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A NOVEL STRATEGY FOR IMPROVING ROOT-MUSIC BASED ON COMPRESSIVE SENSING

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Abstract. *In this work a novel strategy based on compressive sensing (CS) is proposed to improve conventional root-MUSIC for direction-of-arrival (DOA) estimation of coherent signal sources. It provides a new attempt to construct covariance matrix of sources. Steering matrix of signals is converted into the form of trigonometric according to* $e^{j\theta} = \cos\theta + j\sin\theta$*, and then the real and imaginary parts are respectively compressed into a sparse framework by fast Fourier transform (FFT). Orthogonal matching pursuit (OMP) is adopted to be reconstruction algorithm and a random matrix is used as the measurement matrix. In the course of recovering steering matrix, because of the difference of "sparsity", the real and imaginary parts are not recovered in the meantime by assigning a same value to sparsity. So covariance matrix is changed into a new matrix with a feature of full-rank and the number of maximum eigenvalues of covariance matrix equals the number of sources. Consequently, coherent signals are estimated effectively. Simulation results show that this strategy provides a significant performance in estimating the DOAs of coherent signals compared to conventional root-MUSIC, and we get a low estimation bias at closely angle separation. Meanwhile, a lower RMSE is gained at low SNR and small snapshot number.*

Keywords: Direction-of-arrival (DOA), Coherent signals, Compressive sensing (CS), Orthogonal matching pursuit (OMP), Root-MUSIC

1. **Introduction.** Direction-of-arrival (DOA) estimation is an active research area in modern signal processing whose purpose is to be able to distinguish closely single sources in the case of considerable noise [1]. Some well-known existing classical algorithms include Beamforming [2], MUSIC [3], and ESPRIT [4], and they played important roles in solving high resolution problems.

As a derivation of MUSIC algorithm, root-MUSIC utilizes rooting method that a more accurate estimation performance is obtained. However, due to the multipath propagation, there are many coherent signals among the emitted signal sources [5]. Numerous studies have demonstrated that conventional root-MUSIC is not applicable to estimate coherent signals, and there exists a great deviation between the estimated and initial data. Why are coherent source signals hard to estimate? It has been proved that correlation of emitted signals leads to rank deficient of covariance matrix, which makes the signal-noise subspace suffer serious performance degradation [5]. Early works on de-correlation DOA estimation methods hold the idea that the larger number of the sensors is, the higher resolution is. Under this circumstance, a sufficient number of sensors are needed for achieving the goal. However, lavish spending on hardware may get more kicks than halfpence.

Many remarkable methods for coherent signals estimation were presented on the basis of those classical algorithms. Huarng et al. [6,7] proposed a real-valued variant of spectral

MUSIC via a unitary transformation. Haardt et al. derived a unitary ESPRIT, which exploited the centro-Hermitian property of forward and backward covariance matrix, and increased estimation accuracy was gained with a reduced computational burden [7,8]. Gershman and Haardt developed the unitary ESPRIT using estimator bank approach [9,10]. Reference [11] proposed an ESPRIT-Like algorithm based on Toeplitz matrix theory to estimate the coherent signals. In reference [12], Qian et al. put forward a novel unitary root-MUSIC, which utilized the information contained in the root estimator bank to eliminate the abnormal DOA estimator and determine the final DOA estimation result. A good resolution is obtained by using spatial smoothing in [13], of which the covariance matrix was reconstructed into a new matrix with a feature of full-rank.

There is a similarity of these representative algorithms that covariance matrix of signals is reconstructed by some methods, such as Toeplitz matrix and unitary matrix. Consequently, the covariance has a feature of full-rank and coherent sources are estimated successfully. However, these methods are relatively complex especially unitary matrix construction that covariance matrix needs to be changed dramatically. Toeplitz method is often used with fourth-order-cumulant (FOC) algorithm so that a good anti-noise performance can be obtained. However, it usually spends plenty of time on computation. There also exists rank deficient when using spatial smoothing (SS) method, which leads to loss of array aperture, even though coherent sources are estimated effectively. Hence, we have been working to search a simple strategy that can provide a good performance for estimating DOAs of coherent signals.

Compressive sensing (CS) [14-16] theory is used for estimating DOA of signals in many recent works, and applications of CS-based algorithms for DOA estimation possessed good ability to overcome closely spaced sources, few snapshot and sensor numbers requirement and mitigate the effect of coherent signals [17,19]. Bilik [17] proposed a DOA estimation algorithm for multiple sources via spatial compressive sensing (SCS), which used dynamic sensor arrays and allowed exploitation of the array orientation diversity. The experimental result showed that a good super-resolution was obtained by this algorithm and the problem of poor estimation at endfire can be addressed effectively. Carlin et al. [18] derived a DOA method exploited Bayesian compressive sensing for narrow band signals. In that paper, the estimation performance of DOAs was determined directly by the voltages at the output of the receiving sensors. Northard et al. [19] put forward an optimization algorithm based on SCS for DOA estimation via an expected likelihood (EL) method, and a new application of EL method was derived to investigate the SCS bias signals. Simulations showed that DOA estimation accuracy was significantly improved by this method without the requirement for intensive regularization parameter tuning. In [20], Rossi et al. derived a sparse localization framework to reduce the number of transmit and receive elements needed. The authors supplied a bound to the coherence of the measurement matrix to make it satisfy the isotropy property. It is observed that a high resolution can be provided by this virtual array with a small number of antenna elements.

