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THE PARAMETERIZATION OF ALL SEMISTRONGLY STABILIZING CONTROLLERS

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Abstract. *When a control system is stabilized by a stable controller, the controller is said to be a strongly stabilizing controller. Using strongly stabilizing controllers, when an uncertainty in the plant or a step disturbance exists, the output of the control system cannot follow the step reference input without steady state error. From a practical point of view, it is better to stabilize plants by using controllers that have a pole at the origin and other poles in the open left-half plane. Those controllers are called semistrongly stabilizing controllers. From the literature, a semistrongly stabilizing controller does not necessarily exist for a plant. Hoshikawa et al. clarified the parameterization of all semistrongly stabilizable plants. However, the parameterization of all semistrongly stabilizing controllers has not previously been considered. In this paper, we clarify the parameterization of all semistrongly stabilizing controllers.*

Keywords: Robust servo, Semistrong stabilization, Parameterization

1. Introduction.

In the parameterization problem, all stabilizing controllers for a plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and all plants that can be stabilized [11] are sought. Because this parameterization can successfully search for all proper stabilizing controllers, it is used as a tool for many control problems.

For an unstable plant, the parameterization of all stabilizing controllers was solved by Youla et al. [1, 2]. The structure of the parameterization of all stabilizing controllers for unstable plants requires full-order state feedback, including a full-order observer [7]. Glaria and Goodwin [6] gave a simple parameterization for single-input/single-output minimum-phase systems. However, in their parameterization, two difficulties remain. One is that the parameterization of all stabilizing controllers given by Glaria and Goodwin generally includes improper controllers, whereas in practical applications, the controller is required to be proper. The other is that they do not give the parameterization of all internally stabilizing controllers. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for single-input/single-output minimum-phase systems [8].

For a stable plant, the parameterization of all stabilizing controllers has a structure identical to that of Internal Model Control. Two advantages of this are that closed-loop stability is assured simply by choosing a stable Internal Model Controller parameter, and closed-loop performance characteristics are related directly to controller parameters, which makes online tuning of the Internal Model Controller very convenient [5]. However, the question remains whether or not stabilizing controllers for unstable plants can be represented by the Internal Model Control structure. For this question, Morari and Zafiriou

[5] examined the parameterization of all stabilizing Internal Model Controllers for unstable plants. However, their parameterization has difficulties. First, their internal model is not necessarily proper. In addition, their parameterization includes improper Internal Model Controllers. To overcome these problems, Chen et al. proposed the simple parameterization of all proper stabilizing Internal Model Controllers for minimum-phase unstable plants [12]. Mai et al. [13] expanded the result in [12] and proposed a parameterization for nonminimum-phase unstable plants. Zhang et al. [9] proposed a new parameterization, which is a coprime factorization and has a similar form to the parameterization in [1, 2, 4]. In this way, the parameterization of all stabilizing controllers has been shown.

However, little attention has been paid to the stability of stabilizing controllers. In the case of an unstable stabilizing controller, its unstable poles make the closed-loop transfer function have zeros in the right-half plane. This makes the closed-loop system very sensitive to disturbances and reduces the performance when tracking reference inputs [4, 17]. In addition, if the feedback loop of the feedback control system is cut by breakdown, that is, if it becomes a feed-forward control system, the unstable poles of the stabilizing controller become unstable poles of the control system. Thus, the control system becomes unstable even if the plant is stable. For these reasons, it is desirable in practice that the control system is stabilized by a stable stabilizing controller [17]. Therefore, several methods for designing a stable stabilizing controller, which is called a strongly stabilizing controller, have been considered [4, 14, 15, 16, 17, 18].

Youla et al. clarified the necessary and sufficient condition for a plant to be stabilized by stable controllers [4, 14]. This condition is called the parity interlacing property (p.i.p.) and is used as a tool to confirm whether or not a plant is strongly stabilizable. In addition, Youla et al. proposed a method to find strongly stabilizing controllers using Nevanlinna-Pick interpolation [4, 14]. However, there is a problem that the resulting controller may become high-order with irrational functions [15, 19]. To design controllers and tune parameters easily, it is desirable that stabilizing controllers are low order and have rational functions. To overcome this problem, Dorato et al. [15], Ganesh and Pearson [20], and Ito et al. [16] proposed methods to find low-order, rational, strongly stabilizing controllers for single-input/single-output systems using Nevanlinna–Pick interpolation. In addition, Saif et al. proposed a method for multiple-input/multiple-output systems, also using Nevanlinna-Pick interpolation [21].

