

A DISCRETE-TIME STOCHASTIC MODEL OF SEARCHING AND SWITCHING STRATEGY IN A CUSTOMIZED SERVICE PROVIDING COMPANY

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ABSTRACT. *This paper introduces switching strategy between admission control and pricing control policies as well as searching strategy in a customized service providing company. The company should spend the so-called search cost to find customers, which creates the search option on whether to continue the search or not. For an appearing customer by paying the search cost, the company has an option on which policy to choose between the two control policies, that is, it determines whether or not to admit the customer's request for the service (admission control) or decides a price of the customer's request and offers it to the customer (pricing control). We clarify the properties of the optimal switching strategy as well as the optimal search strategy in order to maximize the total expected net profit. In addition, in order to validate the economic effects of our switching strategy we implement numerical study examining the relative difference of the maximum expected profits between our switching model and the model without switching strategy. According to our results, employing the switching strategy can significantly improve the maximum expected profit as high as 13 percent.*

Keywords: Switching strategy, Admission control, Pricing control, Search cost

1. Introduction. Let us consider a company that provides the customized service to meet customers' various requests. A sequentially arriving customer requests his/her offer to the company, and the service for admitted request will be provided according to the service process for a certain period. Suppose that the company admits all the requests from arriving customers regardless of their profitabilities. Then the service capacity would be soon full. As a result, the customer's request arriving thereafter could not be admitted. In contrast, in order to avoid such a situation and keep allowance in the service capacity if the company is excessively reluctant to admit customers who are not highly profitable, then all requests that have been admitted may be completed and the capacity becomes empty, and this leads to being server's idle. Therefore, it is important for a system manager to decide how to admit the consequently arriving customers considering the service capacity. As for this problem, two kinds of policies, the admission control policies and the pricing control policies, have been formulated so far.

Both the admission and pricing control policies have been widely investigated to improve the performance of queueing system in the telecommunications and manufacturing industries [1, 2, 4, 13]. In the admission control, an arriving customer proposes a price for his request, and the decision maker decides the admission merely based upon the proposed price of the customer. In general, it is assumed that the customer has one's own maximum permissible offering price, which is also referred to as the reservation price, and that the stronger the desire the customer is to be served, a price closer to the reservation

price may be proposed. On the other hand, in the pricing control, the decision maker suggests a price to an arriving customer who has a service request. The customer then makes a service request to the company if and only if this suggested price is lower than or equal to the reservation price. Hence, the decision maker should determine the offering price to maximize the expected profit.

Most of the prior researches have adopted exactly one of the two forementioned policies in accordance with their model's characteristics. However, in what follows, both types of policies were considered. First, Yoon and Lewis [14] formulated and analyzed the problems involving both admission and pricing control policies, but as distinct entities. Furthermore, Gans and Savin [3] considered a rental firm with two types of customers where one is controlled by the admission policy and the other by the pricing policy. Hew and White [5] integrated a call admission and dynamic pricing problem with hand-offs and price-affected arrivals. In their formulation, the former arrivals are controlled by only the admission policy, while the latter ones are sequentially controlled by both policies. Son [12] gave separate formulations of the admission and the pricing control problems, yet showing that both problems can be analyzed within an identical framework.

Whether we consider these two policies separately or within an identical framework, the switching strategy between the two has not been introduced so far. In reality, a system manager is not restricted to use only one policy to control the system capacity. For example, let us consider a travel company that provides airline tickets, hotels, car rentals, and the other travel related services on the website. The website allows visitors to buy tickets after comparing the prices provided by the company, so the company should control the offering prices to maximize its profit, using admission control policy. Meanwhile, visitors can also bid for the travel services on the website, and the company should decide whether to admit them or not based on their bid prices, adopting admission control policy. These two types of website are managed separately, which implies that two policies are used independently. However, the company may employ switching strategy that changes the control policy from one type to the other as long as a higher profit is expected. Motivated by these observations, we propose a basic switching strategy between admission control and pricing control policies. We also introduce in the paper the so-called search cost that is the cost a company would spend in order to find customers. The search cost has been introduced in the conventional optimal stopping problem [6, 8, 11]. By introducing the search cost in the system, it would eventually create a search option on whether or not to conduct the search. Therefore, the objective of this paper is to examine the structure of an optimal switching strategy as well as an optimal search policy in order to maximize the total expected net profit over an infinite planning horizon. In addition, in order to validate the economic effects of our switching model we examine the relative difference of the maximum expected profits between our switching model and the model without switching strategy. According to our results, employing the switching strategy can significantly improve the maximum expected profit as high as 13 percent.