Taking the features of array signals into account, we propose a novel strategy based on CS for improving root-MUSIC. Steering matrix is converted into trigonometric form via $e^{j\theta} = \cos\theta + j\sin\theta$ so that it is decomposed into real part and imaginary part. These two parts are respectively compressed by using FFT and we would observe the "sparsity". In our work OMP [21] is used to recover steering matrix. Due to the difference between sparsity of real part and imaginary part, the initial data are not recovered simultaneously if it assigns the same value to the sparsity. Consequently, the covariance matrix will be changed into a new matrix with a feature of full-rank, as well as through computation we find that the number of maximum eigenvalues in the covariance matrix equals that of emitted sources, so that coherent sources are estimated successfully.

Our strategy has dramatically reduced complexity of constructing full-rank covariance matrix compared with other de-correlation methods and seems to be the first attempt to directly combine CS with e conventional root-MUSIC. It is a fairly simple implementation for mitigating estimation biases of coherent signals in the presence of 1-D ULA. Simulation results show a goal of good resolution is achieved at limited number of sensors and a stable root mean square error (RMSE) is obtained at low signal to noise (SNR).

2. **Related Work.** Compressive sensing is regarded as a paradigmatic shift in the way information is represented, transmitted and recovered [22]. It overcomes the bottleneck of conventional Nyqusit sampling theorem and leads to a reduction of sampling rate in a largely extent [14-17,20]. The theory is referred to two parts of work, sparse representation and signal reconstruction.

2.1. **Sparse representation.**

Lemma 2.1. *Consider a discrete signal f with a length of N in a certain space, and it is written with a linear combination of a base vector ψ_i $(i = 1, 2, \cdots, N)$.*

$$f = \sum_{i=1}^{N} \psi_i \alpha_i = \mathbf{\Psi} \boldsymbol{\alpha} \tag{1}$$

where $\mathbf{\Psi}$ and $\boldsymbol{\alpha}$ respectively satisfy

$$\mathbf{\Psi} = [\psi_1, \psi_2, \cdots, \psi_N] \tag{2}$$

$$\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \cdots, \alpha_N]^T \tag{3}$$

It is named K-sparse if there exist K $(K \ll N)$ nonzero coefficients of $\boldsymbol{\alpha}$. Here K is regarded as "spsrsity". The information of f can live at most K dimensions rather than N [22]. Let $\mathbf{\Psi}$ denote sparse basis, and $\mathbf{\Theta}$ $(\mathbf{\Theta} = \mathbf{\Psi}^T f)$ represents the sparse (or projection) coefficient vector. Both $\mathbf{\Theta}$ and f are equivalent expression in the view of $\mathbf{\Psi}$ domain and time domain. Assume there exists a measurement matrix $\mathbf{\Phi}$ of size $M \times N$ $(M \ll N)$, which is independent of the signal f. We need to guarantee the information quality when the signal is reduced from M-D to N-D. It has been demonstrated that f can be recovered perfectly if $\mathbf{\Phi}$ satisfies restricted isometry property (RIP) [22,23]. Let $\boldsymbol{Y}$ denote the measurement vector, written as

$$\mathbf{Y} = \mathbf{\Phi\Theta} = \mathbf{\Phi\Psi}^T f = Af \tag{4}$$

Here the length of $\boldsymbol{Y}$ is far less than that of initial data f. The thought of CS is that signal f is recovered with rare data $\boldsymbol{Y}$ and a measurement matrix $\mathbf{\Phi}$ that are known.

2.2. **Signal reconstruction.** Signal reconstruction is another important aspect of CS. There are noted methods including greedy algorithms, such as matching pursuit (MP) [24] and orthogonal matching pursuit (OMP). Convex optimization algorithm is another reconstructed method, such as basis pursuit (BP) [14,25] and gradient projection for sparse reconstruction (GPSR) [26]. Although a better performance is obtained by convex optimization algorithms, OMP is used in our work relying on its lower complexity, and it provides a faster convergence than MP for non-orthogonal dictionaries. Generally, the idea of OMP is that residual is orthogonal to the selected atom using Gram-Schmidt method in each iteration. All the selected atoms are linearly independent so that repeated selection is avoided.

OMP: *Define a Hilbert space $\boldsymbol{H}$, and assume a dictionary $\boldsymbol{D}$ $(\boldsymbol{D} \in \boldsymbol{H})$. Assume a discrete signal f $(f \in \boldsymbol{H})$. The initial index set $\boldsymbol{D}_0$ satisfies $\boldsymbol{D}_0 = \emptyset$. The initial residual is α_0. Due to $\boldsymbol{Y} = \mathbf{\Phi\Psi}^T f$, let $\alpha_0 = Y$. The sparsity equals K. OMP process is presented in [27], which is as follows.*

(1) *Consider* $\boldsymbol{V} = \boldsymbol{\Phi}\boldsymbol{\Psi}^T$. *Compute maximum value of inner product (represented with* λ_n*) between the measurement vector* v_j *selected from* $\boldsymbol{V}$ *and the initial residual* α_0. *Find out the corresponding vector* v_{λ_n}. *Let* n *denote iteration times, whose initial value equals to 1.*

$$\lambda_n = \arg\max |< \alpha_{n-1}, v_j >|_{j=1,2,\cdots,N} \tag{5}$$

(2) *Update the index set* $\boldsymbol{D}_n$ *and the selected column space* $\boldsymbol{V}_n$

$$\boldsymbol{D}_n = \boldsymbol{D}_{n-1} \cup \{\lambda_n\} \tag{6}$$

$$\boldsymbol{V}_n = [\boldsymbol{V}_{n-1}, v_{\lambda_n}] \tag{7}$$

(3) *Compute projection* $\hat{f}_n$ *with least square method and make sure a minimum residual is obtained.*

$$\hat{f}_n = \arg\min \|\alpha_0 - \boldsymbol{V}_n f\|_2 \tag{8}$$

(4) *Update residual*

$$\alpha_t = \alpha_0 - \boldsymbol{V}_n \hat{f}_n \tag{9}$$

(5) *Let* $n = n + 1$. *Repeat the cycle until* $n > K$.

