converges to zero in about 2.0 seconds. Therefore, it can be concluded that the second control scheme works well when applied to the power system with state delays and without uncertainties.

Next, the performance of the uncertain linearized power system is simulated. The simulation results are presented in Figures 14-16. Figure 14 shows the deviation of the rotor angle versus time; Figure 15 depicts the deviation of the rotor angular speed versus

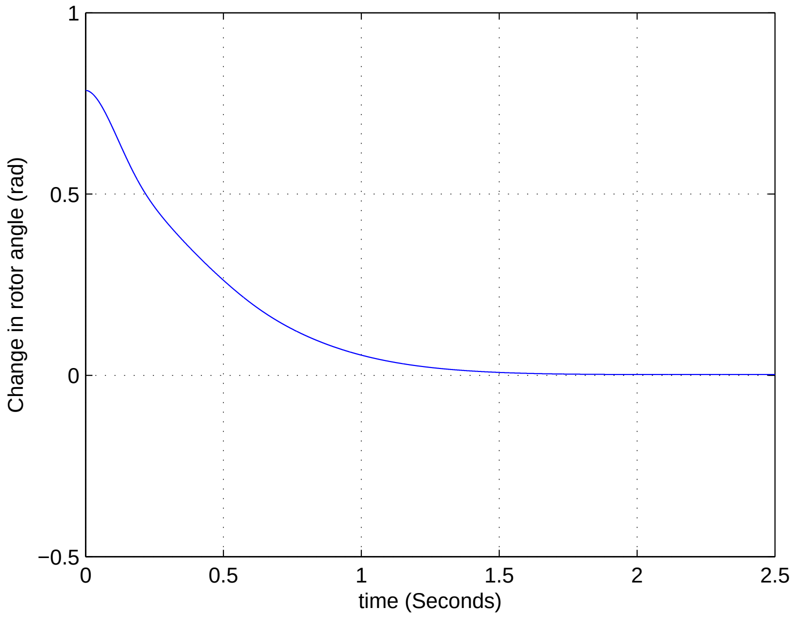

Figure 11. The deviation of the rotor angle $\delta(t)$ versus time (second controller)

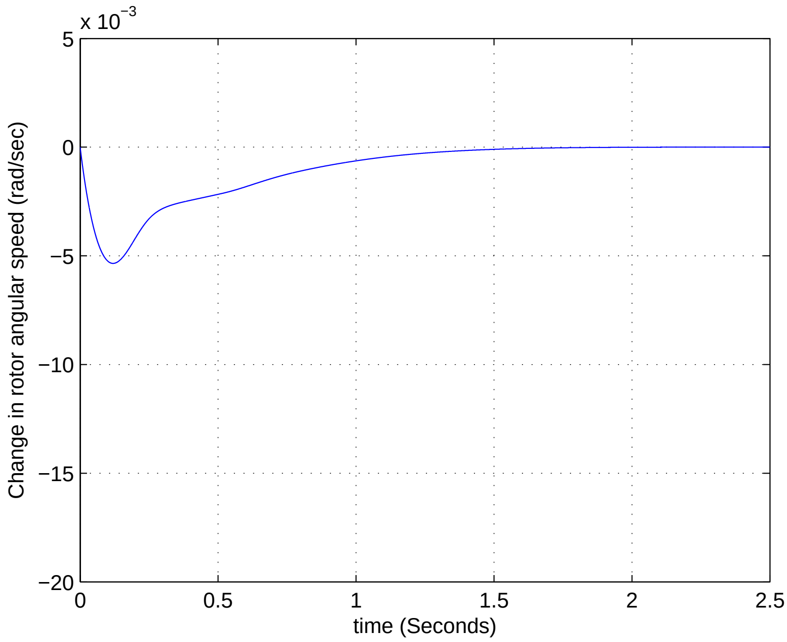

Figure 12. The deviation of the rotor angular speed $\omega(t)$ versus time (second controller)