2. Model Formulation. We consider the following discrete-time queueing model with a single server and the system capacity k ($k \geq 1$). A customer appears with the probability λ ($0 < \lambda \leq 1$) only after the search cost $c \geq 0$ has been paid at the previous point in time. The service for the admitted request is completed with the probability q ($0 < q < 1$) at the next point in time. The sequentially arriving customers are assumed to have their reservation prices, ξ_1, ξ_2, \dots , which are i.i.d random variables determined by a known distribution function $F_\xi(x)$ with the expectation value μ_ξ , and the density function is defined as follows:

$$f_\xi(x) > 0, \quad \text{if } a \leq x \leq b, \quad f_\xi(x) = 0, \quad \text{otherwise,} \quad (1)$$

where a and b ($0 < a < b < \infty$) are certain given numbers. For an arriving customer with the reservation price ξ , the decision maker must decide which policy to adopt between the admission and pricing control policies. Adopting the former, the decision maker then has to determine whether or not to admit the customer based on the price $w = \alpha\xi$ where $\alpha \in [0, 1]$ is a ratio which denotes the customer's degree of desirability for the service, i.e., the greater the customer's desirability, α is closer to 1. These ratios $\alpha_1, \alpha_2, \dots$ are i.i.d random variables from a known distribution function $F_\alpha(x)$ with the expectation μ_α . Therefore, it is clear that $\mu_w = \mu_\alpha\mu_\xi$. Now the distribution function of w will be

$$\begin{aligned} F_w(x) &= \Pr(w \leq x) = \Pr(\alpha\xi \leq x) = \Pr(\xi \leq x/\alpha) \\ &= \int_0^\infty F_\xi(x/\alpha)f_\alpha(\alpha)d\alpha = E_\alpha[F_\xi(x/\alpha)], \end{aligned}$$

and the density function becomes

$$f_w(x) = E_\alpha \left[\frac{1}{\alpha} f_\xi(x/\alpha) \right]. \quad (2)$$

On the other hand, if the pricing control policy is adopted, the decision maker then proposes a price z to the customer's request and the customer makes a service request if and only if the proposed price z is less than or equal to his own reservation price ξ . Accordingly, the probability that the customer requests the service will be $p(z) = \Pr(z \leq \xi)$.

Let us denote by i the number of customers in the system, and let $V(i)$ represent the maximum expected net profit in the current state i . In such case, by using the Markovian Decision Process we can describe the optimality equations of the model as follows:

$$V(0) = \max \left\{ \begin{array}{l} \text{C: } \beta \left(\lambda \max \left\{ \begin{array}{l} E_w[\max\{w + V(1), V(0)\}], \\ \max_z \{p(z)(z + V(1)) + (1 - p(z))V(0)\} \end{array} \right\} + (1 - \lambda)V(0) \right) - c, \\ \text{K: } \beta V(0) \end{array} \right\}, \quad (3)$$

$$V(i) = \max \left\{ \begin{array}{l} \text{C: } \beta(1 - q) \left(\left(\lambda \max \left\{ \begin{array}{l} E_w[\max\{w + V(i + 1), V(i)\}], \\ \max_z \{p(z)(z + V(i + 1)) + (1 - p(z))V(i)\} \end{array} \right\} + (1 - \lambda)V(i) \right) I_{(1 \leq i < k)} \right. \\ \left. + V(i)I_{(i=k)} \right) + \beta q \left(\lambda \max \left\{ \begin{array}{l} E_w[\max\{w + V(i), V(i - 1)\}], \\ \max_z \{p(z)(z + V(i)) + (1 - p(z))V(i - 1)\} \end{array} \right\} + (1 - \lambda)V(i - 1) \right) - c, \\ \text{K: } \beta(1 - q)V(i) + \beta qV(i - 1) \end{array} \right\} \quad (4)$$

for $1 \leq i \leq k$,

where β and $I_{(\cdot)}$ represent the discount factor and the indicator function, respectively. The letters C and K denote the decision of conducting the search and skipping the search, respectively.

To explain this model in more details, suppose that a customer appears with the probability λ in the state i having paid the search cost c . When adopting the admission control policy, in the case that the customer proposes the price w for the service and the company admits it, the profit w would be obtained and the state would increase to $i + 1$; otherwise, the state would remain as i . When the pricing control policy is employed instead, as we mentioned before we first assume that the decision maker offers the price z to an arriving customer in state i . If the customer requests his service with probability $p(z)$, the company obtains the profit z and the state becomes $i + 1$; otherwise, the state remains as i . Hence, the decision maker should offer a price z that pertains to the maximization of $\{p(z)(z + V(i + 1)) + (1 - p(z))V(i)\}$. Note that when $i = k$, if the current service is not yet completed with the probability $(1 - q)$, an arriving customer's request cannot be met due to the service capacity, and so the state remains as i .

3. Transformation of the Optimality Equations. In this section, we transform the optimality equations described in the preceding section and present the optimal switching and search policies. Let us begin by defining

$$h_i = V(i) - V(i + 1), \quad 0 \leq i \leq k - 1. \quad (5)$$

For concise expression and for the convenience of model analysis, let us further define the following functions.