It is shown that at each iteration $\boldsymbol{Y}$ *is optimally projected to selected column vector, and the residual* α *is required to be the minimum. Consequently, the recovered signal* $\hat{f}_n$ *can be approximated gradually to initial signal.*

3. Signal Model and Proposed Strategy.

3.1. Signal model.

According to [12,28,29], the signal model is established.

Lemma 3.1. *Assume a uniform linear array (ULA) composed of* N *isotropic sensors, there are* P ($P \leq N$) *coherent narrowband source signals, from directions* $(\theta_1, \theta_2, \cdots, \theta_p)$ *impinging on array in the far filed. The* $N \times 1$ *signal vector is*

$$\boldsymbol{X}(t) = \boldsymbol{A}(\theta)\boldsymbol{S}(t) + \boldsymbol{N}(t) \tag{10}$$

Here $\boldsymbol{S}(t)$ *is the source signal vector consisting of* P *different incidence signals,* $\boldsymbol{A}(t)$ *denotes the steering matrix consisting of steering vectors, and* $\boldsymbol{G}(t)$ *represents a Gaussian noise vector which is generated by each array element with a zero mean and variance of* σ^2. *They are respectively defined as*

$$\boldsymbol{S}(t) = [s_1(t), s_2(t), \cdots, s_P(t)]^T \tag{11}$$

$$\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), \cdots, \boldsymbol{a}(\theta_P)] \tag{12}$$

$$G(t) = [g_1(t), g_2(t), \cdots, g_N(t)]^T \tag{13}$$

Let λ *denote the carrier wavelength, and the i-th steering vector* $\boldsymbol{a}(\theta_i)$ *can be expressed as*

$$\boldsymbol{a}(\theta_i) = \left[1, e^{-j2\pi \sin\theta_i d/\lambda}, \cdots, e^{-j2\pi(N-1)\sin\theta_i d/\lambda}\right]^T \tag{14}$$

3.2. **The proposed strategy.** The novel strategy is presented in this part, and we have named it OR-MUSIC. It derives from CS and de-correlation DOA estimation theory. The thought of this strategy is constructing a full-rank covariance matrix and make sure that its number of maximum eigenvalue equals that of signal sources. The derivation process is as follows.

Proof: According to that $e^{\pm j\theta}$ satisfies $e^{\pm j\theta} = \cos\theta \pm j\sin\theta$, $e^{-j2\pi(N-1)\sin\theta d/\lambda}$ will be converted into trigonometric form.

$$e^{-j2\pi(N-1)\sin\theta d/\lambda} = \cos[2\pi(N-1)\sin\theta d/\lambda] - j\sin[2\pi(N-1)\sin\theta d/\lambda] \tag{15}$$

We extract the real part of Formula (15), that is $\cos[2\pi(N-1)\sin\theta d/\lambda]$. By that analogy, the real part of vector $\boldsymbol{a}(\theta_i)$ is described as

$$\boldsymbol{a}_{\mathrm{Re}} = \{1, \cos(2\pi\sin\theta d/\lambda), \cdots, \cos[2\pi(N-1)\sin\theta/\lambda]\} \tag{16}$$

According to OMP theory, let $y = \boldsymbol{\Phi}\boldsymbol{\Psi}^T\boldsymbol{a}_{\mathrm{Re}}$. Here $\boldsymbol{\Phi}$ is a random matrix of size $M \times N$ ($M \gg N$), and the sparse matrix $\boldsymbol{\Psi}$ is obtained by doing FFT of an $N \times N$ identity matrix. Meanwhile, FFT is applicable to $\boldsymbol{a}_{\mathrm{Re}}$ so that the sparsity K can be observed, which is represented with K. Let t denote iteration tines, whose initial value is 1.

Let $\boldsymbol{\Lambda}_0$ denote the initial index, which satisfies $\boldsymbol{\Lambda}_0 = \emptyset$. The initial residual is α_0. Here

$$\alpha_0 = y \tag{17}$$

OMP is applied to $\boldsymbol{a}_{\mathrm{Re}}$ in the following.

(1) Define $\boldsymbol{\Gamma} = \boldsymbol{\Phi}\boldsymbol{\Psi}^T$, and then work out inner product between selected vector φ_j in the index set $\boldsymbol{\Gamma}$ and α_0 by (22), which is represented as λ_t. Meanwhile, find out the corresponding vector φ_{λ_t}.

$$\lambda_t = \arg\max_{j=1,2,\cdots,N} |<\alpha_{t-1}, \varphi_j>| \tag{18}$$

(2) Update the index set $\boldsymbol{\Lambda}_t$ and the index set $\boldsymbol{\Gamma}$

$$\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_{t-1} \cup \{\lambda_t\} \tag{19}$$

$$\boldsymbol{\Gamma}_{\lambda_t} = [\boldsymbol{\Gamma}_{\lambda_{t-1}}, \varphi_{\lambda_t}] \tag{20}$$

(3) Ordinary least square (OLS) is used as (21), and the approximate signal $\hat{\boldsymbol{a}}_t$ is obtained in the following.

$$\hat{\boldsymbol{a}}_t = \arg\min \|\alpha_0 - \boldsymbol{\Gamma}_t\boldsymbol{a}_{\mathrm{Re}}\|_2 \tag{21}$$

(4) The updated residual α_t is obtained, and α_t satisfies (25).

$$\alpha_t = y - \boldsymbol{\Gamma}_t\hat{\boldsymbol{a}}_t \tag{22}$$

Repeat the iteration process (1) to (4) until condition $t > K$ is met. Let $\hat{\boldsymbol{a}}_{\mathrm{Re}}$ denote the recovered signal of $\boldsymbol{a}_{\mathrm{Re}}$.