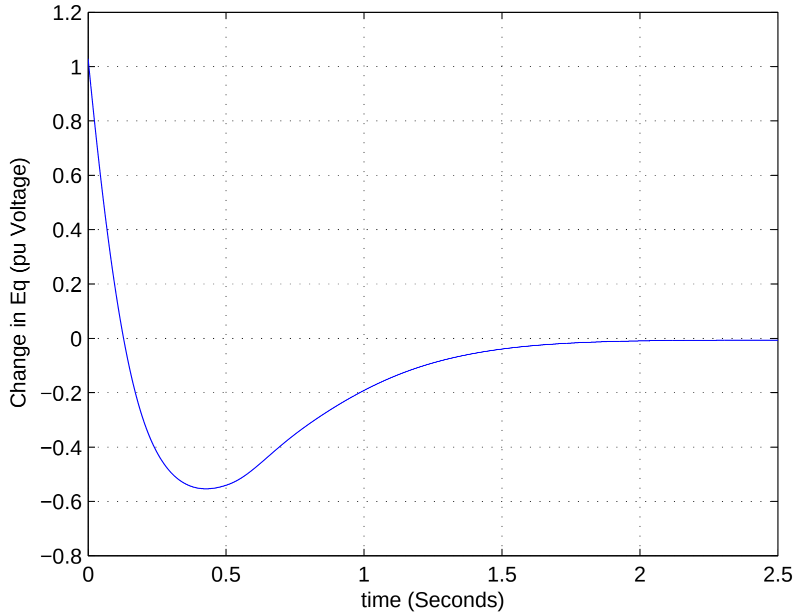

FIGURE 13. The deviation of the direct q-axis voltage $E_q(t)$ versus time (second controller)

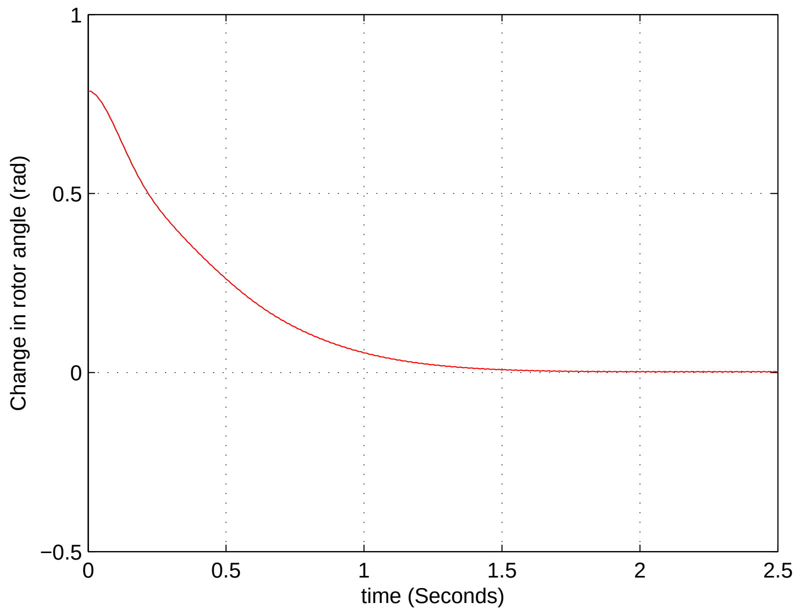

FIGURE 14. The deviation of the rotor angle $\delta(t)$ versus time (second controller + uncertainties)

time. Figure 16 shows the deviation of the direct q-axis voltage versus time. The figures show that the deviations in the rotor angle and in the rotor angular speed converge to zero in almost 1.5 seconds. The deviation of the direct q-axis voltage converges to zero in less than 2 seconds.

Therefore, it can be concluded that the proposed second controller is robust with respect to uncertainties in D and $\zeta(.,.)$ satisfying assumptions **A1-A3**.