$$T_p(x) = \max_z p(z)(z - x), \quad (6)$$

$$T_w(x) = E_w[\max\{w - x, 0\}], \quad (7)$$

$$J(x) = T_w(x) - T_p(x), \quad \text{and} \quad (8)$$

$$K(x) = \max\{T_w(x), T_p(x)\} = \max\{J(x), 0\} + T_p(x). \quad (9)$$

Moreover, we will denote the value z which maximizes $p(z)(z - x)$ by $z(x)$ in Equation (6). Corresponding to Equation (7), we will also define

$$b^o = \sup\{x | T_w(x) > 0\}. \quad (10)$$

Since the expectation of immediate reward at any point in time is finite, by using the conventional method outlined in a Markovian Decision Process [10], we can easily verify that $V(i) \leq M/(1 - \beta)$ for a sufficiently large $M > 0$, i.e., $V(i)$ is finite. Hence, we can see that the system of Equations (3) and (4) has a unique solution, regardless of the details of the optimal decisions. Now, the terms $\max\{w + V(i + 1), V(i)\}$ and $\max_z\{p(z)(z + V(i + 1)) + (1 - p(z))V(i)\}$ can be represented as $\max\{w + V(i + 1) - V(i), 0\} + V(i)$ and $\max_z p(z)(z + V(i + 1) - V(i)) + V(i)$, respectively, i.e., $\max\{w - h_i, 0\} + V(i)$ and $\max_z p(z)(z - h_i) + V(i)$ via the definition in Equation (5). Hence, Equations (3) and (4) now become

$$V(0) = \beta V(0) + \max \left\{ \beta \lambda \max \left\{ E_w[\max\{w - h_0, 0\}], \right. \right. \\ \left. \left. \max_z p(z)(z - h_0) \right\} - c, 0 \right\}, \quad (11)$$

$$V(i) = \beta(1 - q)V(i) + \beta q V(i - 1) + \beta \lambda \max \left\{ (1 - q) \max \left\{ E_w[\max\{w - h_i, 0\}], \right. \right. \\ \left. \left. \max_z p(z)(z - h_i) \right\} I_{(1 \leq i < k)} \right. \\ \left. + q \max \left\{ E_w[\max\{w - h_{i-1}, 0\}], \right. \right. \\ \left. \left. \max_z p(z)(z - h_{i-1}) \right\} - c, 0 \right\}, \quad 1 \leq i \leq k. \quad (12)$$

Then using Equations (6) to (9), we rearrange these equations as follows.

$$V(0) = \max\{\beta \lambda (\max\{J(h_0), 0\} + T_p(h_0)) - c, 0\} / (1 - \beta), \quad (13)$$

$$V(i) = \gamma \beta q V(i - 1) + \gamma \max \left\{ \beta(1 - q) \lambda (\max\{J(h_i), 0\} + T_p(h_i)) I_{(1 \leq i < k)} \right. \\ \left. + \beta q \lambda (\max\{J(h_{i-1}), 0\} + T_p(h_{i-1})) - c, 0 \right\}, \quad 1 \leq i \leq k, \quad (14)$$

where $\gamma = (1 - \beta(1 - q))^{-1} > 1$, which lead to the following optimality equations.

$$V(0) = \max\{Q_0, 0\} / (1 - \beta), \quad (15)$$

$$V(i) = \gamma \beta q V(i - 1) + \gamma \max\{Q_i, 0\}, \quad 1 \leq i \leq k, \quad (16)$$

where $Q_0 = \beta \lambda K(h_0) - c$ and $Q_i = \beta(1 - q) \lambda K(h_i) I_{(1 \leq i < k)} + \beta q \lambda K(h_{i-1}) - c$ for $1 \leq i \leq k$.

In what comes below, we derive some equations related to h_i . By setting $i = 0$ in Equation (5), we obtain $h_0 = V(0) - V(1)$, and replacing $V(1)$ by Equation (16) with $i = 1$ produces $h_0 = (1 - \gamma \beta q)V(0) - \gamma \max\{Q_1, 0\}$. Since $1 - \gamma \beta q = \gamma(1 - \beta)$, this can

be rewritten as $h_0 = \gamma(1 - \beta)V(0) - \gamma \max\{Q_1, 0\}$. Combining this with Equation (15) now leads to

$$h_0 = \gamma \max\{Q_0, 0\} - \gamma \max\{Q_1, 0\}. \quad (17)$$

In a similar way, for $1 \leq i \leq k - 1$, we obtain the following equation.

$$h_i = \gamma\beta q h_{i-1} + \gamma \max\{Q_i, 0\} - \gamma \max\{Q_{i+1}, 0\}, \quad 1 \leq i < k. \quad (18)$$

Based upon our discussions so far, we are now ready to describe the optimal policies with respect to the switching strategy and the search option for a given state as follows:

- (a) *Optimal Switching Strategies.* For $0 \leq i < k$,
1. If $J(h_i) > 0$, adopt the admission control policy (See Equations (13) and (14)). In this case, if the proposed price w of an arriving customer is greater than h_i , then admitting the customer is optimal in state i ; otherwise, rejection is optimal (See Equations (11) and (12)).
 2. If $J(h_i) \leq 0$, adopt the pricing control policy (See Equations (13) and (14)). In this case, the optimal price to offer to an arriving customer is determined by $z(h_i)$ which maximizes $p(z)(z - h_i)$ in state i (See Equations (11) and (12)).
- (b) *Optimal Search Strategies.* For $0 \leq i \leq k$, it is optimal to conduct the search if $Q_i > 0$, and to skip it otherwise (See Equations (15) and (16)).

4. Results. In what follows, we examine the structure of the optimal policies described above and consider their implications. We start with the lemma that represents the properties of the functions of $K(x)$ and $J(x)$, both of which play an important role in clarifying the characteristics of the optimal policies.

