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OPTIMAL BASED ROBUST FUZZY PARAMETRIC UNCERTAIN CONTROLLER DESIGN FOR A SMIB POWER SYSTEM STABILIZER

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Abstract. *This paper addresses the problem of determining an optimal LQR design for a robust power system stabilizer (PSS). For achieving optimal PSS design, first, the different loads of a PSS are described using fuzzy α-cut numbers, and a fuzzy parametric uncertain system is formulated into an interval state-space controllable canonical form system. Second, the maximum uncertainty interval of the system is translated into the weighting matrix Q of the LQR problem to guarantee that the designed optimal controller is robust under worst-case conditions. The designed PSS is applied to a single machine infinite bus (SMIB) system operating under various loading conditions. The simulation results showed that the performance indexes of proposed method are superior to those of typical Phase Lead based method.*
Keywords: Power system stabilizer, Fuzzy parametric uncertain systems, LQR

1. Introduction.

Power systems are often experiencing uncertainty disturbances such as power consumption changes, variation of operating conditions, and faults. The disturbances cause low frequency oscillation which may result in serious consequences. The problem of underdamped oscillations within power systems has received a great deal of attention in past decades [1]. Various power system stabilizers (PSS) have been designed to provide additional damping and maintain stability over a wide range of operating points [2]. According to Kharitonov's theory the nonlinear power system can be converted into an uncertain parametric interval system. Then the classical robust Phase-Lead PSS controller design can be determined by examining the root locus diagrams associated with extreme conditions of interval system [3-5]. However, this type of PSS design is often conservative; it assumes that the parameters of characteristic polynomials vary independently. In addition, the interval representation has another drawback of considering the range of all possible outcomes in the same probability. Last, this kind of controller is suboptimal.

Optimization methods based on evolutionary algorithms or swarm intelligence have also been applied to the PSS design [6-8]. Techniques such as particle swarm optimization (PSO), gravitational search algorithms (GSAs), and genetic algorithms (GAs) are used for solving the optimal problem while working within system constraints. Considering the time or frequency domain specification (e.g., a large overshoot or eigenvalue location), these types of technique can be used to find a near-optimal solution. To simplify the calculation of the objective function, the proposed controller was limited to a typical Phase-Lead configuration, and only a few specific operating points were evaluated.

State feedback controllers, based on linear quadratic control theory (LQR), are an alternative optimal method for PSS design. They have shown improved performance over classical PSS [9,10]. Furthermore, different techniques (e.g., output feedback methods, observer-based controllers and modified Heffron-Phillip models) have been proposed to compensate the limitations of immeasurable state variables [11-14]. However, this type of approach is still difficult because the determination of the weighting matrices Q and R is often arbitrary or based on trial and error. In addition, the LQR controller is not robust; its performance is degraded under condition of system parameter uncertainty.

To solve the aforementioned problem, fuzzy logic control (FLC) offers powerful tools to overcome the uncertainty problem. For example, the Takagi-Sugeno (T-S) fuzzy model is a nonlinear model that uses fuzzy membership functions connected by if-then rules. The analysis and the control of T-S fuzzy systems can be easily applied to some firmly established linear system theories to solve the problem of uncertainty system [15-18]. An alternative FLC strategy is representing uncertainty as a fuzzy number with a membership function, which is a possibility approach based on fuzzy set theory. Such a fuzzy dynamic system can be viewed as an extension of uncertain parametric interval systems [19,20]. Because fuzzy logic has an interpolative characteristic, fuzzy-set-based approaches can describe the uncertainties in PSS design [21]. In addition, an α-cut representation can be used to create a family of crisp sets from a given fuzzy set. Thus, a system with fuzzy uncertainties becomes a system with interval uncertainties for each α-cut. For α-cut $= 0$, we obtain maximum uncertainty. The design of a robust controller must stabilize all the systems corresponding to each $\alpha \in [0, 1]$. In [22], a robust controller design for a fuzzy parametric uncertain system can be converted into an optimal LQR problem. When the solution of the algebraic Riccati equation of LQR problem was determined, the robust controller can be implemented.

In this study, a novel PSS controller design was proposed and applied in a single machine infinite bus (SMIB) power system. The proposed method has the following advantages: 1) the different PSS loads are described using fuzzy α-cut numbers, which is compromised between uncertainty and probability; 2) the maximum fuzzy uncertainty interval of the system is translated into a weighting matrix Q of the LQR problem, which guarantees that the designed optimal controller is robust even under worst-case conditions. Compared with traditional state feedback methods, the proposed method is more intuitive, involves simpler computations, and improves the robustness of the LQR controller. Simulation studies of the PSS designed by the proposed method are also given in the paper. The results show that both robustness and optimization of controller can be considered simultaneously.

