International Journal of Innovative Computing, Information and Control
Volume **8**, Number **6**, June **2012**

pp. 4235–4247

STABILITY ANALYSIS FOR SYSTEMS WITH LARGE DELAY PERIOD: A SWITCHING METHOD

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Received March 2011; revised July 2011

Abstract. *This paper studies an exponential stability problem for a class of time-varying delay systems with large delay period. The time-varying delay is interval-range. Although large delay period (LDP) is often present in practical control systems, the problems of stability analysis for such systems have been unsolved to date. In this paper, a new method based on switching technique is proposed to solve the case of LDP. It addresses how long and how frequent the LDP can be and still maintain the system exponentially stable. We first utilize switched delay systems, which may include an unstable subsystem, to model such a system. Then, the concepts of frequency and length rate of LDP are introduced. Next, based on a novel piecewise Lyapunov functional including the large-delay-integral terms (LDITs), sufficient conditions guaranteeing exponential stability are developed. Without LDP, the criterion obtained in this paper contains results already published as a special case. Two numerical examples are given to show the effectiveness of the proposed method.*
Keywords: Delay-dependent criterion, Switched delay systems, Piecewise Lyapunov functional, Large delay period

1. **Introduction.** Delay-dependent stability analysis has been a very popular topic in the study of delay systems in recent years [1-6]. Based on the traditional Lyapunov Krasovskii functional method, the search for delay-dependent bound h is carried out so that the system stability or other system performance measures can be guaranteed. A basic assumption in these papers is that the delay must satisfy $0 \leq h_1 \leq d(t) \leq h_2$ for $\forall\, t \in [t_0, +\infty)$. Because if the delay bound increases to $h_3 > h_2$, that is $0 \leq d(t) \leq h_3$ for $\forall\, t \in [t_0, +\infty)$, then the system may become unstable. However, for the large delay case, that is $h_2 < d(t) \leq h_3$, if it occurs occasionally in some local interval of $[t_0, +\infty)$ rather than the whole time interval $[t_0, +\infty)$, then the delay system may be still stable. However, the traditional Lyapunov functional method in the above-mentioned papers will fail once this large delay case occurs. In fact, this large delay phenomenon is often encountered in practical systems. For example, it often appears in the study of networked control system for the reason of networked induced-delay and package dropout phenomena [7-10]. Thus, it is important to study the stability problem of systems with a large delay. To the best of our knowledge, this problem has been unsolved to date since traditional methods used in the study of large delays fail for such systems.

On the other hand, it is noted that switching system [15-23] and switched delay systems [24-32] have been a popular topic in recent years. In this paper, we use a switched delay system consisting of two subsystems to describe the system with a large delay. One stable subsystem is used to describe small delay case; the other subsystem, which may be unstable, is used to describe the large delay case. This switched system is different from those in our previous work [31, 32] because a main feature of the considered switched delay system in this paper is that one of the subsystems may be unstable due to the larger delay bound, while all subsystems in these papers are stable. Thus, the stability analysis of switched systems including an unstable subsystem caused by the large delay bound is more difficult to give than those considered in [31, 32].

In this paper, the concepts of the length rate and frequency of the large delay period are introduced. Then, inspired by the switching method in [31], a piecewise Lyapunov functional method is adopted. To apply this method, two problems need to be solved. The first is how to choose some special Lyapunov functional candidates in order to get explicit conditions on their exponential growth and decay change along the corresponding system trajectory. The second, which is the most difficult, is how to calculate the ratio between two Lyapunov functional candidates at each switching time point. In this paper, the first problem is solved by introducing some special Lyaunov functional candidates with exponential terms and adopting the free weighting matrix method [11, 12]. The second problem is solved primarily by introducing some large-delay-integral terms (LDITs) to the piecewise Lyapunov functional candidate. The adding of LDITs makes the second problem solvable and does not increase the conservativeness of the criterion. After these two problems are solved successfully, we develop exponential stability conditions for the system with large delay period (LDP). Without considering LDP, the result in this paper contains existing results [13] as a special case. Two numerical examples are given to show the effectiveness of the proposed method.