Using stable stabilizing controllers, when an uncertainty in the plant or a step disturbance exists, the output of the control system cannot follow the step reference input without steady state error. In many cases, the output is required to follow the step reference input without steady state error, even if there is a step disturbance or an uncertainty in the plant. To realize this requirement, controllers must have a pole at the origin. That is, it is better to stabilize plants by using controllers that have a pole at the origin and other poles in the open left-half plane. We call those controllers semistrongly stabilizing controllers. Because plants that are unstabilizable by strongly stabilizing controllers exist [14], it is expected that plants that are unstabilizable by a semistrongly stabilizing controller also exist. From this viewpoint, Hoshikawa et al. clarified the parameterization of all semistrongly stabilizable plants [22].

In this paper, we propose the parameterization of all semistrongly stabilizing controllers for the semistrongly stabilizable plants clarified in [22]. The control characteristics of the control system using the parameterization of all semistrongly stabilizing controllers are described. A design procedure for semistrongly stabilizing controllers is presented. A numerical example is presented to illustrate the effectiveness of the proposed method.

Notation

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.
$\mathcal{U}$	The set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.

2. **Problem Formulation.** Consider the control system:

$$\begin{cases} y(s) = G(s)u(s) + d(s) \\ u(s) = C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s) \in R$ is the output, $u(s) \in R$ is the control input, $d(s) \in R$ is the disturbance, and $r(s) \in R$ is the reference input.

Using stable stabilizing controllers, when an uncertainty in the plant $G(s)$ or a step disturbance $d(s)$ exists, the output $y(s)$ of the control system in (1) cannot follow the step reference input without steady state error. If the output $y(s)$ must follow the step reference input $r(s)$ without steady state error even if an uncertainty in the plant or a step disturbance exists, the controller must have a pole at the origin. From a practical point of view, it is better to stabilize plants by using controllers that have a pole at the origin and other poles in the open left-half plane. From this viewpoint, Hoshikawa et al. proposed the concept of semistrongly stabilizing controllers as follows.

Definition 2.1. *(Semistrongly stabilizing controllers)* [22]
We call the controller $C(s)$ in (1) a "semistrongly stabilizing controller" if the stabilizing controller has only one pole at the origin and other poles in the open left-half plane. That is, if $C(s)$ in (1) is written:

$$C(s) = \frac{s+\alpha}{s}Q_1(s), \tag{2}$$

then we call $C(s)$ in (1) a semistrongly stabilizing controller, where $\alpha \in R$ is any positive real number and $Q_1(s) \in RH_\infty$ is any function satisfying $Q_1(0) \neq 0$.

Definition 2.2. *(Semistrongly stabilizable plant)* [22]
We call $G(s)$ in (1) a "semistrongly stabilizable plant" if $G(s)$ in (1) can be stabilized by a semistrongly stabilizing controller $C(s)$ in (2).

Because plants unstabilizable by strongly stabilizing controllers exist [14], it is expected that plants unstabilizable by semistrongly stabilizing controllers also exist. According to [22], the parameterization of all semistrongly stabilizable plants is written:

$$G(s) = \frac{sQ_2(s) + \beta}{(s+\alpha)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)}, \tag{3}$$

where $\beta \in R$ is given by:

$$\beta = \frac{\alpha}{Q_1(0)}, \tag{4}$$

$Q_3(s) \in RH_\infty$ satisfies:

$$Q_3(s) = \frac{\alpha - \beta Q_1(s)}{s}, \tag{5}$$

and $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions satisfying $Q_1(0) \neq 0$.

In this paper, we clarify the parameterization of all semistrongly stabilizing controllers for the plant $G(s)$ in (3).