2. Problem Statement and Preliminaries.

2.1. Single machine infinite bus model. The power system was derived using the Heffron-Phillips model. A block diagram of the linearized single machine infinite bus (SMIB) model is shown in Figure 1 [3]. Such a model is common in the literature.

The operating points of the PSS model are loading dependent. The parameters k_1-k_6 can be derived from the different operation points P_{op} and Q_{op}. The formulas of the k parameters are explained in the Appendix. The deviation signal $\Delta\omega$ is used as an input signal for the conventional PSS. The linear state equation for the power system under study is written as follows:

$$\dot{x} = Ax + Bu \tag{1}$$
$$y = Cx$$

where

$$x = \begin{bmatrix} \Delta\delta & \Delta\varpi & \Delta E'_q & \Delta E_{fd} \end{bmatrix} \tag{2}$$

$$A = \begin{bmatrix} 0 & \omega_0 & 0 & 0 \\ -\frac{k_1}{M} & 0 & -\frac{k_2}{M} & 0 \\ -\frac{k_4}{T'_{do}} & 0 & -\frac{1}{k_3 T'_{do}} & -\frac{1}{T'_{do}} \\ -\frac{k_5 k_E}{T_E} & 0 & -\frac{k_6 k_E}{T_E} & -\frac{1}{T_E} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_E}{T_E} \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

The fourth-order transfer function in (1) is written as

$$\frac{\Delta\omega}{U} = \frac{bs}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{3}$$

where the transfer function coefficients expressed in terms of the k parameters are written as

$$b = k_E k_2 k_3, \quad a_4 = MTT_E, \quad a_3 = M(T + T_E), \tag{4}$$

$$a_2 = M + \omega_0 k_1 TT_E + k_E k_3 k_6 M, \quad a_1 = \omega_0 k_1 (T + T_E) - \omega_0 k_2 k_3 k_4 T_E,$$

$$a_0 = \omega_0 (k_1 - k_2 k_3 k_4 - k_E k_2 k_3 k_5 + k_E k_1 k_3 k_6)$$

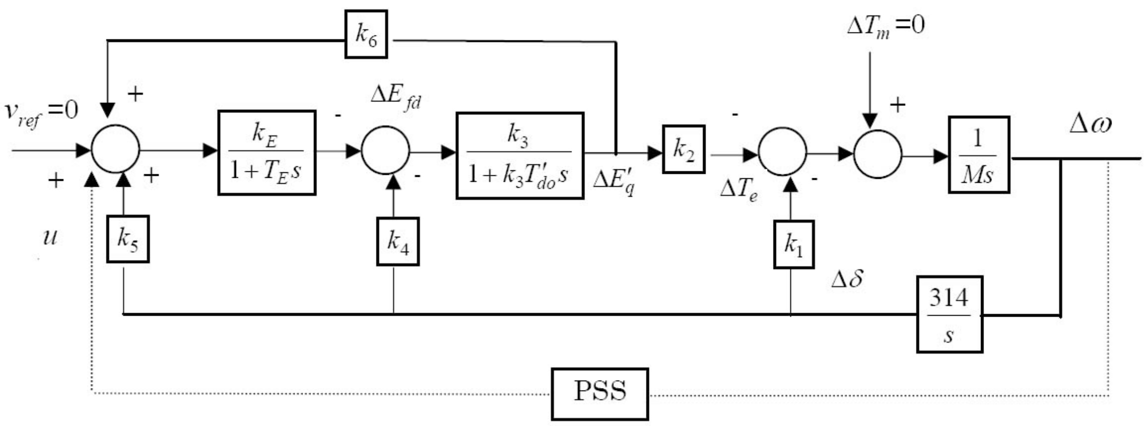

FIGURE 1. Linearized model of a single machine infinite bus power system

2.2. **Fuzzy parametric α-cut representation of different loads of PSS.** The uncertain parameters are represented by a fuzzy number, $\tilde{q}_i$, with membership function α-cut $= \mu(\tilde{q}_i) \in [0, 1]$. The membership function $\mu(\tilde{q}_i)$ may be any nonsymmetrical triangular membership function but decreases to the interval endpoint. The fuzzy parametric uncertainty α-cut is defined using

$$q_i(\alpha_i) = \left[q_i^-(\alpha_i), q_i^+(\alpha_i)\right], \quad q_i^-(0) = q_i^-, \quad q_i^+(0) = q_i^+, \quad q_i^-(1) = q_i^+(1) = q_i^0 \tag{5}$$

where $q_i^-(.)$ is an increasing function and $q_i^+(.)$ is a decreasing function.

