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A NEW MATHEMATICAL MODEL FOR DIMENSIONING OF THE BOUNDARY TRAPEZOIDAL COMBINED FOOTINGS

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Abstract. *This paper presents a mathematical model to obtain the most economical dimension of the contact surface for boundary trapezoidal combined footings, when the load that must support said structural member is applied (axial load and moment in two directions to each column). The classic model considers an axial load and a moment around the axis "X" (transverse axis) applied to each column, i.e., the resultant force from the applied loads is located on the axis "Y" (longitudinal axis), and its position must match with the geometric center of the footing, and when the moment around the axis "Y" appears, this is taken into account through a transverse beam of the axis "Y" (cantilever). Thus the classic model neglects and the proposed model considers the moments around the axis "Y", which is the main part of this research. Then, the proposed model is more suited to the real conditions.*

Keywords: Boundary trapezoidal combined footings, Contact surface, More economical dimension, Bidirectional bending

1. **Introduction.** The function of a footing or a foundation is to transmit the load of the structure to the underlying soil. The choice of suitable type of footing depends on the depth at which the bearing stratum is localized, the soil condition and the type of superstructure. The foundations are classified into superficial and deep ones, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems [1,2].

In the design of superficial foundations in terms of the application of loads are: 1) the footings subjected to concentric axial load, 2) the footings subjected to axial load and moment in one direction (unidirectional bending), 3) the footings subjected to axial load and moment in two directions (bidirectional bending) [1-6].

Superficial foundations may be of various types according to their function: isolated footing, combined footing, strip footing, or mat foundation [1-6].

A combined footing is a long footing supporting two or more columns in (typically two) one row. The combined footing may be rectangular, trapezoidal or T-shaped in plan. Rectangular footing is provided when one of the projections of the footing is restricted or the width of the footing is restricted. Trapezoidal footing or T-shaped is provided when one column load is much more than the other. As a result, both projections of the footing beyond the faces of the columns will be restricted [7-9].

Whenever two or more columns in a straight line are carried on a single spread footing, it is called a combined footing. Isolated footings for each column are generally more economical.

Combined footings are provided only when it is absolutely necessary, as

1. When two columns are close together, causing overlap of adjacent isolated footings

2. Where soil bearing capacity is low, causing overlap of adjacent isolated footings

3. Proximity of building line or existing building or sewer, adjacent to a building column.

Figure 1 shows some solutions through combined footings.