This paper is organized as follows. Section 2 introduces model description and preliminaries. Section 3 gives two lemmas and the main result in this paper. In Section 4, two numerical examples are given to show the effectiveness of the proposed method. Section 5 draws the conclusions.

Notation. $P > 0$ is used to denote a positive definite matrix P. $\lambda_{\min}(P)$ denotes the minimum eigenvalue of symmetric matrix P. $\bar{N}$ is used to denote an integer set $\{0, 1, 2, \dots\}$.

2. **Preliminaries.**

Consider the following system model

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - d(t)), \\ x(\theta) &= \phi(\theta), \quad \theta \in [-h_3, 0], \end{aligned} \tag{1}$$

where $x(t)$ is the state vector; A and A_d are constant matrices; $\phi(\theta)$ denotes continuously differentiable vector-valued initial function of $\theta \in [-h_3, 0]$, $h_3 > 0$; $d(t)$ denotes the time-varying delay satisfying

$$0 \le h_1 \le d(t) \le h_3, \quad \dot{d}(t) \le d < 1. \tag{2}$$

The following two assumptions are adopted:

Assumption 1. System (1) is stable if delay $d(t)$ satisfies $h_1 \le d(t) \le h_2$ for $\forall t \in [0, +\infty)$, where $h_2 < h_3$ and h_2 can be obtained based on existing methods.

Assumption 2. The stability of system (1) is not guaranteed based on existing method or system itself is unstable if delay $d(t)$ satisfies $h_2 < d(t) \le h_3$ for $\forall t \in [0, +\infty)$.

Definition 2.1. *Time interval* $[T_1, T_2)$ *is called large delay period (LDP) if for* $\forall t \in [T_1, T_2)$, *it holds that* $h_2 \le d(t) \le h_3$. *And time interval* $[T_1, T_2)$ *is called small delay period (SDP), if it holds that* $h_1 \le d(t) \le h_2$ *for* $\forall t \in [T_1, T_2)$.

Existing literature [2-6, 11-13] focus on the case of SDP, searching for the bound h_2 to make system (1) stable. For the case of LDP, where the delay system may be unstable, there have been no results available to date. Although system (1) may be unstable if LDP appears in the whole time interval $[0, +\infty)$, it is possible that system (1) is stable if the LDP only appears locally, namely, for some short time intervals within $[0, +\infty)$. Suppose the LDP occurs occasionally, then system (1) can be described by the following switched delay system

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t - d_{\sigma(t)}(t)) \\ x(\theta) = \phi(\theta), \quad \theta \in [-h_3, 0] \end{cases} \tag{3}$$

where $\sigma(t) : [0, +\infty) \to \{1, 2\}$ is a piecewise constant function and called switching signal; $\sigma(t) = 1$ implies that system (3) is running in SDP, and $\sigma(t) = 2$ denotes that system (3) is running in LDP; $h_1 \le d_1(t) \le h_2$ and $h_2 < d_2(t) \le h_3$.

Remark 2.1. *System (3) may include an unstable subsystem 2,* $\dot{x}(t) = Ax(t) + A_d x(t - d_2(t))$, *due to the effect of larger delay* $d_2(t)$.

We use time sequence $t_0 < t_1 < t_2 < \ldots$ to denote time switching sequence of switching signal $\sigma(t)$. The following time sequence, belonging to one of subsequences of $\{t_0, t_1, t_2, \ldots\}$, is also introduced

$$t_0 = p_0 < p_1 < p_2 < p_3 < \ldots, \tag{4}$$

which satisfies the following conditions: $\bigcup_{k=0}^{+\infty} [p_k, p_{k+1}) = [t_0, +\infty)$, $[p_k, p_{k+1}) = [t_i, t_j)$, $k \le i < j$, and k, i, $j \in \bar{N}$, and,

$$p_{k+1} - p_k \le \eta_k \le \eta < +\infty, \quad \forall k \in \bar{N}, \tag{5}$$

for positive constants η_k and η. It is usually difficult to exactly know when and where the switching signal $\sigma(t)$ occurs. However, in some cases, it will be easier to estimate the information of the upper bounds on length and numbers of LDP in some longer time interval $[p_k, p_{k+1})$. This paper just gives the conditions on these bounds in each interval $[p_k, p_{k+1})$ to insure exponential stability of system (3).