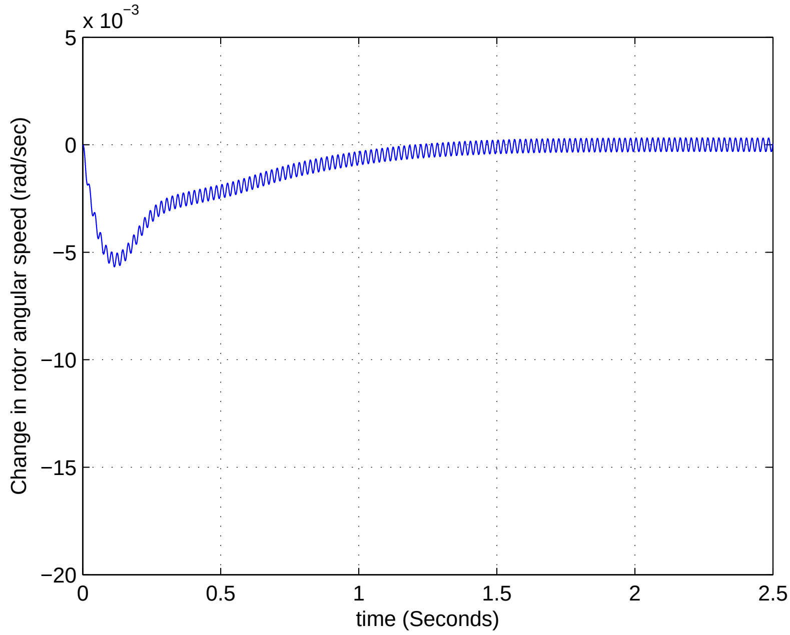

FIGURE 15. The deviation of the rotor angular speed $\omega(t)$ versus time (second controller + uncertainties)

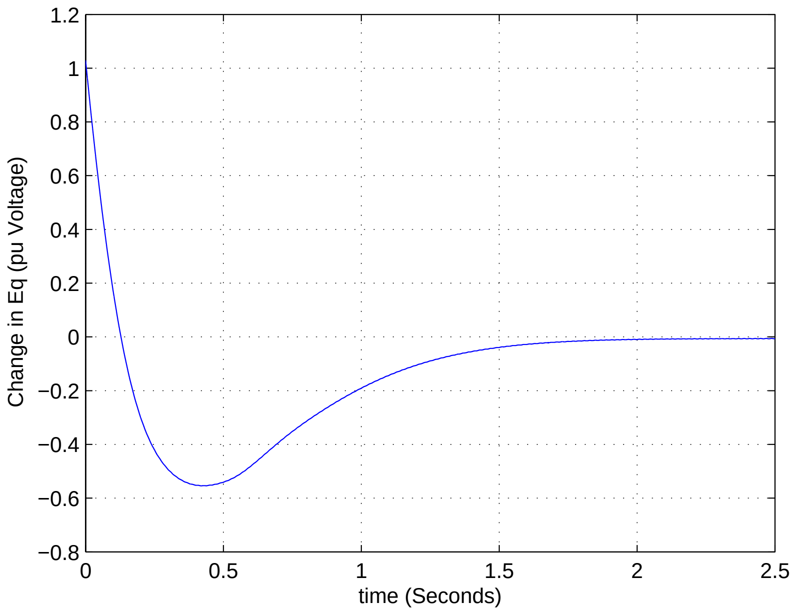

FIGURE 16. The deviation of the direct q-axis voltage $E_q(t)$ versus time (second controller + uncertainties)

6.3. **Comparison of the performances of the proposed control schemes.** For performance comparison purposes Figures 17-22 were generated. Figures 17-19 show the simulation results of the two controllers without uncertainties, while Figures 20-22 show the simulation results of the two controllers with uncertainties applied to the system parameters.