Lemma 4.1. *We have*

- (a) $J(x) = 0$ on $[b, \infty)$, and if $b^o < b$, then $J(x)$ is negative and increasing on (b^o, b) .
- (b) $K(x)$ is convex and decreasing on $(-\infty, \infty)$, and strictly decreasing on $(-\infty, b)$.
- (c) $K(x) > 0$ on $(-\infty, b)$ and $K(x) = 0$ on $[b, \infty)$.
- (d) $x + \nu K(x)$ is increasing on $(-\infty, \infty)$, where $0 \leq \nu \leq 1$.

Proof: (a) From Equation (2) and the definition of $T_w(x)$ given in Equation (7), we have $T_w(x) = \int_0^\infty \max\{w - x, 0\} E_\alpha[\frac{1}{\alpha} f_\xi(w/\alpha)] dw$. Substituting $w = \alpha\xi$ into this equation leads to

$$T_w(x) = E_\alpha \left[\alpha \int_0^\infty \max\{\xi - x/\alpha, 0\} f_\xi(\xi) d\xi \right] = E_\alpha [\alpha T_\xi(x/\alpha)], \quad (19)$$

where $T_\xi(x) = E_\xi[\max\{\xi - x, 0\}]$. Here we note that if $b \leq x$, then $b \leq x/\alpha$ due to the inequality $0 < \alpha \leq 1$. Subsequently, $T_\xi(x/\alpha) = 0$ from Equation (1). Hence, from Equation (19) we have $T_w(b) = E_\alpha[\alpha T_\xi(b/\alpha)] = 0$. Furthermore, if $b \leq x$, we have $z(x) = b$ maximizing $p(z)(z - b)$, so $T_p(x) = \max p(z)(z - x) = p(b)(b - x) = 0$ because $p(b) = 0$ from Equation (1). Therefore, we get $J(x) = 0$ on (b, ∞) from Equation (8). Now, from the definition of b^o in Equation (10) and the fact that $T_w(x)$ is decreasing in x , we see that if $x < b^o$, then $T_w(x) > 0$ and $T_w(x) = 0$ otherwise. This validates the relation $b^o \leq b$. Suppose that $b^o \leq x < b$, then $T_w(x) = 0$ as shown above and $T_p(x) > 0$ due to Equations (6) and (1). Thus, from Equation (8) we have $J(x) = -T_p(x) < 0$, which is increasing on (b^o, b) because $T_p(x)$ is decreasing on $(-\infty, \infty)$ from Equation (6).

(b) This is immediate from Equation (9) and the fact that both $T_p(x)$ and $T_w(x)$ are convex and decreasing on $(-\infty, \infty)$ from Equations (6) and (7).

(c) The proof follows from (a), Equation (9), and the fact that $T_p(x) > 0$ for $x < b$ and $T_p(x) = 0$ for $b \leq x$, as shown in the proof of (a).

(d) We note that $x + \nu K(x) = \max\{x + \nu T_w(x), x + \nu T_p(x)\}$. Accordingly, we can prove the assertion by showing that each of $x + \nu T_w(x)$ and $x + \nu T_p(x)$ are increasing in x . Let $x_1 < x_2$. Since $T_w(x) = E_\alpha[\alpha T_\xi(x/\alpha)]$ from Equation (19), we have

$$\begin{aligned} \nu T_w(x_2) + x_2 - \nu T_w(x_1) - x_1 &= (x_2 - x_1) + \nu E_\alpha [\alpha (T_\xi(x_2/\alpha) - T_\xi(x_1/\alpha))] \\ &= (x_2 - x_1) + \nu E_\alpha \left[\alpha \left(\int_0^\infty \max\{\xi - x_2/\alpha, 0\} dF(\xi) \right. \right. \\ &\quad \left. \left. - \int_0^\infty \max\{\xi - x_1/\alpha, 0\} dF(\xi) \right) \right] \\ &\geq (x_2 - x_1) + \nu E_\alpha \left[\alpha \left(\int_{x_1/\alpha}^\infty (\xi - x_2/\alpha) dF(\xi) \right. \right. \\ &\quad \left. \left. - \int_{x_1/\alpha}^\infty (\xi - x_1/\alpha) dF(\xi) \right) \right] \\ &= (x_2 - x_1) - \nu E_\alpha [\alpha (x_2/\alpha - x_1/\alpha)(1 - F_\xi(x_1/\alpha))] \\ &= (x_2 - x_1)(1 - \nu) E_\alpha [1 - F_\xi(x_1/\alpha)] \geq 0, \end{aligned}$$

and so $T_w(x_1) + x_1 \leq T_w(x_2) + x_2$. Thus, $T_w(x) + x$ is increasing on $(-\infty, \infty)$. The proof for $T_p(x) + x$ can be found in [7]. □

The lemma given below guarantees the property of nonnegativity of h_i .

Lemma 4.2. $h_i \geq 0$ for i ($0 \leq i \leq k$).

Proof: Since h_i is given by Equation (5), to prove that the assertion is true we will show that $V(i)$ is decreasing in all i . Consider a value iteration algorithm corresponding to Equations (3) and (4) for $t \geq 1$ with $V_0(i) = 0$ for all i . Clearly, $V_0(i)$ is decreasing in i . If we assume that $V_{t-1}(i)$ is decreasing in i , then it is immediate that $V_t(i)$ is decreasing in i as well, so the assertion holds. □

Theorem 4.1. If $\beta\lambda K(0) \leq c$, then $Q_i \leq 0$ for $0 \leq i \leq k$.