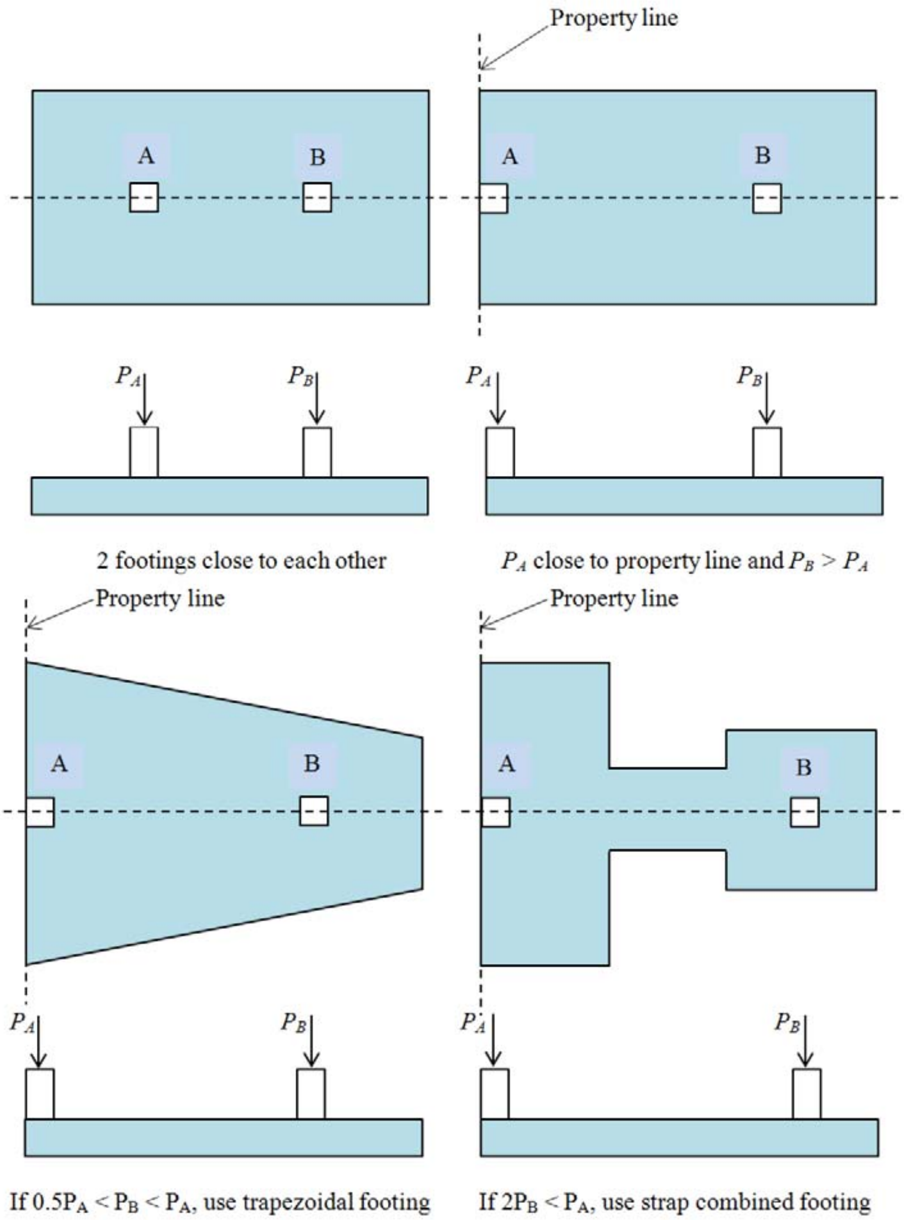

FIGURE 1. Combined footings

The hypothesis used in the classic model considers an axial load and a moment around the axis "X" (transverse axis) applied to each column, i.e., the resultant force from the applied loads is located on the axis "Y" (longitudinal axis), and its position must match with the geometric center of the footing, and when the moment around the axis "Y" appears, this is taken into account through a transverse beam of the axis "Y" (cantilever). This results in uniform pressure below the entire area of footing. Then the equation of the bidirectional bending is used to obtain the stresses acting on the contact surface of the combined footings, which must meet the following conditions: 1) the minimum stress should be equal to or greater than zero, because the soil is not capable of withstanding tensile stresses; 2) the maximum stress must be equal to or less than the allowable capacity that can withstand the soil [1,2,5,6].

Mathematical models have been developed to obtain the dimensions of rectangular, square and circular isolated footings subjected to axial load and moments in two directions (bidirectional bending) [10-12]. Also a comparison was presented between the rectangular footings, square and circular in terms of the contact area with soil [13].

A mathematical model was presented for design of isolated footings of rectangular form using a new model [14].

This paper presents a mathematical model to obtain the most economical dimension of the contact surface for boundary trapezoidal combined footings subjected to axial load and moment in two directions (bidirectional bending) applied to each column, where there are two conditions: the first condition is that the minimum stress should be equal to zero, because the soil is not able to withstand tensile stresses, and the second condition is that the maximum stress should be equal to the allowable soil load capacity. Also, examples are presented to validate the proposed model, and these problems are: 1) two property lines are considered at opposite ends; 2) a property line is taken into account.

2. **Proposed Mathematical Model.**

2.1. **General equation of the bidirectional bending.**

Figure 2 shows a footing of general shape under axial load and moment in two directions (bidirectional bending), where pressures are different in contact surface; such pressures vary linearly [10-14].