Figure 17 and Figure 20 show the deviation of the rotor angle versus time; Figure 18 and Figure 21 depict the deviation of the rotor angular speed versus time. Figure 19 and

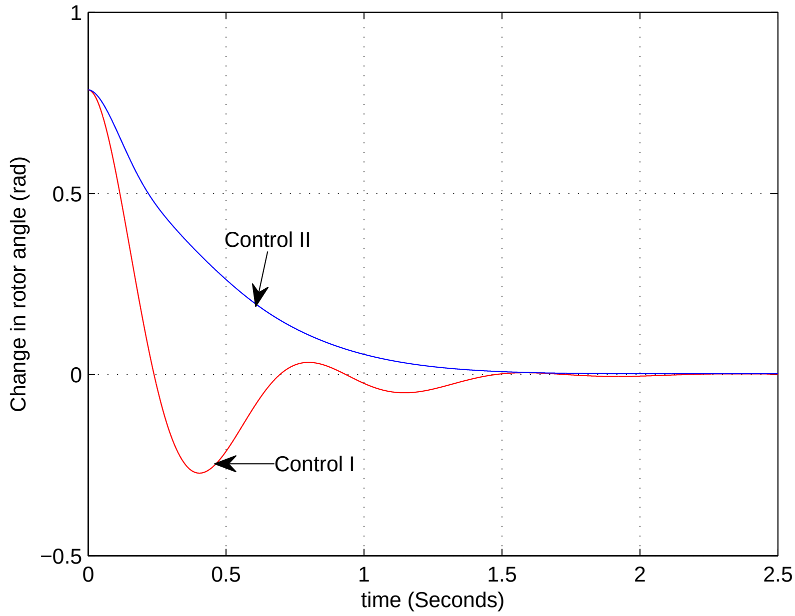

FIGURE 17. The deviation of the rotor angle $\delta(t)$ versus time (two controllers)

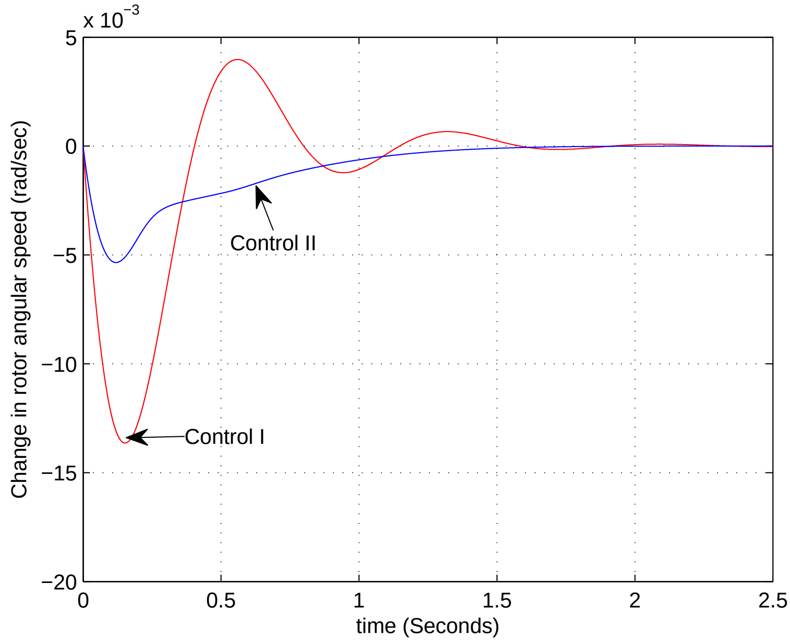

FIGURE 18. The deviation of the rotor angular speed $\omega(t)$ versus time (two controllers)

Figure 22 show the deviation of the direct q-axis voltage versus time. These figures clearly indicate that the second controller gave better results than the first controller. This is an expected result as the second controller contains the extra term $\bar{K}x(t-d)$; this term feeds back the delayed state of the system which enhances the performance of the system.