Consider a power system with two uncertain operating parameters, P_{op} and Q_{op}. Assume the linguistic information of operating condition: "high load", "normal load", and "low load" are represented as fuzzy sets with triangular membership function. For α-cut = 1, we obtain nominal condition { normal load: $P_{op}(1) = 1$, $Q_{op}(1) = 0$ }; for α-cut = 0, we obtain maximum uncertainty { high load: $P^+_{op}(0) = 1.15$, $Q^+_{op}(0) = 0.4$ } or { low load: $P^-_{op}(0) = 0.2$, $Q^-_{op}(0) = -0.4$ }. These total membership function can be described in terms of the uncertain operating parameters P_{op} and Q_{op} (shown in Figure 2), where $\tilde{P}_{op} = tri(0.2, 1, 1.15)$ and $\tilde{Q}_{op} = tri(-0.4, 0, 0.4)$.

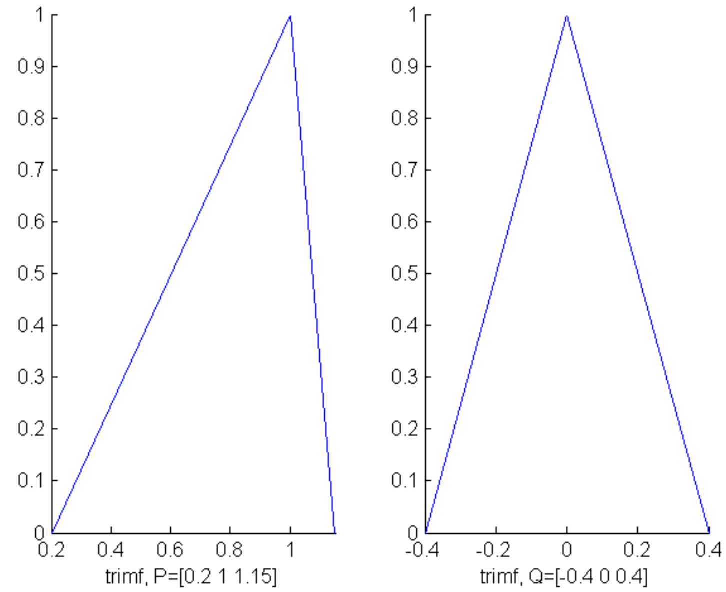

FIGURE 2. Membership function for P_{op} and Q_{op}

The nonlinear power system is represented as an uncertainty interval system with a degree of confidence of $\alpha \in [0, 1]$. For α-cut $= 0$, using (4), the extreme values of interval transfer function coefficients are calculated as follows:

$$a_i^- = \min_{\tilde{P}^-(0), \tilde{Q}^-(0)} a_i, \quad a_i^+ = \max_{\tilde{P}^+(0), \tilde{Q}^+(0)} a_i \tag{6}$$

This study focuses on the question of converting the fuzzy uncertainty robustness problem into an optimal LQR control problem for a worst-case condition of α-cut $= 0$.

3. Optimal-Approach-Based Robust Controller Design for a Fuzzy Parametric System.

3.1. **Kharitonov stability theory.** Using the Kharitonov theory to demonstrate the robust Hurwitz stability of an interval uncertain system is efficient and intuitive. An interval polynomial, in which each coefficient a_i is independent of the others and varies within the bounds of an interval, can be written as

$$p = a_n s^n + a_{n-1} s^{n-1} + \ldots + a_0, \quad a_i = \left[\begin{array}{cc} a_i^- & a_i^+ \end{array}\right] \tag{7}$$

This type of uncertain interval polynomial is robust and stable if and only if the following four extreme characteristic polynomials (8) possess Hurwitzian stability; in other words, if they all have no roots in the right half plane.

$$\begin{aligned} p_1^{--} &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + \ldots \\ p_2^{++} &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + \ldots \\ p_3^{+-} &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + \ldots \\ p_4^{-+} &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + \ldots \end{aligned} \tag{8}$$

The Kharitonov theorem assumes that the coefficients in the interval polynomial perturb independently. Even the interval characteristic polynomial does not conform to the assumptions made by the Kharitonov theorem in real-world problems; the theorem can still obtain conservative stability bounds and a sufficient condition of stability for polynomials with dependent coefficients [23].