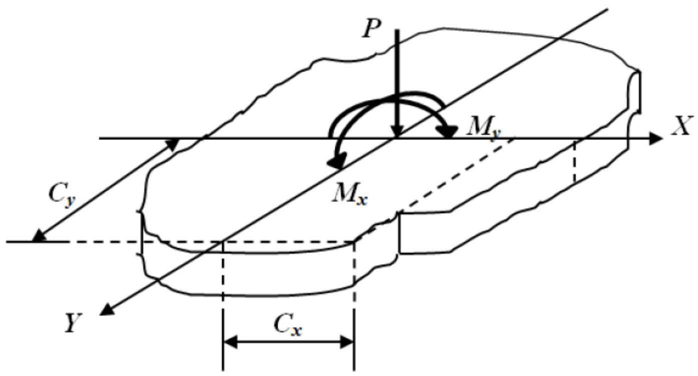

FIGURE 2. General shape footing

General equation for any type of footings subjected to bidirectional bending [10-14]:

$$\sigma = \frac{P}{A} \pm \frac{M_x C_y}{I_x} \pm \frac{M_y C_x}{I_y} \tag{1}$$

where: σ is the stress exerted by the soil on the footing (soil pressure), A is the contact area of the footing, P is the axial load applied at the center of gravity of the footing, M_x is the moment around the axis "X", M_y is the moment around the axis "Y", C_x is the distance in the direction "X" measured from the axis "Y" up to the farthest end, C_y is the distance in direction "Y" measured from the axis "X" up to the farthest end, I_y is the moment of inertia around the axis "Y" and I_x is the moment of inertia around the axis "X".

Equation (1) is used to find the stresses in the farthest end for any type of footing, and these are:

$$\sigma_1 = \frac{P}{A} + \frac{M_x C_y}{I_x} + \frac{M_y C_x}{I_y} \tag{2}$$

$$\sigma_2 = \frac{P}{A} + \frac{M_x C_y}{I_x} - \frac{M_y C_x}{I_y} \tag{3}$$

$$\sigma_3 = \frac{P}{A} - \frac{M_x C_y}{I_x} + \frac{M_y C_x}{I_y} \tag{4}$$

$$\sigma_4 = \frac{P}{A} - \frac{M_x C_y}{I_x} - \frac{M_y C_x}{I_y} \tag{5}$$

where: $\sigma_1 = \sigma_{\max}$ is the maximum stress and $\sigma_4 = \sigma_{\min}$ is the minimum stress.

2.2. **Boundary trapezoidal combined footing.** Figure 3 shows a boundary trapezoidal combined footing supporting two rectangular columns of different dimensions subjected to axial load and moments in two directions (bidirectional bending) applied to each column.

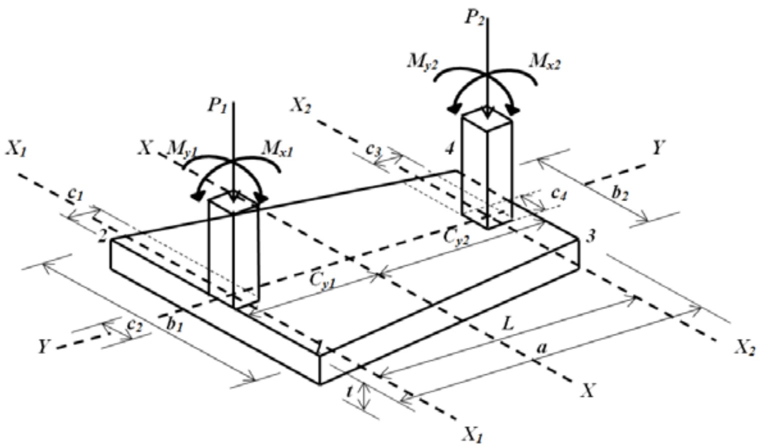

FIGURE 3. Boundary trapezoidal combined footing subjected to the real loads

The geometric properties of a trapezoidal section are:

$$A = \frac{a(b_1 + b_2)}{2} \tag{6}$$

$$C_{y1} = \frac{a(b_1 + 2b_2)}{3(b_1 + b_2)} \tag{7}$$

$$C_{y2} = \frac{a(2b_1 + b_2)}{3(b_1 + b_2)} \tag{8}$$

$$I_x = \frac{a^3(b_1^2 + 4b_1 b_2 + b_2^2)}{36(b_1 + b_2)} \tag{9}$$

$$I_y = \frac{a(b_1 + b_2)(b_1^2 + b_2^2)}{48} \tag{10}$$

For a footing there are two conditions: the first is that the minimum stress should be zero, since the soil is not capable of withstanding tensile stresses and the second is that the maximum stress is the load capacity that can withstand the soil.