7. **Conclusion.** The control of a power system with state-delay and mismatched uncertainties is investigated in this paper. Two control schemes are proposed to achieve the exponential stability of the system. The first controller uses the instantaneous states for

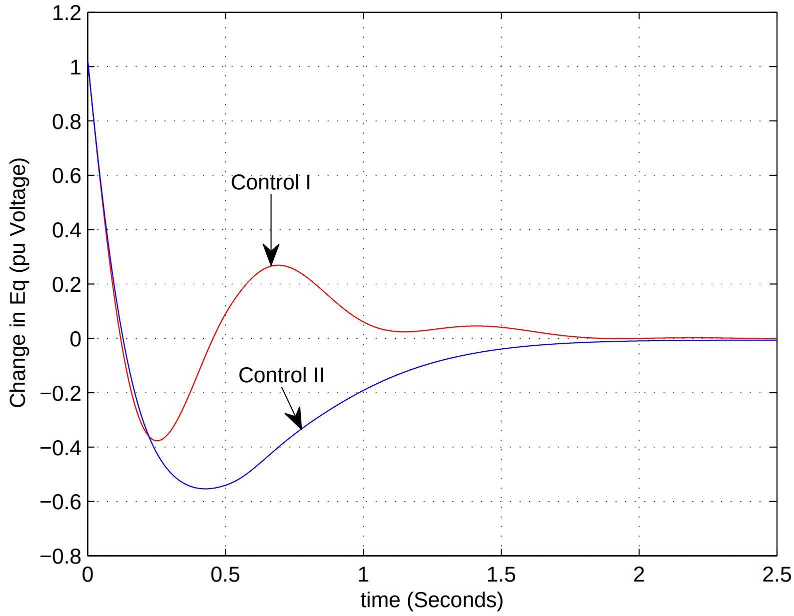

FIGURE 19. The deviation of the direct q-axis voltage $E_q(t)$ versus time (two controller)

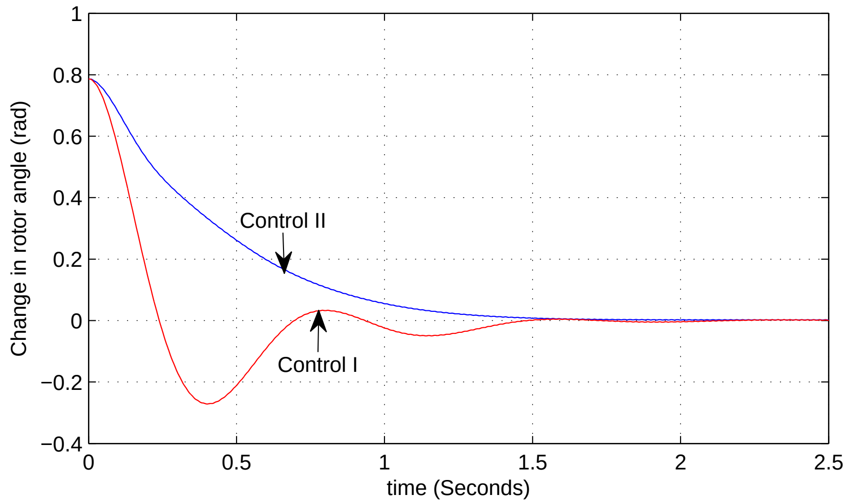

FIGURE 20. The deviation of the rotor angle $\delta(t)$ versus time (two controllers + uncertainties)

feedback; the second controller combines the effects of the instantaneous as well as the delayed states. The exponential stability of the closed loop system is shown using Lyapunov theory. The simulation results clearly show that the control schemes work well. Moreover, simulation results show that the proposed controllers are robust to mismatched and cone-bounded uncertainties.

Future work will address the design of observer based controllers for uncertain power systems with state delays.