Similarly, we extract the imaginary part of $e^{-j2\pi(N-1)\sin\theta d/\lambda}$, which is denoted with $\boldsymbol{a}_{\mathrm{Im}}$ and satisfies

$$\boldsymbol{a}_{\mathrm{Im}} = \{0, \sin(2\pi\sin\theta d/\lambda), \cdots, \sin[2\pi(N-1)\sin\theta/\lambda]\} \tag{23}$$

We set the sparsity equal to K, and use $\hat{\boldsymbol{a}}_{\mathrm{Im}}$ to represent the approximate signal of $\boldsymbol{a}_{\mathrm{Im}}$ through CS, so the steering vector is expressed as

$$\hat{\boldsymbol{a}}(\theta_i) = [\hat{\boldsymbol{a}}_{\mathrm{Re}} - j\hat{\boldsymbol{a}}_{\mathrm{Im}}]^T, \quad i = 1, 2, \cdots, P \tag{24}$$

According to (12), that is

$$\hat{\boldsymbol{A}}(\theta) = [\hat{\boldsymbol{a}}(\theta_1), \hat{\boldsymbol{a}}(\theta_2), \cdots, \hat{\boldsymbol{a}}(\theta_P)] \tag{25}$$

The new array output $\hat{\boldsymbol{X}}(t)$ satisfies

$$\hat{\boldsymbol{X}}(t) = \hat{\boldsymbol{A}}\boldsymbol{S}(t) + \boldsymbol{N}(t) = \sum \hat{\boldsymbol{a}}_i\boldsymbol{s}_i + \boldsymbol{N}(t) \tag{26}$$

The new covariance of $\hat{\boldsymbol{X}}(t)$ is denoted as

$$\hat{\boldsymbol{R}} = E\left[\hat{\boldsymbol{X}}(t)\hat{\boldsymbol{X}}(t)^H\right] = \hat{\boldsymbol{A}}\boldsymbol{R}_s\left(\hat{\boldsymbol{A}}\right)^H + \sigma^2\boldsymbol{I}_N \tag{27}$$

Here $\boldsymbol{R}_s = E[\boldsymbol{S}(t)\boldsymbol{S}(t)^H]$ is the covariance matrix of source signal vector. Let L represent the snapshot number, and then the covariance matrix is denoted as

$$\hat{\boldsymbol{R}} = \frac{1}{L}\sum_{i=1}^{L}\hat{\boldsymbol{X}}(i)\hat{\boldsymbol{X}}(i)^H \tag{28}$$

We compute MUSIC spectrum by performing an eigen-decomposition on the matrix $\hat{\boldsymbol{R}}$, and the space spanned by N^2 eigenvectors produces into two disjoint subspaces: those are signal-noise subspaces [11].

$$\boldsymbol{E_s} = [e_1, e_2, \cdots, e_p] \tag{29}$$

$$\boldsymbol{E_N} = [e_{P+1}, e_{P+2}, \cdots, e_{N^2}] \tag{30}$$

Here $\boldsymbol{E_s}$ represents the signal subspace consisting of P eigenvectors, and $\boldsymbol{E_N}$ is the noise subspace consisting of N^2-P eigenvectors, and then MUSIC spectrum is defined as

$$P(\theta) = \frac{1}{\left|(\hat{\boldsymbol{a}})^H\boldsymbol{E}_N\boldsymbol{E}_N\hat{\boldsymbol{a}}\right|} \tag{31}$$

Reference [30] presents root-MUSIC in detail. Let $\boldsymbol{C} = \boldsymbol{E_N}\boldsymbol{E_N^H}$, where $\boldsymbol{C}$ is a hermitian matrix, so Formula (31) is written as

$$P(\theta) = \frac{1}{\left|(\hat{\boldsymbol{a}})^H\boldsymbol{C}\hat{\boldsymbol{a}}\right|} \tag{32}$$

Let m denote the sequence number of sensors. According to (14), the denominator of (32) is written as

$$(\hat{\boldsymbol{a}})^H\boldsymbol{C}\hat{\boldsymbol{a}} = \sum_{m=1}^{N}\sum_{n=1}^{N}e^{j2\pi d(m-1)\sin\theta/\lambda}C_{mn}e^{-j2\pi d(n-1)\sin\theta/\lambda} = \sum_{l=-N+1}^{N-1}ce^{-j2\pi dl\sin\theta/\lambda} \tag{33}$$

Let $c_l = \sum_{n-m=l} C_{mn}$. Here c_l represents summation of the l-th diagonal's elements in matrix $\boldsymbol{C}$. According to the Z-Transform theory

$$D(z) = \sum_{l=-N+1}^{N-1}c_lz_l, \quad z_i = |z_i|e^{j\arg(z_i)} \tag{34}$$

There exist exact zeros under the condition of $|z_i| = 1$, and the final estimation angel is computed as follows