Inspired by average dwell time used in switched systems [15, 16], we will introduce a concept: the frequency of LDP to denote the number of LDP within a certain time interval.

Definition 2.2. *For any* $T_2 > T_1 \ge 0$, *let* $N_f(T_1, T_2)$ *denote the number of LDP in time interval* $[T_1, T_2)$. $F_f(T_1, T_2) = \frac{N_f(T_1, T_2)}{T_2 - T_1}$ *is referred to as frequency of LDP in time interval* $[T_1, T_2)$.

In this paper, it is assumed that the time interval $\bigcup_{k=0}^{+\infty} [t_{2k}, t_{2k+1})$ denotes SDP and $\bigcup_{k=0}^{+\infty} [t_{2k+1}, t_{2(k+1)})$ denotes LDP. If $N_\sigma(T_1, T_2)$ is used to denote the number of switchings of $\sigma(t)$ in time interval $[T_1, T_2)$, then it holds that

$$N_\sigma(t_0, t) \le 2N_f(t_0, t). \tag{6}$$

Similar to the concept of the unavailability rate of the controller in [17], we introduce the concept of length rate of LDP.

Definition 2.3. *For time interval* $[T_1, T_2)$, *denote the total time length of LDP during* $[T_1, T_2)$ *by* $T^+(T_1, T_2)$, *and denote the total time length of SDP by* $T^-(T_1, T_2)$. *We call* $\frac{T^+(p_k, p_{k+1})}{T^-(p_k, p_{k+1})}$ *the length rate of LDP in time interval* $[p_k, p_{k+1})$.

3. **Stability Analysis.** In this section, two lemmas will be first developed. Consider the following delay system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - d_1(t)), \\ x(\theta) &= \phi(\theta), \theta \in [-h_2, 0], \end{aligned} \tag{7}$$

where A, A_d are constant matrices; $d_1(t)$ satisfies $0 \le h_1 \le d_1(t) \le h_2$. Choose the Lyapunov functional candidate of the following form

$$V_1(t) = V^1(t) + V^2(t) + V^3(t) + V^4(t) + V^5(t) + V^6(t) + V^7(t), \tag{8}$$

where
$V^1(t) = x^T(t)P_1x(t)$,
$V^2(t) = \sum_{i=1}^{2}\int_{t-h_i}^{t} x^T(s)e^{\alpha_1(s-t)}Q_ix(s)ds$,
$V^3(t) = \int_{t-d_1(t)}^{t} x^T(s)e^{\alpha_1(s-t)}Q_3x(s)ds$,
$V^4(t) = \int_{t-h_3}^{t-h_2} x^T(s)e^{\alpha_1(s-t)}Q_4x(s)ds$,
$V^5(t) = \int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_1\dot{x}(s)dsd\theta$,
$V^6(t) = \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_2\dot{x}(s)dsd\theta$,
$V^7(t) = \int_{-h_3}^{-h_2}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_3\dot{x}(s)dsd\theta$, $h_3 > h_2 > h_1 \ge 0$, and P_1, Q_i, $i = 1, 2, 3, 4$, and Z_j, $j = 1, 2, 3$ are positive definite matrices to be determined.

Lemma 3.1. *For given constants $\alpha_1 > 0$, $h_3 > h_2 > h_1 \ge 0$ and $d < 1$, if there exist matrices $P_1 > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$, $Z_j > 0$ $(j = 1, 2, 3)$, and any matrices N_i, M_i and S_i $(i = 1, 2)$ with appropriate dimensions such that*

$$\begin{bmatrix} \Delta & c_0N & c_1S & c_1M \\ * & -c_0Z_1 & 0 & 0 \\ * & * & -c_1(Z_1+Z_2) & 0 \\ * & * & * & -c_1Z_2 \end{bmatrix} < 0, \tag{9}$$