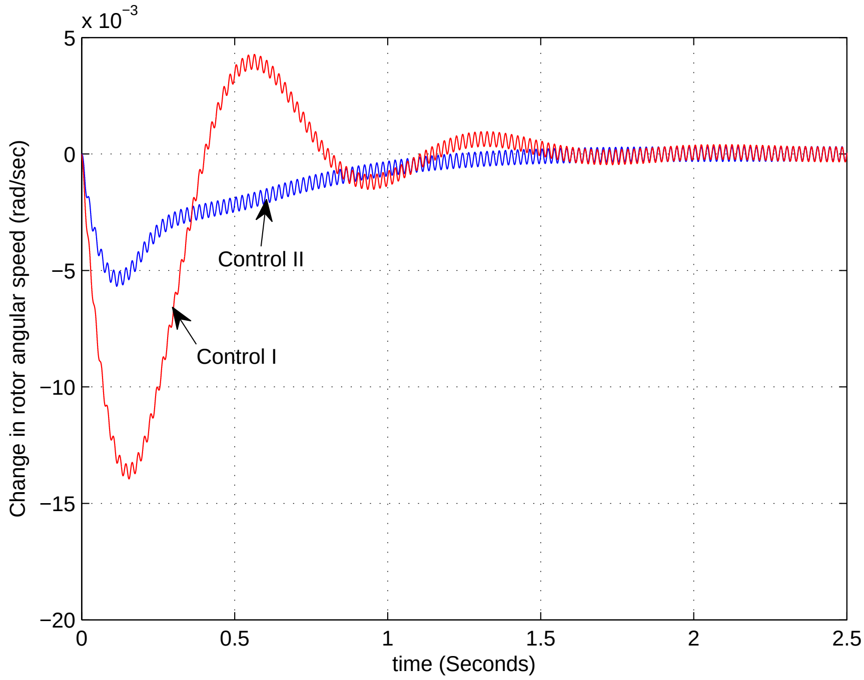

FIGURE 21. The deviation of the rotor angular speed $\omega(t)$ versus time (two controllers + uncertainties)

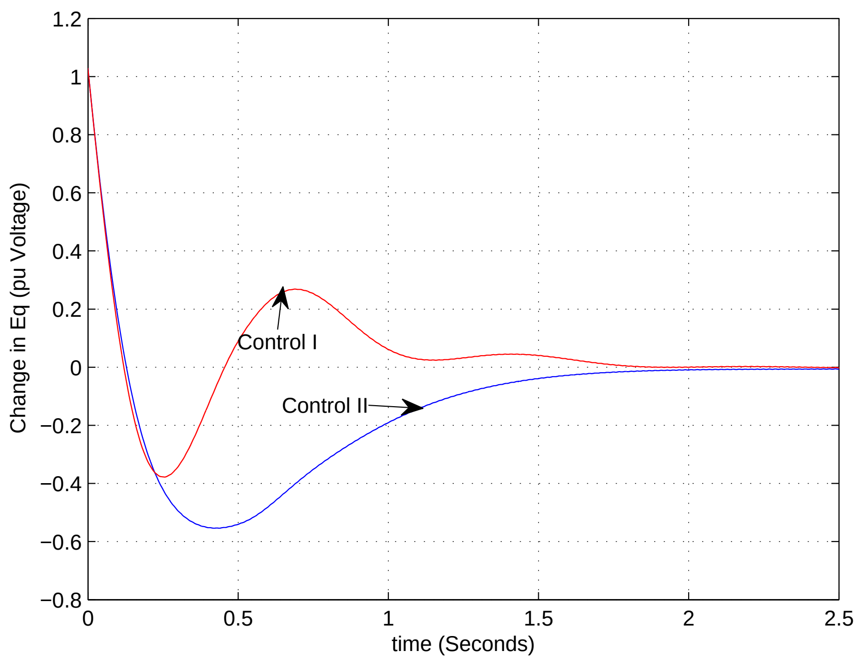

FIGURE 22. The deviation of the direct q-axis voltage $E_q(t)$ versus time (two controller + uncertainties)

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