Figure 4 presents a boundary trapezoidal combined footing due to the equivalent loads. The mechanical elements of the components P_1, M_{x1}, M_{y1} are equivalent to a normal force

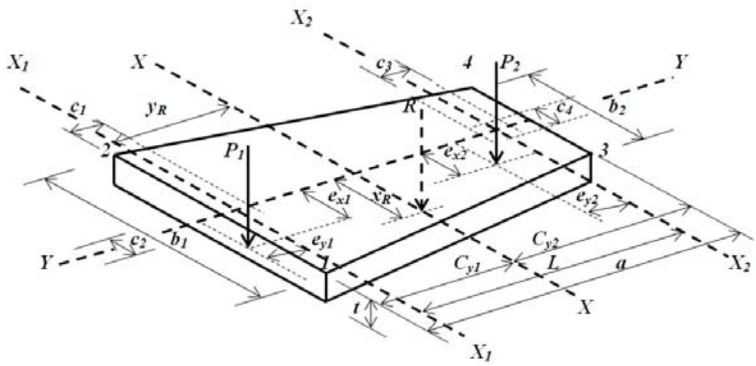

FIGURE 4. Boundary trapezoidal combined footing due to equivalent loads

"P_1" acting on the point with coordinates (e_{x1}, e_{y1}), and the components of P_2, M_{x2}, M_{y2} are equivalent to a normal force "P_2" acting on the point with coordinates (e_{x2}, e_{y2}).

If the sum of moments around the axis "X_1" of the footing 1 is obtained to find the position of the resulting "$R = P_1 + P_2$", consider that is located between the two columns:

$$P_1 e_{y1} - P_2(L - e_{y2}) = -R y_R \tag{11}$$

Now we obtain "y_R" of Equation (11):

$$y_R = \frac{P_2(L - e_{y2}) - P_1 e_{y1}}{R} \tag{12}$$

where: y_R is the distance measured from "X_1" up to the resultant force.

If "$M_{x1} = P_1 e_{y1}$" and "$M_{x2} = P_2 e_{y2}$" are substituted into Equation (12), present the equation in terms of the mechanical elements acting on the footing:

$$y_R = \frac{P_2 L - (M_{x1} + M_{x2})}{R} \tag{13}$$

Substituting "$M_{xT} = M_{x1} + M_{x2}$" into Equation (13) obtained:

$$y_R = \frac{P_2 L - M_{xT}}{R} \tag{14}$$

Now the resultant force is made to coincide with the gravity center of the area of the footing with the position of the resultant force in the direction "Y", and then the value of "C_{y1}" is:

$$C_{y1} = \frac{c_1}{2} + y_R \tag{15}$$

Substituting Equation (14) into Equation (15) obtained:

$$C_{y1} = \frac{R c_1 + 2 P_2 L - 2 M_{xT}}{2R} \tag{16}$$

Once known the value "C_{y1}" of Equation (16) is substituted into Equation (7), obtain "b_1" in function of "b_2":

$$b_1 = \left(\frac{2a - 3C_{y1}}{3C_{y1} - a} \right) b_2 \tag{17}$$

where the value of "a" should be:

$$\frac{3}{2} C_{y1} < a < 3C_{y1} \tag{18}$$

If we consider a trapezoidal combined footing with two boundaries in opposite ends, the value of "a" is:

$$a = \frac{c_1}{2} + L + \frac{c_3}{2} \tag{19}$$

If we take account of a trapezoidal combined footing with a boundary, the value of "a" is proposed according to Equation (18).