3.2. **Optimal robust controller design for a system with fuzzy parametric uncertainty.** Nonlinear dynamic equations can be represented as linear models at specific operating points. When a nonlinear system can be stabilized at different operating points, it can be equivalent to stabilize the parametrical uncertain linear model. Consider an uncertain system represented as a system with fuzzy parametric uncertainty, as described by the transfer function.

$$G(s,\tilde{p},\tilde{q}) = \frac{\tilde{p}_{n-1}(\alpha)s^{n-1} + \cdots + \tilde{p}_1(\alpha)s + \tilde{p}_0(\alpha)}{s^n + \tilde{q}_{n-1}(\alpha)s^{n-1} + \cdots + \tilde{q}_1(\alpha)s + \tilde{q}_0(\alpha)} \tag{9}$$

where $\tilde{p}_i(\alpha)$, $\tilde{q}_i(\alpha)$ represent the fuzzy interval number. The α-cut confidence is given as $\alpha \in [0,1]$.

Furthermore, the fuzzy parametric uncertain system is realized in state-space representation by a controllable canonical form [22]:

$$\dot{x} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\tilde{q}_0(\alpha) & -\tilde{q}_1(\alpha) & \cdots & -\tilde{q}_{n-1}(\alpha) \end{bmatrix} x + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \tag{10}$$

$$y = \begin{bmatrix} \tilde{p}_0(\alpha) & \tilde{p}_1(\alpha) & \cdots & \tilde{p}_{n-1}(\alpha) \end{bmatrix} x$$

The compact representation of (10) is

$$\dot{x} = A(\tilde{q}(\alpha))x + Bu \tag{11}$$

$$y = C(\tilde{p}(\alpha))x$$

Assume that there exists a nominal value, $q_{nom} \in \tilde{q}(\alpha)$ such that $(A(q_{nom}), B)$ is stable; there exists a $1 \times n$ matrix $\phi(\tilde{q}(\alpha))$. The uncertainty in A is represented as

$$A(\tilde{q}(\alpha)) - A(q_{nom}) = B\phi\left(\tilde{q}(\alpha)\right) \tag{12}$$

The fuzzy parametric uncertain system can then be rewritten as

$$\dot{x} = A(q_{nom})x + B\phi\left(\tilde{q}(\alpha)\right) + Bu \tag{13}$$

The problem of designing a robust controller for a system with fuzzy parametric uncertainty lies in finding a feedback control law $u = -kx$ such that the closed loop system

$$\dot{x} = A(q_{nom})x + B\phi\left(\tilde{q}(\alpha)\right) - Bkx \tag{14}$$

is stable for all $\alpha \in [0,1]$. Thus, the aforementioned robust control problem is translated into an optimal control problem by using an LQR approach, as follows.

For the system with fuzzy parametric uncertainty in (14), the cost function is designed as

$$J = \int_0^\infty \left(x^T F x + x^T x + u^T R u\right) dt \tag{15}$$

where F is an upper bound on the uncertainty.

When the uncertain system $\phi\left(\tilde{q}(\alpha)\right)$ is bounded, the upper bound on F can be written as $\phi\left(\tilde{q}(\alpha)\right)^T \phi\left(\tilde{q}(\alpha)\right) \le F$.

When $Q = [F + I]$, the cost function is rewritten as

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) dt \tag{16}$$

The LQR optimal control problem is finding the optimal feedback gain $u = -kx$ that minimizes the cost function. If there exists a feedback control law $u = -kx$ such that (14) is stable for all $\tilde{q}(\alpha)$, $\alpha \in [0,1]$, the design of a robust controller is considered to be successful. For a system with fuzzy parametric uncertainty, the solution to the LQR problem is the solution of the robust control problem. The following proposition demonstrates how to determinate the weighting matrix Q in the LQR problem.

3.3. **Determination of the uncertainty weighting matrix.** In (13), the worst case condition ($\alpha = 0$) is the largest interval in the uncertain system. An optimal controller guarantees stabilization of the entire uncertain interval.