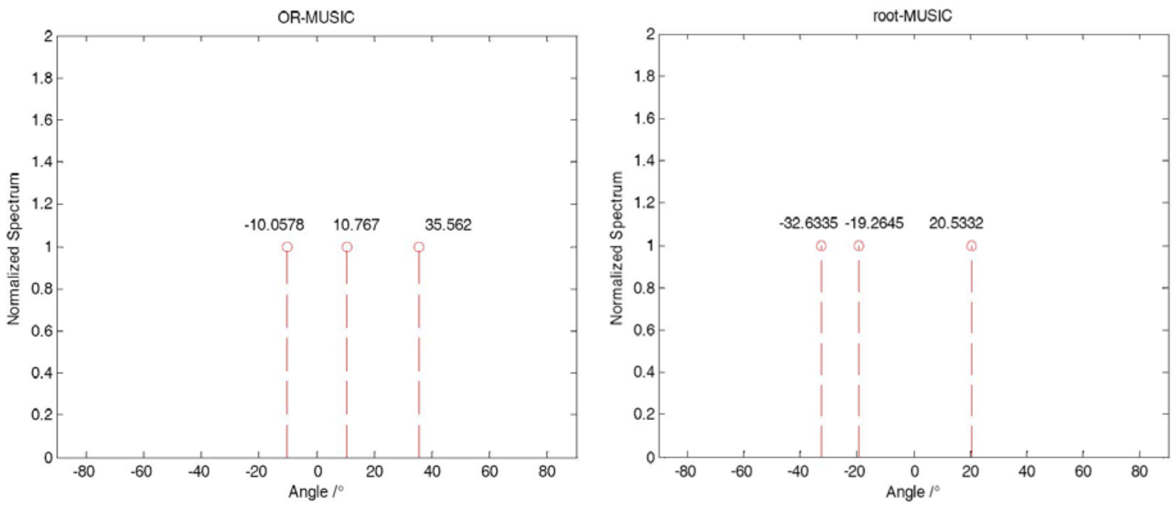
$$\hat{\theta}_i = -\arcsin\left(\frac{\lambda}{2\pi d}\arg(z_i)\right) \tag{35}$$

Here $\hat{\theta}_i$ is the final estimation performance.

We make sure that array matrix $\hat{\boldsymbol{R}}$ has a full-rank feature in this strategy that the problem of loss of array aperture generated by the spatial smoothing is solved. Due to there being P maximum eigenvalues of matrix $\hat{\boldsymbol{R}}$, coherent signals are estimated effectively.

4. **Simulations.** This section presents numerical experiments to illustrate the performance of OR-MUSIC. In the first experiment, compare the estimation results between OR-MUSIC and root-MUSIC through estimating the same sources. Next, in the second experiment we analyze the performance of estimation bias under the condition of different angle separation by comparing with some other de-correlation algorithms (such as Toeplitz-ESPRIT, Spatial-smoothing FOC and L1-SVD). In the third experiment we present root mean square errors (RMSEs) of different methods from the perspective of SNR, sensor number and snapshot number.

Experiment 1. Accuracy analysis. Consider a uniform linear array with an inter-element spacing of $\lambda/2$ and $N = 12$ isotropic sensors. There are three narrowband signal sources $\theta_1 = -10.37$, $\theta_2 = -10.26$, $\theta_3 = 35.54$ in far-field impinging on the array, and their frequency equals $\pi/4$. Let snapshot number $L = 1024$. Let SNR = 0dB. We set the sparsity K equal to 5 in this work. Let zeromean white Gaussian be the noise. We compare the estimation results between the OR-MUSIC and root-MUSIC, which is shown in Figure 1.



(a) Estimation performance of OR-MUSIC (b) Estimation performance of root-MUSIC

FIGURE 1. Comparison of estimation results between OR-MUSIC and root-MUSIC (source number = 3, SNR = 0dB, sensor number = 12)

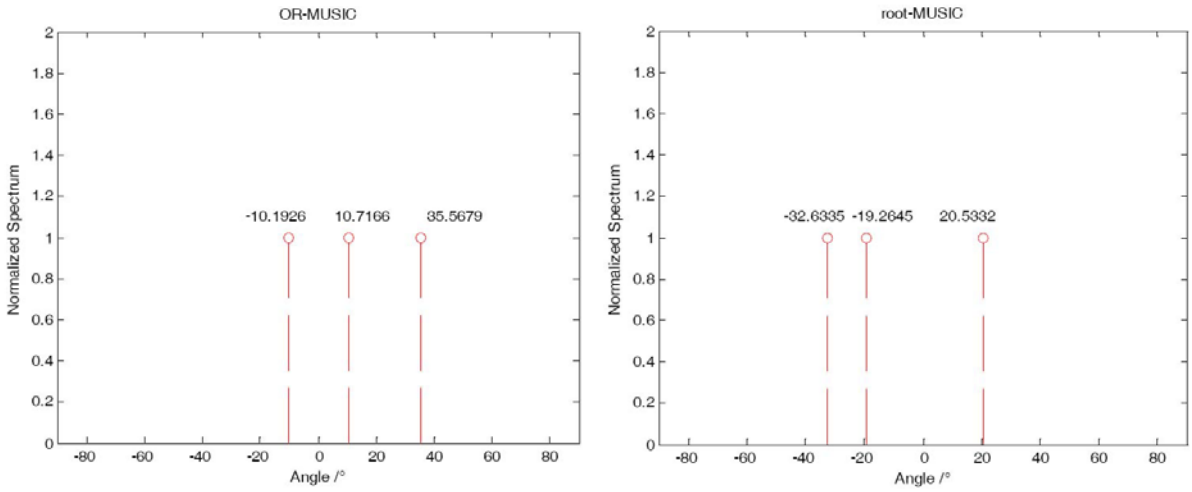
Next SNR is decreased to -10dB and we observe the estimation result, which is shown in Figure 2.