The general equation of the bidirectional bending for a trapezoidal combined footing is transformed as follows:

$$\sigma(x,y) = \frac{R}{\frac{a(b_1+b_2)}{2}} + \frac{Ry_c y}{\frac{a^3(b_1^2+4b_1b_2+b_2^2)}{36(b_1+b_2)}} + \frac{Rx_c x}{\frac{a[(b_1+b_2)(b_1^2+b_2^2)]}{48}} \tag{20}$$

where: $\sigma(x,y)$ is stress generated in anywhere with coordinates (x,y) on the soil, R is the resultant force of the forces, y_c is the distance from the center of the contact area of the footing in the direction "Y" to the resultant force, and x_c is the distance from the center of the contact area of the footing in the direction "X" to the resultant force.

Now the resultant force is made to coincide with the gravity center of the area of the footing with the position of the resultant force in the direction "Y", therefore, there is not moment around the axis "X" and the value of "y_c" is zero, and "x_c" is the sum of moments around the axis "Y" divided by the resultant, which is:

$$x_c = \frac{M_{y1} + M_{y2}}{P_1 + P_2} \tag{21}$$

Substituting "$M_{yT} = M_{y1} + M_{y2}$" into Equation (21) found:

$$x_c = \frac{M_{yT}}{R} \tag{22}$$

Then, substituting Equation (22) into Equation (20) and taking into account that "y_c" is zero, this equation is transformed in a unidirectional bending system, thus being presented of the following way:

$$\sigma(x,y) = \frac{R}{\frac{a(b_1+b_2)}{2}} + \frac{M_{yT} x}{\frac{a[(b_1+b_2)(b_1^2+b_2^2)]}{48}} \tag{23}$$

2.2.1. *First condition.* The minimum stress is zero:

$$\sigma_{\min} = \sigma_4 = 0 \tag{24}$$

Substituting Equation (24) and "$x = -b_1/2$" into Equation (23) obtained:

$$0 = \frac{2R}{a(b_1+b_2)} + \frac{48M_{yT}(-b_1/2)}{a(b_1+b_2)(b_1^2+b_2^2)} \tag{25}$$

Equation (17) is substituted into Equation (25) to obtain "b_2":

$$b_2 = \frac{12M_{yT}(2a - 3C_{y1})(3C_{y1} - a)}{R(5a^2 - 18aC_{y1} + 18C_{y1}^2)} \tag{26}$$

After, Equation (26) is substituted into Equation (17) to find "b_1":

$$b_1 = \frac{12M_{yT}(2a - 3C_{y1})^2}{R(5a^2 - 18aC_{y1} + 18C_{y1}^2)} \tag{27}$$

Therefore, the dimensions of a boundary trapezoidal combined footing are obtained by Equations (26) and (27), when the pressure is zero.

2.2.2. *Second condition.* The maximum stress is the allowable soil load capacity:

$$\sigma_1 = \sigma_{adm} \tag{28}$$

where: σ_{adm} is the available allowable soil load capacity.

Substituting Equation (28) and "$x = b_1/2$" into Equation (23) obtained:

$$\sigma_{adm} = \frac{2R}{a(b_1 + b_2)} + \frac{48M_{yT}(b_1/2)}{a(b_1 + b_2)(b_1^2 + b_2^2)} \tag{29}$$

Equation (17) is substituted into Equation (29) to find "b_2":

$$\begin{aligned}\sigma_{adm}a^2(5a^2 - 18aC_{y1} + 18C_{y1}^2)b_2^2 - 2R(3C_{y1} - a)(5a^2 - 18aC_{y1} + 18C_{y1}^2)b_2 \\ - 24M_{yT}(2a - 3C_{y1})(3C_{y1} - a)^2 = 0\end{aligned} \tag{30}$$

Then, Equation (30) is solved for the value "b_2", and this is substituted into Equation (17) to find the value "b_1". These are the dimensions of a boundary trapezoidal combined footing, when the pressure is the allowable soil load capacity.

Therefore, the proposal minimum dimension of a boundary trapezoidal combined footing is the following: the greater dimension obtained from the first condition by Equations (26) and (27) or the second condition by Equations (30) and (17).