For $\alpha = 0$, consider the maximum uncertainty described by $q_i \in \begin{bmatrix} q_i^- & q_i^+ \end{bmatrix}$. For $\alpha \in [0, 1]$, the uncertainty $\begin{bmatrix} q_i^-(\alpha_i) & q_i^+(\alpha_i) \end{bmatrix}$ can be written as any value in $\begin{bmatrix} q_i^- & q_i^+ \end{bmatrix}$. For the sake of demonstration, assume that the nominal value is $q_{nom} = \begin{bmatrix} q_0^- & q_1^- & \cdots & q_{n-1}^- \end{bmatrix}$. The system in (10) can then be written as

$$\dot{x} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -q_0^- & -q_1^- & \cdots & -q_{n-1}^- \end{bmatrix} x + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \tag{17}$$

In (12), the uncertain system $B\phi(\tilde{q})$ can be written as

$$\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -\tilde{q}_0 & -\tilde{q}_1 & \cdots & -\tilde{q}_{n-1} \end{bmatrix} - \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ -q_0^- & -q_1^- & \cdots & -q_{n-1}^- \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} q_0^- - \tilde{q}_0 \\ q_1^- - \tilde{q}_1 \\ \vdots \\ q_{n-1}^- - \tilde{q}_{n-1} \end{bmatrix}^T \tag{18}$$

The maximum uncertainty $\phi(\tilde{q})$ is bounded by

$$F = \phi^T\phi \leq \begin{bmatrix} [q_0^+ - q_0^-][q_0^+ - q_0^-] & \cdots & [q_0^+ - q_0^-][q_{n-1}^+ - q_{n-1}^-] \\ \vdots & \vdots & \vdots \\ [q_{n-1}^+ - q_{n-1}^-][q_0^+ - q_0^-] & \cdots & [q_{n-1}^+ - q_{n-1}^-][q_{n-1}^+ - q_{n-1}^-] \end{bmatrix} \tag{19}$$

Let $Q = [F + I]$, and designate Q as the cost function for the LQR optimal control problem. With the feedback control law $u = -kx$, the characteristic equation of closed loop system in (14) can be written as

$$s^n + [k_n + \tilde{q}_{n-1}]\, s^{n-1} + \cdots + [k_2 + \tilde{q}_1]\, s + [k_1 + \tilde{q}_0] = 0 \tag{20}$$

The Kharitonov theorem in (8) can be used to determine whether the interval polynomial is stable. If and only if the four Kharitonov extreme characteristic polynomials all have roots in the left half plane (LHP), the controller designed is both optimal and robust.

4. **Simulation Results.** This section illustrates how to design a robust power system stabilizer based on an optimal LQR approach in an SMIB system.

The transfer function of an uncertain SMIB power system is expressed as (3).

$$G(s) = \frac{[\ 21 \quad 96.1\ ]s}{8.34s^4 + 167.3s^3 + [\ 427.1 \quad 744.4\ ]s^2 + [\ 2941 \quad 7500\ ]s + [\ 1393 \quad 16758\ ]} \tag{21}$$

For the sake of convenience, we translate this equation into its controllable canonical form, and the transfer function of the uncertain system in (21) is expressed as

$$G(s) = \frac{[\ 2.5 \quad 11.5\ ]s}{s^4 + 20.1s^3 + [\ 51.2 \quad 89.3\ ]s^2 + [\ 352.6 \quad 899.3\ ]s + [\ 167.0 \quad 2009.4\ ]} \tag{22}$$

The uncertain system in (22) can be expressed in its state-space controllable canonical form as

$$\overset{\bullet}{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\begin{bmatrix} 167.0 & 2009.4 \end{bmatrix} & -\begin{bmatrix} 352.6 & 899.3 \end{bmatrix} & -\begin{bmatrix} 51.2 & 89.3 \end{bmatrix} & -20.1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \tag{23}$$

Assuming the nominal operating condition is $P_{op} = 1$ p.u. and $Q_{op} = 0$ p.u., the transfer function of the nominal system is given by

$$G(s) = \frac{85.8s}{8.34s^4 + 167.3s^3 + 639s^2 + 6252.6s + 12287} \tag{24}$$

The nominal system in (24) can be expressed in its state-space controllable canonical form as

$$\overset{\bullet}{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1473.4 & -749.7 & -76.7 & -20.1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \tag{25}$$

Using (23), the uncertainty is given by $\phi(\tilde{q}) = \begin{bmatrix} q_0^+ - q_0^- & q_1^+ - q_1^- & q_2^+ - q_2^- & 0 \end{bmatrix}$.