Proof: Assume $\beta\lambda K(0) \leq c$. Then from Lemmas 4.2 and 4.1(b) we have $0 \geq \beta\lambda K(0) - c \geq \beta\lambda K(h_0) - c = Q_0$, $0 \geq \beta\lambda K(0) - c = (1 - q)(\beta\lambda K(0) - c) + q(\beta\lambda K(0) - c) \geq (1 - q)(\beta\lambda K(h_i) - c) + q(\beta\lambda K(h_i - 1) - c) = Q_i$ for $1 \leq i < k$, and $0 \geq \beta\lambda K(0) - c > \beta q\lambda K(h_{k-1}) - c = Q_k$. □

This result indicates that when the search cost c is sufficiently large as $c \geq \beta\lambda K(0)$, it is optimal not to conduct the search for customers; hence, no customer is at present in the system.

Lemma 4.3. If $Q_i \leq 0$ for a given i ($1 \leq i < k$), then $h_{i-1} \geq h_i$.

Proof: Let $Q_i \leq 0$ for a given i ($1 \leq i < k$). Then from Equation (16) we have $V(i) = \gamma q\beta V(i - 1)$, and hence $V(i + 1) = \gamma q\beta V(i) + \gamma \max\{Q_{i+1}, 0\} = (\gamma q\beta)^2 V(i - 1) + \gamma \max\{Q_{i+1}, 0\}$. Accordingly, we get

$$\begin{aligned} h_i - h_{i-1} &= 2V(i) - V(i - 1) - V(i + 1) \\ &= 2\gamma q\beta V(i - 1) - V(i - 1) - (\gamma q\beta)^2 V(i - 1) - \gamma \max\{Q_{i+1}, 0\} \\ &= -(1 - \gamma q\beta)^2 V(i - 1) - \gamma \max\{Q_{i+1}, 0\} \leq 0 \end{aligned}$$

due to $V(i - 1) \geq 0$ from Equations (15) and (16) and the fact that $1 > \gamma q\beta$. Therefore, $h_{i-1} \geq h_i$. □

Theorem 4.2. Suppose $h_{i-1} \leq h_i$ for a given i ($1 \leq i < k$). Then we have $Q_j > 0$ for j with $i \leq j < k$ and

- (a) $h_{i-1} \leq h_i \leq \dots \leq h_{n-1} < b$,
- (b) $z(h_{i-1}) \leq z(h_i) \leq \dots \leq z(h_{n-1}) < b$.

Proof: (a) Let $h_{i-1} < h_i$ for a given i ($1 \leq i < k$), so $h_{i-1} \leq h_i$. Then from the contrapositions of Lemma 4.3 we get $Q_i > 0$; accordingly, using Lemma 4.1(b) we have $0 < Q_i = \beta(1-q)\lambda K(h_i) + \beta q\lambda K(h_{i-1}) - c \leq \beta(1-q)\lambda K(h_{i-1}) + \beta q\lambda K(h_{i-1}) = \beta\lambda K(h_{i-1}) - c$, which gives $K(h_{i-1}) > c/\beta\lambda \geq 0$, and hence $h_{i-1} < b$ due to Lemma 4.1(c). Further, from Equation (18) we have

$$\begin{aligned} h_i &= \gamma q\beta h_{i-1} + \gamma\beta(1-q)\lambda K(h_i) + \gamma\beta q\lambda K(h_{i-1}) - \gamma c - \gamma \max\{Q_{i+1}, 0\} \\ &\leq \gamma q\beta(h_{i-1} + \lambda K(h_{i-1})) + \gamma\beta(1-q)\lambda K(h_i) - \gamma c. \end{aligned} \quad (20)$$

Assume $h_i \geq b$. Then the above inequality becomes $h_i \leq \gamma q\beta(h_{i-1} + \lambda K(h_{i-1})) - \gamma c$ due to $K(h_i) = 0$ from Lemma 4.1(c). Since $h_{i-1} < b$, using Lemma 4.1(d), we have $h_i \leq \gamma q\beta(b + K(b)) - \gamma c = \gamma q\beta(b - 0) - \gamma c \leq \gamma q\beta b < b$ due to $\gamma q\beta < 1$, which is a contradiction. Hence, it must be $h_{i-1} < (\leq) h_i < b$. Noting this result and Lemma 4.1(d), we arrange Equation (20) as $h_i \leq \gamma q\beta h_i + \gamma\beta\lambda K(h_i) - \gamma c$, and this inequality becomes $(1 - \gamma q\beta)h_i \leq \gamma\beta\lambda K(h_i) - \gamma c$. Since $h_{i-1} \geq 0$ from Lemma 4.2, we have $h_i > 0$ due to the assumption $h_{i-1} < (\leq) h_i$. From this and the fact that $1 > \gamma q\beta$ we obtain $(1 - \gamma q\beta)h_i > 0 \dots (1^*)$, so $\gamma(\beta\lambda K(h_i) - c) > 0 \dots (2^*)$. Now, suppose $Q_{i+1} \leq 0$. Then Lemma 4.3 and the above result give $h_{i+1} \leq h_i < b$. Moreover, from Lemma 4.1(c) we have $0 \geq Q_{i+1} = \beta(1-q)\lambda K(h_{i+1}) + \beta q\lambda K(h_i) - c \geq \beta(1-q)K(h_i) + \beta q\lambda K(h_i) - c = \beta\lambda K(h_i) - c$, which contradicts (2^*) , so $Q_{i+1} > 0$. Because both Q_i and Q_{i+1} are positive, we can rewrite Equation (18) as $h_i = \gamma\beta q(h_{i-1} + \lambda K(h_{i-1})) + \gamma\beta(1-2q)K(h_i) - \gamma\beta(1-q)K(h_{i+1})$. Noting the assumption $h_{i-1} < (\leq) h_i$, from Lemma 4.1(d) we get $h_i \leq \gamma\beta q(h_i + \lambda K(h_i)) + \gamma\beta(1-2q)\lambda K(h_i) - \gamma\beta(1-q)K(h_{i+1}) = \gamma\beta q h_i + \gamma\beta(1-q)(K(h_i) - K(h_{i+1}))$, which leads to $(1 - \gamma\beta q)h_i \leq \gamma\beta(1-q)(K(h_i) - K(h_{i+1}))$. Since $(1 - \gamma\beta q)h_i > 0$ from (1^*) , we get $K(h_{i+1}) \leq K(h_i)$, implying $h_i \leq h_{i+1}$ due to Lemma 4.1(b). Repeating the same procedure leads to the completion of the induction.