From Figure 2 we find that our strategy provides an accurate result. Additionally, it is shown that a good performance is also obtained even though at low SNR condition.

Experiment 2. Bias analysis. Estimation biases are presented in this part at different angle separation. Let SNR = 0 and 10 dB respectively. Consider a uniform linear array with an inter-element spacing of $\lambda/2$ and $N = 24$ isotropic sensors. A narrowband signal impinges on the array ($\theta = 20.15$), whose frequency is equal to $\pi/4$. Let snapshot number $L = 1024$. There exists another source with the same parameters. Their angle separation θ_{se} satisfies $[5°, 10°, 15°, 20°, 25°, 30°]$ respectively. Work out absolute values between experimental separations and initial data θ_{se}. Zero-mean white Gaussian is used to be noise, and the results are shown in Table 1 and Table 2.

It is shown that a more accurate result is obtained by our strategy at low angle separation situation. However, if the separation satisfies $\theta_{se} \geq 10°$, the bias in our work is larger than that of other decorrelation algorithms. From these two tables we also observed that the biases in other algorithms are affected greatly with the change of SNR, but our approach seemingly provides a more stationary performance. The biases from Table 1 and Table 2 are presented intuitively in Figure 3.

Experiment 3. RMSE analysis. Work out RMSEs, which is defined as (36), so that we can find that different conditions have different impacts on the experimental



(a) Estimation performance of OR-MUSIC (b) Estimation performance of root-MUSIC

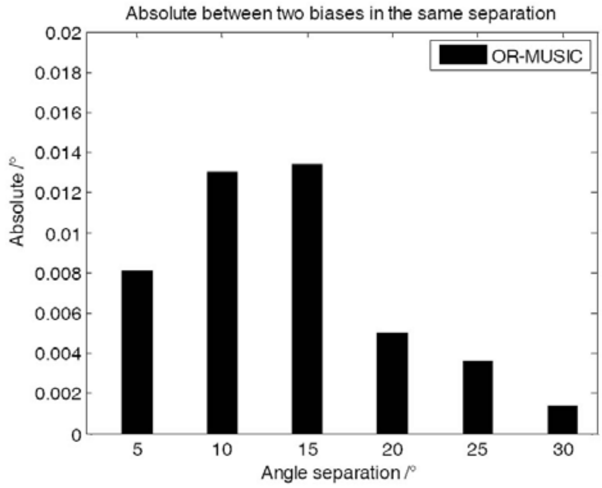
FIGURE 2. Comparison of estimation performance between OR-MUSIC and root-MUSIC (source number = 3, SNR = -10dB, sensor number = 12)

TABLE 1. Estimation bias at different separation (SNR = 0dB, sensor number = 24, snapshot number = 1024)

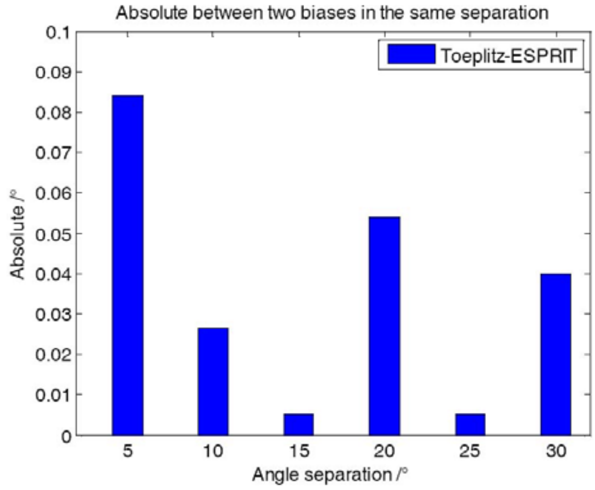
Angle separation	Maximum bias /° (SNR = 0dB)				
	OR-MUSIC	Toeplitz-ESPRIT	Spatial smoothing	FOC	L1-SVD
5°	0.8749	1.0065	2.3303	1.3765	1.2500
10°	0.4684	0.2730	0.2764	0.1730	0.8500
15°	0.4272	0.1500	0.1491	0.2535	0.8500
20°	0.4092	0.1840	0.1457	0.2183	0.1500
25°	0.2528	0.1555	0.1019	0.2460	0.2500
30°	0.2315	0.1500	0.1074	0.2760	0.1500

TABLE 2. Estimation bias at different separation (SNR = 10dB, sensor number = 24, snapshot number = 1024)