The upper bound of the uncertainty in (21) can be expressed as

$$F = \begin{bmatrix} 1842.3 \\ 546.7 \\ 38.1 \\ 0 \end{bmatrix} \begin{bmatrix} 1842.3 & 546.7 & 38.1 & 0 \end{bmatrix} \tag{26}$$

The LQG weighting matrix Q can be written as

$$Q = [F + I] = \begin{bmatrix} 3394144.0 & 1007085.8 & 70100.3 & 0 \\ 1007085.8 & 298816.3 & 20799.7 & 0 \\ 70100.3 & 20799.7 & 1448.8 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

Considering $R = 1$ and solving the feedback control gain by using the LQR approach, we obtain K_{LQR} as follows:

$$K_{LQR} = \begin{bmatrix} 885.6 & 413.8 & 105.4 & 4.7 \end{bmatrix} \tag{28}$$

With the feedback control law $u = -K_{LQR}x$, the characteristic equation over all of the operating conditions (20) can be expressed as

$$s^4 + 24.78s^3 + \begin{bmatrix} 156.6 & 194.7 \end{bmatrix} s^2 + \begin{bmatrix} 766.5 & 1313.1 \end{bmatrix} s + \begin{bmatrix} 1052.7 & 2895 \end{bmatrix} = 0 \tag{29}$$

The roots of the four Kharitonov polynomials given below are located in the left half plane (LHP), which proves that the controller designed can stabilize all interval plants.

$$\begin{aligned} p_1^{--} &= s^4 + 24.8s^3 + 194.7s^2 + 766.5s + 1052.7 \\ p_2^{++} &= s^4 + 24.8s^3 + 156.6s^2 + 1313.1s + 2895 \\ p_3^{+-} &= s^4 + 24.8s^3 + 156.6s^2 + 766.5s + 2895 \\ p_4^{-+} &= s^4 + 24.8s^3 + 194.7s^2 + 1313.1s + 1052.7 \end{aligned} \tag{30}$$

We simulated the optimal-LQR-based robust controller (28) for nominal and worst-case closed-loop systems. The responses of each state x_1, x_2, x_3, and x_4 for $\alpha = 1$ and 0 are shown in Figure 3. It is clear that all the interval plants are stable.

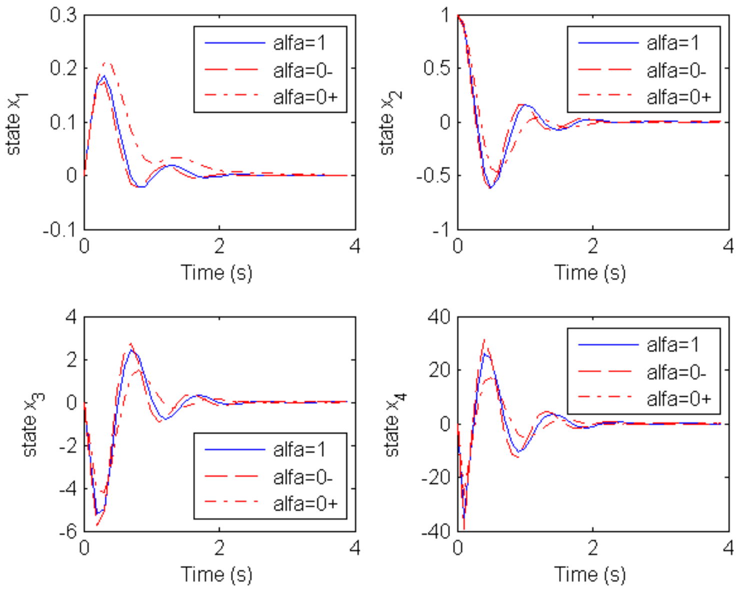

FIGURE 3. States x_1, x_2, x_3, and x_4 for α-cut = 1 and 0

To demonstrate that the proposed approach has the capability to consider the robustness and the optimization simultaneously, we evaluated the proposed approach by comparing two methods with different loading regimes.

Example 4.1. *A PSS, designed based on a robust phase-lead approach [5]. The design procedure is summarized as follows.*

1) Set up the operating range to determine the polynomial coefficients of the uncertainty interval.

2) Design the transfer function representation of the phase-lead controller and the controller time constant.

3) Use the generalized Kharitonov theorem and the Routh-Hurwitz criterion to plot the boundaries of stability region in parametric space.

4) Choose the property controller gain and time constant from stability region of parametric space.

The transfer function of the robust phase-lead-based PSS can be written as

$$G_c(s) = 50\frac{1 + 0.5s}{1 + 0.05s} \tag{31}$$

Example 4.2. *A PSS, designed based on a PSO approach [6]. The design procedure is summarized as follows.*

1) Set up the nominal operating point of the state-space representation of the power system.