Remark 2.1. *Semistrongly stabilizable plants in (3) have five parameters, and so appear complicated. However, the problem that $G(s)$ is semistrongly stabilizable is equivalent to the problem that $(s+\alpha)G(s)/s$ is strongly stabilizable. Therefore, while Equation (3) seems complicated, its meaning is simple.*

3. **The Parameterization of All Semistrongly Stabilizing Controllers for Semistrongly Stabilizable Plants.** In this section, we propose the parameterization of all semistrongly stabilizing controllers $C(s)$ for the semistrongly stabilizable plant $G(s)$ in (3).

This parameterization is summarized in the following theorem.

Theorem 3.1. *The controller $C(s)$ is a semistrongly stabilizing controller for the semistrongly stabilizable plant $G(s)$ in (3) if and only if $C(s)$ is given by:*

$$C(s) = \frac{Q_1(s) + (1 + Q_3(s) - Q_1(s)Q_2(s))\, P(s)}{\dfrac{s}{s+\alpha} - \left(\dfrac{\beta}{s+\alpha} + \dfrac{sQ_2(s)}{s+\alpha}\right) P(s)}, \tag{6}$$

where $P(s)$ is given by:

$$P(s) = \frac{s}{s+\alpha} Q(s), \tag{7}$$

$Q(s) \in RH_\infty$ is given by:

$$Q(s) = \frac{1 - \hat{Q}(s)}{\dfrac{\beta}{s+\alpha} + \dfrac{s}{s+\alpha} Q_2(s)}, \tag{8}$$

$\hat{Q}(s) \in \mathcal{U}$ is any function that makes $Q(s)$ in (8) proper and satisfies:

$$\left.\frac{1}{(s-s_i)^{m_i-1}} \left(1 - \hat{Q}(s)\right)\right|_{s=s_i} = 0 \quad (\forall i = 1, \dots, n), \tag{9}$$

s_i $(i = 1, \dots, n)$ are unstable zeros of $\beta + sQ_2(s)$, and the multiplicities of s_i $(i = 1, \dots, n)$ are denoted by m_i $(i = 1, \dots, n)$.

Proof: From [4], the parameterization of all stabilizing controllers for $G(s)$, which are not necessarily semistrongly stabilizing controllers, is given by:

$$C(s) = \frac{X(s) + D(s)P(s)}{Y(s) - N(s)P(s)}, \tag{10}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying:

$$G(s) = \frac{N(s)}{D(s)}, \tag{11}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are any functions satisfying:

$$N(s)X(s) + D(s)Y(s) = 1 \tag{12}$$

and $P(s) \in RH_\infty$ is any function. Because the semistrongly stabilizable plant $G(s)$ takes the form of (3), $G(s)$ in (3) is factorized by (11), where:

$$N(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha} Q_2(s) \tag{13}$$

and

$$D(s) = 1 + Q_3(s) - Q_1(s)Q_2(s). \tag{14}$$

From (13) and (14), a pair of $X(s)$ and $Y(s)$ satisfying (12) is given by:

$$X(s) = Q_1(s) \tag{15}$$

and

$$Y(s) = \frac{s}{s+\alpha}. \tag{16}$$

Substituting (13), (14), (15), and (16) for (10), we have (6), where $P(s) \in RH_\infty$ is any function.

We now show that $C(s)$ in (6) is a semistrongly stabilizing controller if and only if $P(s)$ in (6) is given by (7), $Q(s)$ in (7) is given by (8), and $\hat{Q}(s)$ in (8) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (9).

To prove necessity, we show that if $C(s)$ in (6) is a semistrongly stabilizing controller, then $P(s)$ in (6) is given by (7), $Q(s)$ in (6) is given by (8), and $\hat{Q}(s)$ in (8) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (9). From the assumption that $C(s)$ in (6) is a semistrongly stabilizing controller and (6):

$$\frac{s}{s+\alpha} - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right) P(s)\bigg|_{s=0} = 0 \tag{17}$$

is satisfied. This equation yields:

$$P(0) = 0. \tag{18}$$

This equation implies that $P(s)$ is given by (7), where $Q(s) \in RH_\infty$. Substituting (7) and (5) for (10), (10) is rewritten as:

$$C(s) = \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q(s)}{1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q(s)}\right\}. \tag{19}$$