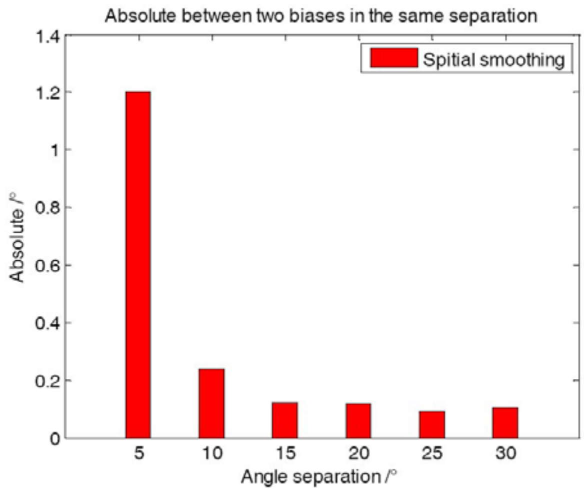
Angle separation	Maximum bias /° (SNR = 10dB)				
	OR-MUSIC	Toeplitz-ESPRIT	Spatial smoothing	FOC	L1-SVD
5°	0.8830	0.9224	1.1270	1.3050	1.1500
10°	0.4454	0.2985	0.0350	0.1650	0.8500
15°	0.4138	0.1550	0.0285	0.1713	0.4500
20°	0.4042	0.1300	0.0270	0.1525	0.1500
25°	0.2492	0.1555	0.0085	0.1480	0.1500
30°	0.2301	0.1260	0.0030	0.1515	0.8500



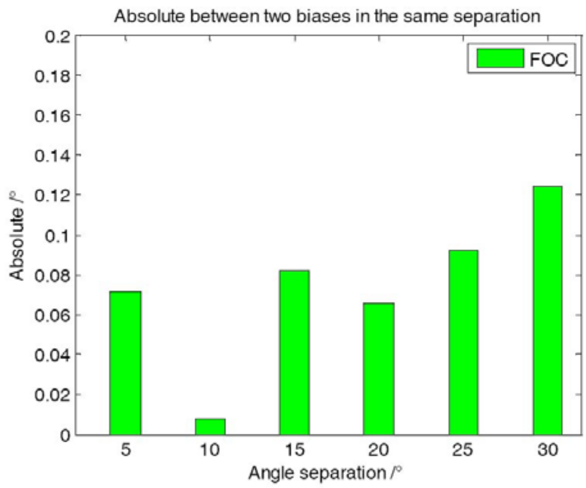
(a) OR-MUSIC



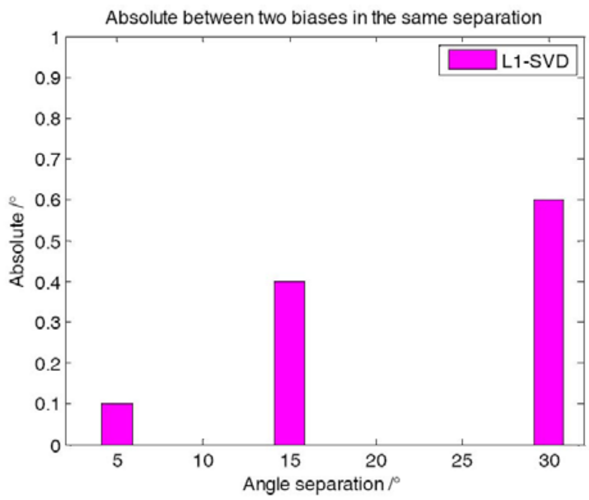
(b) Toeplitz-ESPRIT



(c) Spatial smoothing



(d) FOC



(e) L1-SVD

FIGURE 3. Absolute between two angle biases in the same separation

performance.

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left|\theta_n - \hat{\theta}_n\right|^2} \tag{36}$$

Here $\hat{\theta}_n$ represents the estimated performance and θ_n is the initial data respectively. In this part, 200 Monte Carlo experiments are carried out to calculate the RMSE in terms of SNR, sensor number, and snapshot number.

1) Assume a uniform linear array with an inter-element spacing of $\lambda/2$ and 10 isotropic sensors. There is one narrowband signal ($\theta = 50.15$) in the far-field impinging on the array, and frequency is equal to $\pi/4$. The snapshot number is $L = 1024$. Let SNR $= [-15, -10, -5, 0, 5, 10, 15]$dB. We use zero-mean white Gaussian to be noise. The relation between RMSE and SNR is shown in Figure 4.

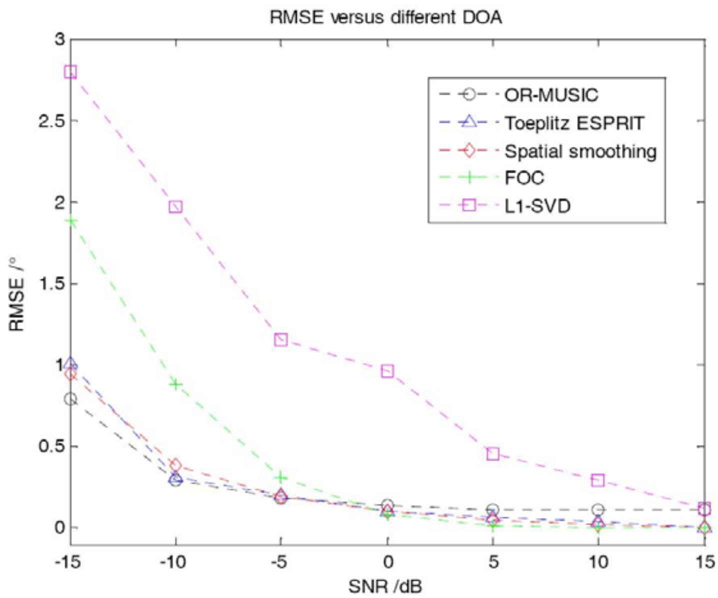


FIGURE 4. RMSE performance versus SNR (incidence angle = 50.15°, snapshot number = 1024, sensor number = 10)

It is observed from Figure 4 that a lower RMSE is obtained by our work under the condition of SNR ≤ -10dB. When the SNR satisfies the condition of -10dB $<$ SNR $\leq$ 0dB, the RMSE of our work is approximately equal to those of other algorithms, so we can get the conclusion that our strategy gets a good estimation performance at a low SNR situation.