2.3. **Special case (rectangular combined footings).** If we consider in Equation (17) "$b = b_1 = b_2$" to obtain a rectangular combined footing, then the value of "C_{y1}" is:

$$C_{y1} = \frac{a}{2} \tag{31}$$

Substituting Equation (31) into Equation (26) or (27), the value of "b" is obtained:

$$b = \frac{6M_{yT}}{R} \tag{32}$$

Therefore, the dimensions of a boundary rectangular combined footing are obtained by Equations (31) and (32), when the pressure is zero.

Now substituting Equation (31) and "$b = b_2$" into Equation (30) found the following equation:

$$\sigma_{adm}ab^2 - Rb - 6M_{yT} = 0 \tag{33}$$

Equation (33) is solved to obtain the value of "b", which is:

$$b = \frac{R + \sqrt{R^2 + 24\sigma_{adm}aM_{yT}}}{2\sigma_{adm}a} \tag{34}$$

Then, the dimensions of a boundary rectangular combined footing are obtained by Equations (31) and (34), when the pressure is the allowable soil load capacity.

Therefore, the proposal minimum dimension of a boundary rectangular combined footing is: the greater dimension obtained from the first condition by Equations (31) and (32) or the second condition by Equations (31) and (34).

3. **Application.** Table 1 presents five cases for dimensioning of trapezoidal combined footings with two boundaries in opposite ends. Table 2 shows five cases for dimensioning of trapezoidal combined footings with a boundary considering one meter between the outer face of column 2 and end of the footing. Each case has four types of footings. The two tables make the following considerations: 1) the dimensions of the two columns are of $40 \times 40 cm$ in all cases; 2) the soil load capacity varies for each type; 3) the axial load of column 1 varies in each case; 4) the distance between columns varies in each type.

TABLE 1. Dimensioning of trapezoidal combined footings with two boundaries in opposite ends

Load capacity the soil	Loads of the column 1			Loads of the column 2			Distance between columns	Dimension in the direction of the columns	Dimensions for zero minimum pressure		Dimensions for the maximum pressure		Proposed dimensions			Stresses generated by loads	
σ_{adm} (kN/m^2)	P_1 (kN)	M_{x1} $(kN\text{-}m)$	M_{y1} $(kN\text{-}m)$	P_2 (kN)	M_{x2} $(kN\text{-}m)$	M_{y2} $(kN\text{-}m)$	L (m)	a (m)	b_2 (m)	b_1 (m)	b_2 (m)	b_1 (m)	a (m)	b_2 (m)	b_1 (m)	$\sigma_{\max}$ (kN/m^2)	$\sigma_{\min}$ (kN/m^2)
Case 1																	
250	1400	140	200	800	70	100	7.00	7.40	0.13	1.63	0.26	3.24	7.40	0.40	3.30	239.23	82.17
200	1400	140	200	800	70	100	6.00	6.40	0.13	1.63	0.35	4.42	6.40	0.40	4.50	190.93	89.69
150	1400	140	200	800	70	100	5.00	5.40	0.12	1.63	0.45	6.30	5.40	0.50	6.40	148.10	88.08
100	1400	140	200	800	70	100	4.00	4.40	0.11	1.63	0.73	10.74	4.40	0.80	10.80	99.20	73.22
Case 2																	
250	1200	140	200	800	70	100	7.00	7.40	0.37	1.72	0.61	2.86	7.40	0.70	2.90	238.22	62.08
200	1200	140	200	800	70	100	6.00	6.40	0.36	1.73	0.78	3.76	6.40	0.80	3.80	197.50	74.24
150	1200	140	200	800	70	100	5.00	5.40	0.35	1.73	1.09	5.45	5.40	1.10	5.50	147.55	76.92
100	1200	140	200	800	70	100	4.00	4.40	0.33	1.74	1.72	9.09	4.40	1.80	9.10	99.28	67.53
Case 3																	
250	1000	140	200	800	70	100	7.00	7.40	0.71	1.71	0.98	2.36	7.40	1.00	2.40	244.68	41.49
200	1000	140	200	800	70	100	6.00	6.40	0.70	1.72	1.26	3.10	6.40	1.30	3.10	197.98	57.70
150	1000	140	200	800	70	100	5.00	5.40	0.68	1.73	1.75	4.43	5.40	1.80	4.50	146.36	65.28
100	1000	140	200	800	70	100	4.00	4.40	0.65	1.76	2.72	7.42	4.40	2.80	7.50	98.03	60.84
Case 4																	
250	800	140	200	800	70	100	7.00	7.40	1.10	1.36	1.39	1.72	7.40	1.40	1.80	240.38	29.89
200	800	140	200	800	70	100	6.00	6.40	1.09	1.40	1.77	2.27	6.40	1.80	2.30	195.94	47.97
150	800	140	200	800	70	100	5.00	5.40	1.08	1.45	2.43	3.26	5.40	2.50	3.30	146.43	57.91
100	800	140	200	800	70	100	4.00	4.40	1.06	1.52	3.81	5.47	4.40	3.90	5.50	98.43	56.31
Case 5																	
250	600	140	200	800	70	100	7.00	7.40	1.09	0.61	1.63	0.91	7.40	1.70	1.00	232.78	47.50
200	600	140	200	800	70	100	6.00	6.40	1.12	0.66	2.11	1.24	6.40	2.20	1.30	188.99	61.01
150	600	140	200	800	70	100	5.00	5.40	1.16	0.72	2.96	1.85	5.40	3.00	1.90	146.82	64.82
100	600	140	200	800	70	100	4.00	4.40	1.20	0.83	4.73	3.25	4.40	4.80	3.30	98.21	58.92