From the assumption that $C(s)$ in (6) is a semistrongly stabilizing controller,

$$\begin{aligned} \bar{C}(s) &= \frac{s}{s+\alpha}C(s) \\ &= Q_1(s) + \frac{Q(s)}{1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q(s)} \end{aligned} \tag{20}$$

must be included in RH_∞. Because $Q_1(s) \in RH_\infty$ and $Q(s) \in RH_\infty$, the condition of $\bar{C}(s) \in RH_\infty$ in (20) is equivalent to:

$$1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q(s) \in \mathcal{U}. \tag{21}$$

Using $\hat{Q}(s) \in \mathcal{U}$, let

$$1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q(s) = \hat{Q}(s). \tag{22}$$

Equation (22) corresponds to (8). Because s_i $(i = 1, \dots, n)$ are unstable zeros of $\beta + sQ_2(s)$ and the multiplicities of s_i $(i = 1, \dots, n)$ are denoted by m_i $(i = 1, \dots, n)$,

$$\frac{1}{(s-s_i)^{m_i - 1}}\left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)Q(s)\bigg|_{s=s_i} = 0 \quad (\forall i = 1, \dots, n) \tag{23}$$

holds true. From (22) and (23), (9) is satisfied. Thus, the necessity has been shown.

Next, to prove sufficiency, we show that if $Q(s)$ in (6) is given by (8) and $\hat{Q}(s)$ in (8) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (9), then $C(s)$ in (6) is a semistrongly stabilizing controller. Substituting (8) for (6), we have:

$$\begin{aligned} C(s) &= \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{1-\hat{Q}(s)}{\left(\dfrac{\beta}{s+\alpha} + \dfrac{s}{s+\alpha}Q_2(s)\right)\hat{Q}(s)}\right\} \\ &= \frac{s+\alpha}{s}\left\{Q_1(s) + \frac{Q(s)}{\hat{Q}(s)}\right\}. \end{aligned} \tag{24}$$

Because $\hat{Q}(s) \in \mathcal{U}$ and $Q_1(s) \in RH_\infty$, if $Q(s) \in RH_\infty$, then $C(s)$ in (24) has a pole at the origin and other poles in the open left-half plane. Therefore, we show that $Q(s) \in RH_\infty$. From $\hat{Q}(s) \in \mathcal{U}$, if $Q(s)$ in (8) is unstable, unstable poles of $Q(s)$ are equal to unstable zeros s_i $(i = 1, \dots, n)$ of $\beta + sQ_2(s)$. Because $\hat{Q}(s)$ satisfies (9), unstable zeros s_i $(i = 1, \dots, n)$ of $\beta + sQ_2(s)$ are not equal to unstable poles of $Q(s)$. Therefore, $Q(s)$ is stable. In addition, $\hat{Q}(s)$ is selected to make $Q(s)$ in (8) proper, and $Q(s)$ in (8) satisfies $Q(s) \in RH_\infty$. Thus, $C(s)$ in (24) has a pole at the origin and other poles in the open left-half plane.

Next, we show that $C(s)$ in (24) makes the control system in (1) stable. By simple manipulation, we have:

$$\frac{G(s)C(s)}{1+G(s)C(s)} = 1 - \frac{s}{s+\alpha}\hat{Q}(s)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right), \tag{25}$$

$$\frac{G(s)}{1+G(s)C(s)} = \frac{s\left(\beta + sQ_2(s)\right)}{(s+\alpha)^2}\hat{Q}(s), \tag{26}$$

$$\frac{C(s)}{1+G(s)C(s)} = \left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)\left(Q_1(s)\hat{Q}(s) + Q(s)\right) \tag{27}$$

and

$$\frac{1}{1+G(s)C(s)} = \frac{s}{s+\alpha}\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)\hat{Q}(s). \tag{28}$$

Because $\alpha > 0$, $Q_1(s) \in RH_\infty$, $Q_2(s) \in RH_\infty$, $Q_3(s) \in RH_\infty$, $\hat{Q}(s) \in \mathcal{U}$, and $Q(s) \in RH_\infty$, the transfer functions in (25), (26), (27), and (28) are stable. This implies that the control system in (1) is stable.