2) Design the transfer function representation of the phase-lead controller.

3) Design the PSO parameters and objective function.

4) When the objective function has converged, stop the PSO search iteration.

The transfer function of the PSO-based PSS can be written as

$$G_c(s) = 47.95\frac{1 + 0.3176s}{1 + 0.077s} \tag{32}$$

Three designed PSSs were evaluated under two operating conditions (α-cut = 1 and 0). The typical PSS parameter values are summarized in the Appendix. Figure 4 represents *the load angle variations caused by a 0.1-p.u.-step increment in mechanical torque under the operating condition of* α*-cut* $= 1$ *(*$P_{op} = 1$*,* $Q_{op} = 0$*). Because the operating conditions are located in the design loading regimes of all PSSs, all types of PSS can dampen low frequency oscillation.*

Figure 5 represents the load angle variations caused by a 0.1-p.u.-step increment in mechanical torque under the operating condition of α-cut $= 0$ ($P_{op} = 1.15$, $Q_{op} = -0.4$). As shown in Figure 4, the proposed robust PSS can dampen the low frequency oscillation in each $\alpha \in [0, 1]$.

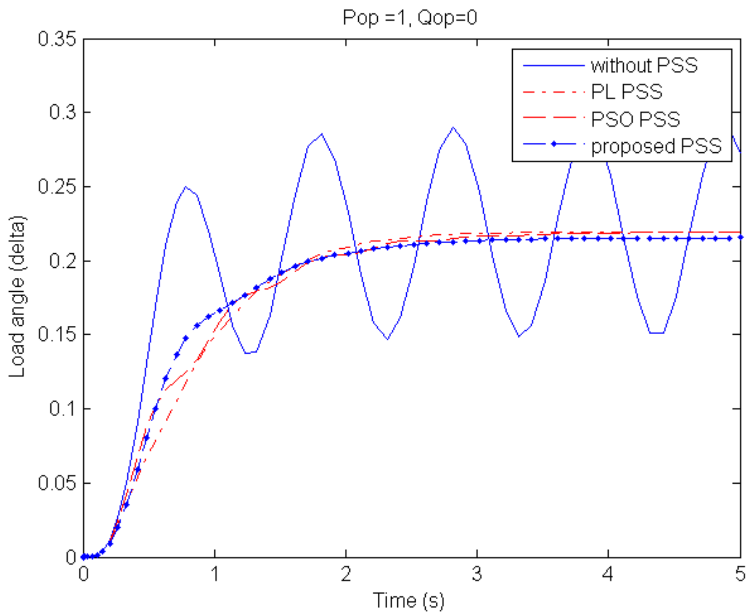

FIGURE 4. Response of system under the $P_{op} = 1$, $Q_{op} = 0$ condition

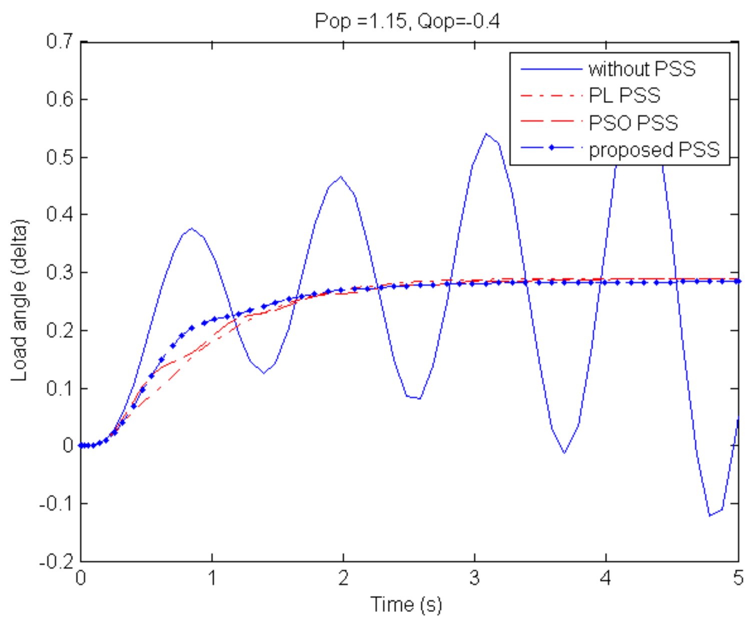

FIGURE 5. Response of the system under the $P_{op} = 1.15$, $Q_{op} = -0.4$ condition