TABLE 2. Dimensioning of trapezoidal combined footings with a boundary

Load capacity the soil	Loads of the column 1			Loads of the column 2			Distance between columns	Dimension in the direction of the columns	Dimensions for zero minimum pressure		Dimensions for the maximum pressure		Proposed dimensions			Stresses generated by loads	
σ_{adm} (kN/m^2)	P_1 (kN)	M_{x1} $(kN\text{-}m)$	M_{y1} $(kN\text{-}m)$	P_2 (kN)	M_{x2} $(kN\text{-}m)$	M_{y2} $(kN\text{-}m)$	L (m)	a (m)	b_2 (m)	b_1 (m)	b_2 (m)	b_1 (m)	a (m)	b_2 (m)	b_1 (m)	$\sigma_{\max}$ (kN/m^2)	$\sigma_{\min}$ (kN/m^2)
								Case 1									
250	1400	140	200	800	70	100	7.00	7.90	0.01	1.64	0.02	3.14	7.90	0.40	3.20	232.61	76.81
200	1400	140	200	800	70	100	6.00	6.80	0.01	1.64	0.04	4.56	6.80	0.40	4.60	175.10	83.72
150	1400	140	200	800	70	100	5.00	5.70	0.02	1.64	0.08	6.61	5.70	0.40	6.70	135.18	82.26
100	1400	140	200	800	70	100	4.00	4.60	0.03	1.64	0.18	10.83	4.60	0.40	10.90	97.34	71.96
								Case 2									
250	1200	140	200	800	70	100	7.00	8.40	0.06	1.80	0.10	2.85	8.40	0.40	2.90	232.19	56.41
200	1200	140	200	800	70	100	6.00	7.40	0.02	1.80	0.05	3.90	7.40	0.40	4.00	177.59	68.11
150	1200	140	200	800	70	100	5.00	6.20	0.03	1.80	0.08	5.76	6.20	0.40	5.80	136.20	71.92
100	1200	140	200	800	70	100	4.00	5.00	0.03	1.80	0.16	9.25	5.00	0.40	9.30	98.41	66.54
								Case 3									
250	1000	140	200	800	70	100	7.00	8.40	0.32	1.95	0.42	2.56	8.40	0.50	2.60	240.80	35.70
200	1000	140	200	800	70	100	6.00	7.40	0.26	1.97	0.44	3.39	7.40	0.50	3.40	196.56	52.92
150	1000	140	200	800	70	100	5.00	6.40	0.17	1.98	0.43	4.90	6.40	0.50	5.00	142.78	61.77
100	1000	140	200	800	70	100	4.00	5.40	0.07	2.00	0.28	7.97	5.40	0.40	8.00	99.16	59.57
								Case 4									
250	800	140	200	800	70	100	7.00	8.40	0.74	1.97	0.81	2.14	8.40	0.90	2.20	230.55	15.23
200	800	140	200	800	70	100	6.00	7.40	0.66	2.04	0.91	2.82	7.40	1.00	2.90	187.77	33.99
150	800	140	200	800	70	100	5.00	6.40	0.54	2.11	1.04	4.06	6.40	1.10	4.10	145.38	46.93
100	800	140	200	800	70	100	4.00	5.40	0.38	2.18	1.17	6.67	5.40	1.20	6.70	99.42	50.60
								Case 5									
250	600	140	200	800	70	100	7.00	8.40	1.26	1.56	1.21	1.50	8.40	1.30	1.60	226.21	3.67
200	600	140	200	800	70	100	6.00	7.40	1.21	1.73	1.43	2.06	7.40	1.50	2.10	190.33	19.88
150	600	140	200	800	70	100	5.00	6.40	1.11	1.94	1.73	3.04	6.40	1.80	3.10	144.67	33.90
100	600	140	200	800	70	100	4.00	5.40	0.92	2.18	2.19	5.17	5.40	2.20	5.20	99.46	40.68