2) Consider a uniform linear array with an inter-element spacing of $\lambda/2$ and $N = [10, 12, 14, 16, 18, 20, 22, 24]$ isotropic sensors. There is one narrowband signal ($\theta = 50.15$) in the far-field impinging on the array, and frequency is equal to $\pi/4$. The snapshot number satisfies $L = 1024$. Let SNR = 0dB. We use zero-mean white Gaussian to be noise. The relation between sensor number and RMSE is shown in Figure 5.

We find from Figure 5 that the RMSE in our work is just smaller than that of L1-SVD, and the performance is no better than that of Toeplitz-ESPRIT, Spatial smoothing and FOC, even though a good stability is obtained with our strategy.

3) Consider a uniform linear array with an inter-element spacing of $\lambda/2$ and 10 isotropic sensors. There is a narrowband signal ($\theta = 50.15$) in the far-field impinging on the array, whose frequency is equal to $\pi/4$. The number of snapshots is $L = [100, 200, 300, 400, 500,$

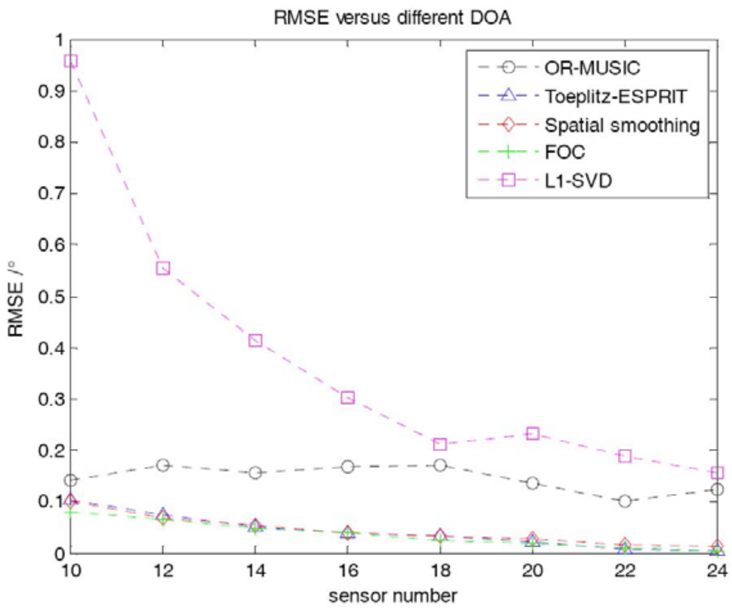


Figure 5. RMSE performance versus sensor number (incidence angle = 50.15°, snapshot number = 1024, SNR = 0dB)

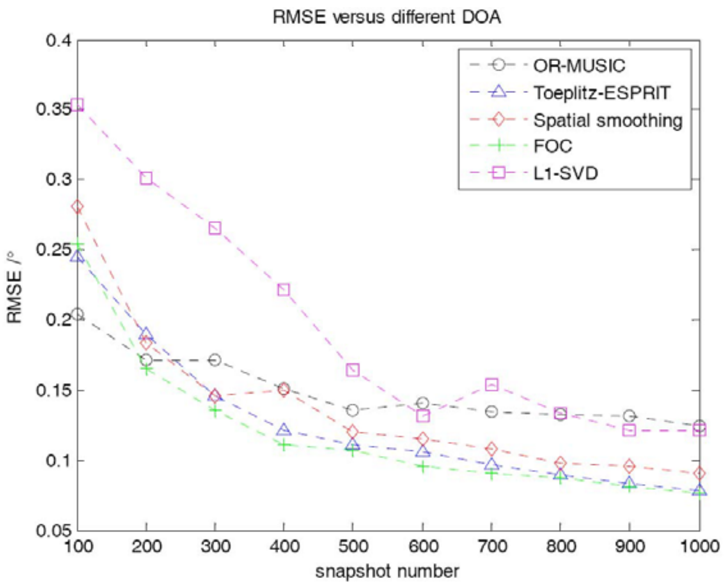


Figure 6. RMSE performance versus snapshot number (incidence angle = 50.15°, sensor number = 10, SNR = 0dB)

$600, 700, 800, 900, 1000]$. Let SNR = 0dB. We use zero-mean white Gaussian to be noise. The relation between snapshot number and RMSE is shown in Figure 6.

From Figure 6 we find that the RMSE in the OR-MUSIC is no more than those in other algorithms when snapshot number satisfies $L < 200$. Therefore, our strategy provides a good estimation performance at a low snapshot number situation.

5. **Conclusions.** A novel strategy based on compressive sensing is proposed in this work for improving conventional root-MUSIC. The steering matrix is compressed and recovered by CS that covariance matrix is changed into a full-rank matrix, of which the number of maximum eigenvalues equals that of emitted sources. Consequently, coherent sources are estimated significantly. In contrast to other de-correlation methods, the proposed strategy

reduces the complexity of calculation. Simulation results show that a good accuracy has been obtained at close space, and a stationary performance of estimation bias is gained at a low SNR. Lastly, we can also get a low RMSE at a small snapshots number situation.

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