A comparison of the performance indices in terms of the rise time (RT) and delay time (DT) that correspond to PSS transient state characteristics is shown in Table 1. Whether α-cut = 1 or 0, the proposed robust PSS outperforms the other two approaches regarding both rise time and delay time. The robust Phase-Lead controller (Example 4.1) was designed by selecting parameters arbitrarily from stability region of parametric space. It needs a lot of trial and error to determine the proper parameters and has to compromise in optimism. Conversely, the PSO controller (Example 4.2) was evaluated only under specific operating conditions. It has suffered the performance degradation in the whole operation regime. Generally speaking, in order to guarantee the stability in the whole operational condition, Phase-Lead controller has to check each extreme polynomial of the parametric space; PSO controller has to evaluate iteratively different specific operating points. However, the proposed method requires few computations and strengthens the robust property of the LQR. It also provides the most favorable tradeoff between performance and stability.

Table 1. Comparisons of performance index

	$P_{op} = 1$, $Q_{op} = 0$		$P_{op} = 1.15$, $Q_{op} = -0.4$	
Controller	Rise Time	Delay Time	Rise Time	Delay Time
Phase Lead	1.72	0.78	1.88	0.85
PSO	1.72	0.62	1.88	0.70
Proposal	1.62	0.62	1.68	0.62

5. **Conclusions.** In this study, we proposed the optimal LQR approach to design a fuzzy parametric uncertain controller for a PSS. Using the proposed algorithm, the α-cut coefficients of a fuzzy parametric uncertain power system are approximated by uncertainty interval. The maximum uncertainty interval is then translated into the weighting matrix Q of the LQR problem to guarantee that the designed optimal PSS controller is robust under various values of $\alpha \in [0, 1]$. The advantages of the proposed approach are that it is intuitive and that it requires only modest computational effort. The results of our simulation demonstrate that both robustness and optimization can be considered simultaneously. Furthermore, the methodology can be applied for any other parametric-uncertainty-related engineering model.

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Appendix A.

$\Delta\omega$: Machine's speed deviation,
δ: Angle between q-axis and infinite bus bar,
E_{fd}: Generator field voltage,
E_q: Induced EMF proportional to field current,
k_E: Exciter gain,
$k_1, \ldots, k_6$: k-parameters of power system block diagram,
M: Inertia coefficient,
T_m: Disturbance mechanical torque,
T'_{do}: Open circuit d-axis time constant,
T_e: Electrical torque,
T_E: Exciter time constant,
v_t: Terminal voltage.

Appendix B.

$$K_1 = C_3 \frac{P^2}{P^2 + (Q + C_1)^2} + Q + C_1,$$

$$K_2 = C_4 \frac{P}{(P^2 + (Q + C_1)^2)^{1/2}},$$

$$K_3 = \frac{x'_d + x_e}{x_d + x_e},$$

$$K_4 = C_5 \frac{P}{(P^2 + (Q + C_1)^2)^{1/2}},$$

$$K_5 = C_4 x_e \frac{P}{V^2 + Q x_e} \left[C_6 \frac{C_1 + Q}{P^2 + (C_1 + Q)^2} \right],$$

$$K_6 = C_7 \frac{\sqrt{P^2 + (C_1 + Q)^2}}{V^2 + Q x_e} \left[x_e + \frac{C_1 x_q (C_1 + Q)}{P^2 + (C_1 + Q)^2} \right],$$

$$C_1 = \frac{V^2}{x_e + x_q}, \quad C_3 = C_1 \frac{x_q - x'_d}{x_e + x'_d}, \quad C_4 = \frac{V}{x_e + x'_d},$$

$$C_5 = \frac{x_d - x'_d}{x_e + x'_d}, \quad C_6 = C_1 \frac{x_q (x_q - x'_d)}{x_e + x_q}, \quad C_7 = \frac{x_e}{x_e + x'_d}.$$

x_e: Line reactance,
V: Infinite bus voltage,
x'_d, x_d, x_q: Generator, d-axis and q-axis synchronous reactance, respectively,
$x_e = 0.4$ pu, $x_q = 1.55$ pu, $x_d = 1.6$ pu, $x'_d = 0.32$ pu, $V = 1$ pu, $\omega_0 = 314.2$ rad/s, $T'_{do} = 6$ s, $M = 10$ s, $k_E = 25$, $T_E = 0.05$ s.