We have thus proved Theorem 3.1. □

Next, we explain the control characteristics of the control system in (1) using the parameterization of all semistrongly stabilizing controllers in (1).

The transfer functions from the reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output $y(s)$ of the control system in (1) are written as:

$$\frac{y(s)}{r(s)} = 1 - \frac{s}{s+\alpha}\hat{Q}(s)\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right) \tag{29}$$

and

$$\frac{y(s)}{d(s)} = \frac{s}{s+\alpha}\left(1 + Q_3(s) - Q_1(s)Q_2(s)\right)\hat{Q}(s), \tag{30}$$

respectively. Therefore, using a semistrongly stabilizing controller $C(s)$ in (6), the output $y(s)$ follows the step reference input $r(s) = 1/s$ without steady state error and the step disturbance $d(s) = 1/s$ is attenuated effectively.

4. **Design Method for $\hat{Q}(s)$.** From Theorem 3.1, to design a semistrongly stabilizing controller $C(s)$, $\hat{Q}(s)$ in (8) must be designed to be $\hat{Q}(s) \in \mathcal{U}$ to satisfy (9) and to make $Q(s)$ in (8) proper. In this section, we present a design method to ensure that $\hat{Q}(s) \in \mathcal{U}$ has these characteristics.

The design method is summarized as follows.

1) We factorize:

$$\tilde{Q}(s) = \frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)$$

as

$$\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s) = \tilde{Q}_i(s)\tilde{Q}_o(s), \tag{31}$$

where $\tilde{Q}_i(s) \in RH_\infty$ is an inner function satisfying $\tilde{Q}_i(0) = 1$ and $\tilde{Q}_o(s) \in RH_\infty$ is an outer function.

2) Using $\tilde{Q}_o(s)$, we make $\bar{Q}(s) \in RH_\infty$:

$$\bar{Q}(s) = \frac{q(s)}{\tilde{Q}_o(s)}, \tag{32}$$

where:

$$q(s) = \frac{k}{(\tau s+1)^m}, \tag{33}$$

$\tau \in R$ is an arbitrary positive number, m is an arbitrary positive integer to make $\bar{Q}(s)$ proper, and $k \in R$ is a real number satisfying $0 < k < 1$.

3) Using $\bar{Q}(s)$, $\hat{Q}(s) \in \mathcal{U}$ is designed as:

$$\hat{Q}(s) = 1 - \left(\frac{\beta}{s+\alpha} + \frac{s}{s+\alpha}Q_2(s)\right)\bar{Q}(s). \tag{34}$$

Next, we show that $\hat{Q}(s)$ in (34) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (9), and makes $Q(s)$ in (8) proper. First, we show that $\hat{Q}(s)$ in (34) satisfies $\hat{Q}(s) \in \mathcal{U}$ and (9). Substituting (32) for (34), $\hat{Q}(s)$ in (34) is rewritten as:

$$\hat{Q}(s) = 1 - \tilde{Q}_i(s)q(s). \tag{35}$$

Because $\tilde{Q}_i(s)$ is an inner function, $\tilde{Q}_i(s)$ is biproper. That is, $\tilde{Q}_i(s)q(s)$ is strictly proper. In addition, from (33) and $0 < k < 1$,

$$\left\|\tilde{Q}_i(s)q(s)\right\|_\infty < 1. \tag{36}$$

This implies that $\hat{Q}(s) \in \mathcal{U}$.

Next, we show that (9) holds true. Because s_i $(i = 1, \ldots, n)$ are unstable zeros of $\beta + sQ_2(s)$, m_i $(i = 1, \ldots, n)$ denotes the multiplicities of s_i $(i = 1, \ldots, n)$, and $\tilde{Q}_i(s)$ is an inner function of $\beta/(s+\alpha) + sQ_2(s)/(s+\alpha)$,

$$\left.\frac{1}{(s-s_i)^{m_i-1}}\tilde{Q}_i(s)\right|_{s=s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{37}$$