4. **Results.** The value of "a" is found by Equation (19) for Table 1, and for Table 2 is considered the value of Equation (19) more one meter between the outer face of column 2 and end of the footing with the exception of the case 1 in the four types, and for case 2 in types 3 and 4, because the distance is restricted by Equation (18). The dimensions of "b_2" and "b_1" for zero minimum pressure are obtained by Equations (26) and (27). The dimensions for maximum pressure (soil load capacity) are found by Equation (30) for "b_2" and Equation (17) for "b_1". The proposed dimensions are found taking account of the larger of the two above conditions. Once the dimensions of the footing are defined, the stresses generated by loads applied to the foundation are obtained to verify that these stresses are within the established parameters, i.e., the maximum stress is equal to or less than the soil load capacity, and the minimum stress is equal to or greater than zero, since the soil is not capable of withstanding the tensile stresses.

According to results in all cases the second condition prevails for the two tables, with the exception in Table 2 for the case 5 the type 1 in which the first condition prevails. The first condition for footings should be dimensioned on the basis of the minimum pressure that is zero, because the soil is not capable of withstanding tensile stresses. The second condition for footings should be dimensioned on the basis of the soil load capacity.

5. **Conclusions.** Footings are structural elements that transmit column or wall loads to the underlying soil below the structure. Footings are designed to transmit these loads to the soil without exceeding its safe bearing capacity, to prevent excessive settlement of the structure to a tolerable limit, to minimize differential settlement, and to prevent sliding and overturning. The settlement depends upon the intensity of the load, type of soil, and foundation level. The different footings should be designed in such way to solve these problems.

Due to the importance of the footing, it is forced to meet certain geometrical parameters, pressure, conformation that respond to the characteristics of soil and loads of the structure.

Trapezoidal footing, it is provided when one column load is much more than the other. As a result, the both projections of footing beyond the faces of the columns will be restricted.

The mathematical approach suggested in this paper produces results having tangible accuracy for all problems under investigation for finding the solution more economical.

Then the proposed model is recommended for dimensioning of trapezoidal combined footings with a boundary (a property line) and two boundaries in opposite ends (two property lines) subjected to axial load and bidirectional bending. Furthermore this adheres more to the real conditions of the soil pressure that are applied to the foundation.

The mathematical model developed in this paper applies only to rigid soils that meet expression of the bidirectional bending, i.e., the variation of the pressure is linear. For future research, when presented another type of soil, by example in totally cohesive soils (clay soils) and totally granular soils (sandy soils), the pressure diagram is not linear and should be treated differently.

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