(b) It is immediate from the fact that $z(x)$ is increasing in x [7]. \square

From the above result we see that if $h_{i-1} < h_i$ for a given state i ($1 \leq i < k$), then the optimal admission threshold h_j and the optimal pricing $z(h_j)$ are given as increasing functions in the number of customers j with $i \leq j < k$. Therefore, h_i and $z(h_i)$ appear as one of the following functions. 1) Both h_i and $z(h_i)$ are decreasing in i . 2) For some value $m > 0$, both h_i and $z(h_i)$ are decreasing in $i < m$ and increasing in $i \geq m$, which means that both h_i and $z(h_i)$ are convex unimodal in i . 3) Both h_i and $z(h_i)$ are increasing in i .

Theorem 4.3. If $Q_i > 0$ for a given i ($0 \leq i < k$), then $Q_j > 0$ for $i \leq j < k$.

Proof: Let $Q_i > 0$ for a given i ($1 \leq i < n$). First, assume $h_{i-1} < h_i$. Then $Q_{i+1} > 0$ from Theorem 4.2. Next, let $h_{i-1} \geq h_i$. Then since $K(h_{i-1}) \leq K(h_i)$ due to Lemma 4.1(c), we get $0 < Q_i = \beta(1-q)\lambda K(h_i) + \beta q\lambda K(h_{i-1}) - c \leq \beta\lambda K(h_i) - c$. Suppose $Q_{i+1} \leq 0$. Then $h_i \geq h_{i+1}$ due to Lemma 4.3. Noting $K(x)$ is convex on $(-\infty, \infty)$ from Lemma 4.1(b), we have $\beta\lambda K((1-q)h_{i+1} + qh_i) - c \leq \beta\lambda((1-q)K(h_{i+1}) + qK(h_i)) - c \leq 0 < \beta\lambda K(h_i) - c$, implying $K((1-q)h_{i+1} + qh_i) < K(h_i)$, and hence $(1-q)h_{i+1} + qh_i > h_i$ due to Lemma 4.1(b). Therefore, $h_i < h_{i+1}$ due to the assumption $q < 1$, which is a contradiction. Thus, $Q_{i+1} > 0$. We can complete the induction by repeating the same procedure. Now, if $Q_0 > 0$ and $Q_1 \leq 0$, we get to a contradiction in quite the same way as above, so it must be that $Q_1 > 0$. \square

The above result states that if it is optimal to continue the search in a given state i , then it will be so in all states $j \geq i$. This means that starting from the initial state 0, if $Q_0 > 0$, then the optimal policy is to conduct the search in all states.

Theorem 4.4. *If $\beta\lambda K(0) > c$, we have*

- (a) $Q_i > 0$ for $0 \leq i < k$.
- (b) h_i and $z(h_i)$ are increasing in i ($0 \leq i < k$).

Proof: (a) Since $V(i) \geq 0$ from Equations (15) and (16), we have $V(0) \geq \max\{\beta\lambda K(0) - c, 0\} = \beta\lambda K(0) - c > 0$ due to the assumption. Therefore, $V(0) > 0$. If $Q_0 \leq 0$ in Equation (15), then $V(0) = 0$ (note $\beta < 1$), which contradicts $V(0) > 0$. Hence, $Q_0 > 0$. This and Theorem 4.3 give the stated result.

(b) From Equation (17) we have $h_0 = \gamma(Q_0 - Q_1) = \gamma\beta(1 - q)\lambda(K(h_0) - K(h_1))$ due to (a). If $h_0 = 0$, then $K(h_1) = K(h_0) = K(0)$, so $h_0 = h_1 = 0$. And if $h_0 > 0$, then $K(h_0) > K(h_1)$, so $h_0 < h_1$ due to Lemma 4.1(b). Applying Theorem 4.2 on this completes the proof. \square

An implication of the result (a) is that when $\lambda\beta K(0) > c$, it is optimal to search for customers in all states i ($0 \leq i < k$). The result (b) means that as i increases, the decision maker will become more selective whether to choose the admission control policy or the pricing control policy.