holds true. From this equation and (33),

$$\left.\frac{1}{(s-s_i)^{m_i-1}}\tilde{Q}_i(s)q(s)\right|_{s=s_i} = 0 \quad (\forall i = 1, \ldots, n) \tag{38}$$

are also satisfied. From (35) and (38), $\hat{Q}(s)$ in (34) satisfies (9). Next, we show that $\hat{Q}(s)$ in (34) makes $Q(s)$ proper. Substituting (35) for (8), $Q(s)$ in (8) is rewritten as:

$$Q(s) = \bar{Q}(s). \tag{39}$$

Because $\bar{Q}(s) \in RH_\infty$, $Q(s)$ is proper. Therefore, $\hat{Q}(s)$ in (34) makes $Q(s)$ proper. Thus, we have shown that, using the method described above, we can design $\hat{Q}(s) \in \mathcal{U}$ to satisfy (9) and make $Q(s)$ in (8) proper.

5. **Numerical Example.** In this section, we present a numerical example to show the effectiveness of the proposed parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants.

Consider the problem of designing a semistrongly stabilizing controller $C(s)$ for the angular velocity control of the two-inertia system in Figure 1. Here, τ_M is the torque of the motor, J_M is the moment of inertia of the motor, D_M is the coefficient of friction of the motor, J_L is the moment of inertia of the load, D_L is the coefficient of friction of the load, K is the torsional spring constant, and ω_L is the angular velocity of the load. For our example, we use the values $J_M = 2.0 \cdot 10^{-4}$, $D_M = 0.8 \cdot 10^{-3}$, $J_L = 2.2 \cdot 10^{-2}$, $D_L = 1.8 \cdot 10^{-3}$, and $K = 0.4$. This plant is then given by:

$$G(s) = \frac{90.9 \cdot 10^3}{(s + 0.117)(s^2 + 3.97s + 2.02 \cdot 10^3)}. \tag{40}$$

First, we show that the plant $G(s)$ in (40) can be rewritten in the form of (3). $\alpha \in R$ is set to:

$$\alpha = 1 \tag{41}$$

and $Q_1(s) \in RH_\infty$ is set to:

$$Q_1(s) = 0.26 \cdot 10^{-2}. \tag{42}$$

Substituting (41) and (42) for (4) and (5), $\beta \in R$ is given by:

$$\beta = 3.85 \cdot 10^2 \tag{43}$$

and $Q_3(s) \in RH_\infty$ is given by:

$$Q_3(s) = 0. \tag{44}$$

Therefore, $Q_2(s) \in RH_\infty$ is given by:

$$Q_2(s) = -\frac{3.85 \cdot 10^2(s^2 + 4.08s + 1.78 \cdot 10^3)}{(s^2 + 0.234s + 0.117)(s^2 + 3.85s + 2.02 \cdot 10^3)}. \tag{45}$$

Using (41), (42), (43), (45), and (44), the plant $G(s)$ in (40) is rewritten in the form of (3). That is, $G(s)$ in (40) is semistrongly stabilizable.

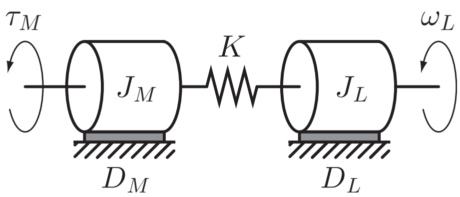

Figure 1. Two-inertia system

For the plant $G(s)$ in (40), we design a semistrongly stabilizing controller. $\hat{Q}(s) \in \mathcal{U}$ in (8) must satisfy (9) and make $Q(s)$ in (8) proper. Using the method in Section 4, we design $\hat{Q}(s)$. $\tilde{Q}(s)$ in (31) is factorized by (31), where:

$$\tilde{Q}_i(s) = 1 \tag{46}$$

and

$$\tilde{Q}_o(s) = \frac{90.9 \cdot 10^3 (s+1)}{(s^2 + 0.234s + 0.117)(s^2 + 3.85s + 2.02 \cdot 10^3)}, \tag{47}$$

respectively. $\bar{Q}(s)$ is made (32), where $q(s)$ is given by (33),

$$\tau = 0.02, \tag{48}$$

$$m = 3, \tag{49}$$

and

$$k = 0.99, \tag{50}$$

respectively. Using this $\bar{Q}(s)$, $\hat{Q}(s)$ is set to (34). In summary, $\hat{Q}(s) \in \mathcal{U}$ becomes:

$$\hat{Q}(s) = \frac{0.99(s^2 + 0.234s + 0.117)(s^2 + 3.85s + 2.02 \cdot 10^3)}{90.9(s+1)(0.2s+1)^3}. \tag{51}$$

We find that the designed $\hat{Q}(s)$ is a unimodular function. Substituting (51) and (8) for (6), we have a semistrongly stabilizing controller for the semistrongly stabilizable plant $G(s)$ in (40):

$$C(s) = \frac{1.36(s^2 + 0.241s + 0.118)(s^2 + 4.12s + 2.03 \cdot 10^3)}{s(s+0.167)(s^2 + 1.50 \cdot 10^2 s + 7.48 \cdot 10^3)}. \tag{52}$$

It is obvious that $C(s)$ in (52) has a pole at the origin and other poles in the open left-half plane, that is, $C(s)$ in (52) is a semistrongly stabilizing controller for $G(s)$ if $C(s)$ in (52) stabilizes $G(s)$ in (40).

Using this semistrongly stabilizing controller $C(s)$ in (52), the response of the output $y(t)$ of the control system in (1) for the step reference input $r(t) = 1$ is shown in Figure 2. Figure 2 shows that the control system in (1) is stable and the output $y(t)$ follows the step reference input $r(t) = 1$ without steady state error.

We have thus confirmed that the controller designed using the method in Section 4 is a semistrongly stabilizing controller. In addition, we have also confirmed that we can design semistrongly stabilizing controllers systematically by considering the angular velocity control of our two-inertia system, which is a real application.

6. **Conclusions.**

In this paper, we have clarified the parameterization of all semistrongly stabilizing controllers for semistrongly stabilizable plants. The control characteristic using semistrongly stabilizable plants is presented. A design method for $\hat{Q}(s) \in \mathcal{U}$ that satisfies (9) and makes $Q(s)$ proper is also presented. In addition, we have presented a numerical example and illustrated the effectiveness of the proposed method.

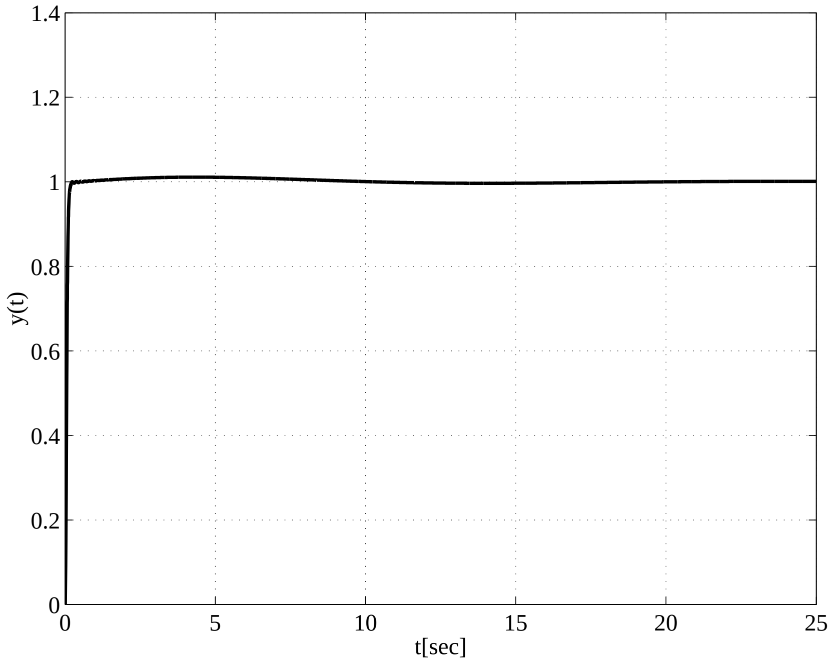

FIGURE 2. Response of the output $y(t)$ of the control system in (1) for the step reference input $r(t) = 1$

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