5. Numerical Studies. In this section, we demonstrate cases where the switching occurs between the admission and the pricing control policies and examine the optimal search policies through some numerical experiments. The experiments have been made under the following conditions: $\beta = 0.97$, $\lambda = 0.99$, $c = 0.05$, $k = 15$, and $F_\xi(x)$ and $F_\alpha(x)$ are the uniform distributions on $[0, 1]$ and $[0.5, 0.9]$, respectively. Note that $\beta\lambda K(0) - c \cong 0.29 > 0$ in this case.

5.1. Optimal switching strategy. In Figure 1, we present the graphs of $J(x)$ and h_i where if $J(h_i) > 0$, adopting the admission control policy is optimal; otherwise, the pricing control policy is optimal. The graph in Figure 1(a) depicts the function $J(x)$ on $[0, 1]$, where $J(x)$ is less than zero and increasing on $(0.89, 1)$ (note $b^o = 0.89$) by Lemma 4.1(a). There exists $x^* = 0.27$ a solution of $J(x) = 0$, such that $J(x) > 0$ if $x < x^*$, and $J(x) \leq 0$ otherwise.

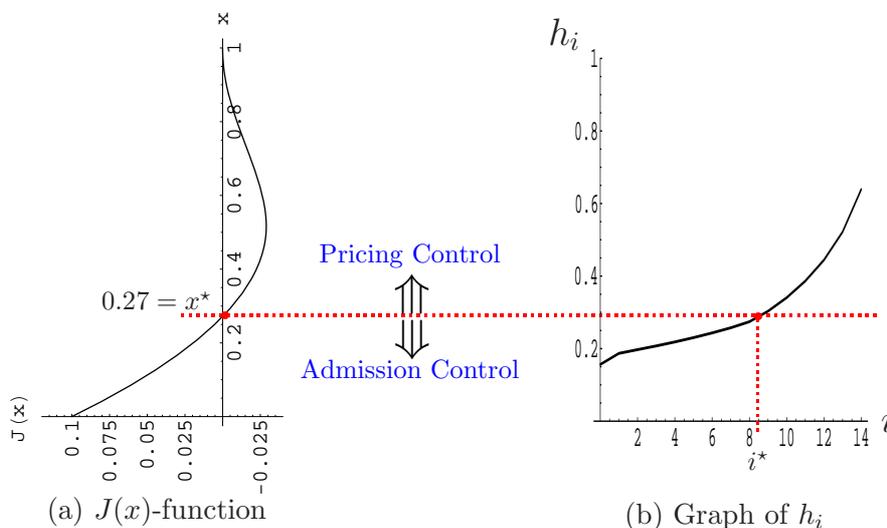


FIGURE 1. Graphs of $J(x)$ and h_i . If $J(h_i) > 0$, adopting the admission control policy is optimal; otherwise, the pricing control policy is optimal.

Figure 1(b) demonstrates the fact that h_i indeed increases in i as expected from Theorem 4.4(b). This creates a switching threshold i^* ($= 9$) such that if $i < i^*$, since $h_i < x^*$, we have $J(h_i) > 0$, and hence it is optimal to adopt the admission control policy. On the other hand, if $i \geq i^*$, since $h_i \geq x^*$, we get $J(h_i) \leq 0$, so it is optimal to employ the pricing control policy. Therefore, when $i < i^*$ ($i \geq i^*$), since adopting the admission control (pricing control) policy is optimal, the optimal threshold h_i (optimal pricing $z(h_i)$) should be set such a way that it increases in the number of customers in the system. The implications of the monotonicity of h_i and $z(h_i)$ are discussed in [12].

5.2. Optimal search strategy. Table 1 below represents the optimal search policies for a given state i ($0, 1, \dots, 15$). When $c = 3.0$ ($\geq \lambda\beta K(0) = 0.24$), it is optimal not to conduct the search for customer as proven in Theorem 4.1(b), while when $c = 0.05$ ($< \lambda\beta K(0) = 0.24$), it is always optimal to conduct the search for customers in all states except the state $i = k$ where the system capacity is full (Theorem 4.4(a)). In state $i = k$, both C and K can be optimal even though it appears in the table to skip the search.

TABLE 1. Optimal search policies

i	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$c = 0.05$	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	K
$c = 0.30$	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K	K

5.3. Economic effects of the switching strategy. To start, let $\tilde{V}(i)$ be the maximum expected profit when the switching strategy is not in use. Subsequently, let $\varphi(i)$ be the relative difference between $V(i)$ and $\tilde{V}(i)$, i.e., $\varphi(i) = (V(i) - \tilde{V}(i))/V(i)$. For convenience, let $\varphi(i)$ for employing admission control and pricing control policies be denoted by, respectively, $\varphi_a(i)$ and $\varphi_p(i)$.

Figure 2 depicts the graph of $\varphi_a(i)$ and $\varphi_p(i)$. We see that $\varphi_a(i)$ and $\varphi_p(i)$ can be as high as, respectively, 13% and 7.6%. The bold dotted red line indicates that when the switching strategy is not in use, if $i \leq 2$ or $i \geq 8$, it is optimal to adopt the pricing control policy; otherwise, it is optimal to use the admission control policy. Despite using the best policy as mentioned above, the maximum relative difference is approximately 5.6%, which occurs near $i = 8$. This implies that the decision maker bears the risk of creating a large opportunity cost by not employing the switching strategy.

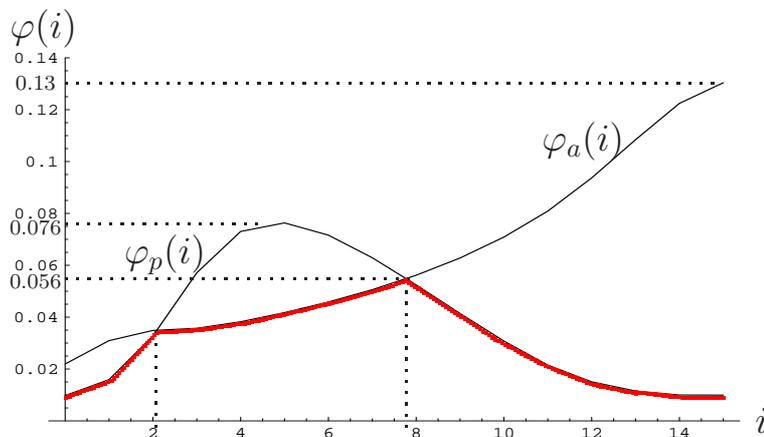


FIGURE 2. Graph of $\varphi_a(i)$ and $\varphi_p(i)$

6. Concluding Remarks. We proposed a switching strategy between the admission and pricing control policies as well as searching strategy in a customized service providing company. In this research we have clarified the structural properties of the optimal switching and searching strategies. According to the results, the optimal switching strategy depends on the shape of the function $J(x)$, and the optimal switching threshold exists in terms of the number of customers in the system under a certain condition. As for the optimal search strategy, it is optimal to skip the search when the search cost is sufficiently large. However, when the search cost is sufficiently small, it is optimal to continue the search except the state where the capacity is full. Furthermore, we have examined the relative difference of the maximum expected profits between our switching model and the model without switching strategy, the results of which can be as high as 13 percent. This validates the economic effects of our switching model as well as representing the opportunity cost of not adopting the switching strategy.

As a general framework for the derivation of monotonicity properties, Koole [9] proposes a unified treatment of the various queueing models by concentrating on system events and the *form* of the value function instead of focusing on the value function itself. To investigate in what extent our model with the notion of search skipping fit within the Koole's framework would be an interesting topic for a further research. Moreover, one could incorporate the following conditions which would make our model more practical: 1) future availability of once rejected customers, 2) customer's renegeing from the queue, 3) strategic interaction between the customer and the decision maker by introducing game theory; and further investigate them.

REFERENCES

- [1] S. B. Crabill, D. Gross and M. Magazine, A classified bibliography of research on optimal design and control of queues, *Oper. Res.*, vol.25, no.2, pp.219-232, 1997.
- [2] L. DaSilva, Pricing for QoS-enabled networks: A survey, *IEEE Commun. Surv.*, pp.2-8, 2000.
- [3] N. Gans and S. Savin, Pricing and capacity rationing in rentals, *Working Paper*, Wharton School of Business, University of Pennsylvania, 2005.
- [4] M. Ghaderi and R. Boutaba, Call admission control in mobile cellular networks: A comprehensive survey, *Wirel. Commun. Mob. Com.*, vol.6, pp.69-93, 2006.
- [5] S. L. Hew and L. B. White, Optimal integrated call admission control and dynamic pricing with handoffs and price-affected arrivals, *Proc. of 2005 Asia-Pacific Conf. Commun.*, pp.396-400, 2005.
- [6] S. Ikuta, Optimal stopping problem with uncertain recall, *J. Oper. Res. Soc. Japan*, vol.31, no.2, pp.145-170, 1988.
- [7] S. Ikuta, An integration of the optimal stopping problem and the optimal pricing problem – Model with no recall, *U. Tsukuba, Inst. Policy Plann. Sc.*, No.1084, 2004.
- [8] B. K. Kang, Optimal stopping problem with recall cost, *Eur. J. Oper. Res.*, vol.117, pp.222-238, 1999.
- [9] G. Koole, Structural results for the control of queueing systems using event-based dynamic programming, *Queueing Syst.*, vol.30, pp.323-339, 1998.
- [10] S. M. Ross, *Introduction to Stochastic Dynamic Programming*, Academic Press, 1983.
- [11] T. Saito, Optimal stopping problem with controlled recall, *Probab. Eng. Inform. Sc.*, vol.12, pp.91-108, 1998.
- [12] J. D. Son, Customer selection problem with profit from a sideline, *Eur. J. Oper. Res.*, vol.176, no.2, 1084-1102, 2007.
- [13] S. Stidham and R. Weber, A survey of Markov decision models for control of networks of queue, *Queueing Syst.*, vol.13, pp.291-314, 1993.
- [14] S. H. Yoon and M. E. Lewis, Optimal pricing and admission control in a queueing system with periodically varying parameters, *Queueing Syst.*, vol.47, no.3, pp.177-199, 2004.