then, along the trajectory of the system (7), we have

$$V_1(t) \le e^{-\alpha_1(t-t_0)}V_1(t_0),$$

where
$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & M_1 & -S_1 \\ * & \Delta_{22} & M_2 & -S_2 \\ * & * & -e^{-\alpha_1h_1}Q_1 & 0 \\ * & * & * & e^{-\alpha_1h_2}(Q_4 - Q_2) \end{bmatrix},$$
$\Delta_{11} = P_1A + A^TP_1 + \alpha_1P_1 + \sum_{i=1}^{3} Q_i + A^T\Lambda_1A + N_1 + N_1^T$,
$\Delta_{12} = P_1A_d + A^T\Lambda_1A_d + N_2^T - N_1 + S_1 - M_1$,
$\Delta_{22} = A_d^T\Lambda_1A_d - (1-d)e^{-\alpha_1h_2}Q_3 + S_2 + S_2^T - N_2 - N_2^T - M_2 - M_2^T$,
$\Lambda_1 = h_2Z_1 + h_{12}Z_2 + h_{23}Z_3$,
$$N = \begin{bmatrix} N_1 \\ N_2 \\ 0 \\ 0 \end{bmatrix}, S = \begin{bmatrix} S_1 \\ S_2 \\ 0 \\ 0 \end{bmatrix}, M = \begin{bmatrix} M_1 \\ M_2 \\ 0 \\ 0 \end{bmatrix},$$
$c_0 = \frac{1}{\alpha_1}(e^{\alpha_1h_2} - 1)$, $c_1 = \frac{1}{\alpha_1}(e^{\alpha_1h_2} - e^{\alpha_1h_1})$, $h_{12} = h_2 - h_1$, $h_{23} = h_3 - h_2$.

Proof: From the Leibniz-Newton formula, for matrices N_i, S_i and M_i, $i = 1, 2$, we have

$$2\left[x^T(t)N_1 + x^T(t-d_1(t))N_2\right] \times \left[x(t) - x(t-d_1(t)) - \int_{t-d_1(t)}^{t} \dot{x}(s)ds\right] = 0,$$

$$2\left[x^T(t)S_1 + x^T(t-d_1(t))S_2\right] \times \left[x(t-d_1(t)) - x(t-h_2) - \int_{t-h_2}^{t-d_1(t)} \dot{x}(s)ds\right] = 0,$$
$$2\left[x^T(t)M_1 + x^T(t-d_1(t))M_2\right] \times \left[x(t-h_1) - x(t-d_1(t)) - \int_{t-d_1(t)}^{t-h_1} \dot{x}(s)ds\right] = 0, \quad (10)$$

where N_i, S_i and M_i are called Free-weighting matrices [11-13]. Also, the following equations are true:

$$\begin{aligned} -\int_{t-h_2}^{t} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_1\dot{x}(s)ds &= -\int_{t-d_1(t)}^{t} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_1\dot{x}(s)ds \\ &\quad -\int_{t-h_2}^{t-d_1(t)} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_1\dot{x}(s)ds, \\ -\int_{t-h_2}^{t-h_1} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_2\dot{x}(s)ds &= -\int_{t-d_1(t)}^{t-h_1} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_2\dot{x}(s)ds \\ &\quad -\int_{t-h_2}^{t-d_1(t)} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_2\dot{x}(s)ds. \end{aligned} \quad (11)$$

Based on (11), along the trajectory of system (7) we have

$$\begin{aligned} &\dot{V}_1(t) + \alpha_1 V_1(t) \\ &\leq 2x^T(t)P_1[Ax(t) + A_d x(t-d_1(t))] + \sum_{i=1}^{2}[x^T(t)Q_i x(t) - x^T(t-h_i)e^{-\alpha_1 h_i}Q_i x(t-h_i)] \\ &\quad + x^T(t)Q_3 x(t) - (1-d)x^T(t-d_1(t))e^{-\alpha_1 h_2}Q_3 x(t-d_1(t)) \\ &\quad + x^T(t-h_2)e^{-\alpha_1 h_2}Q_4 x(t-h_2) \\ &\quad - x^T(t-h_3)e^{-\alpha_1 h_3}Q_4 x(t-h_3) + h_2\dot{x}^T(t)Z_1\dot{x}(t) - \int_{t-h_2}^{t} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_1\dot{x}(s)ds \\ &\quad + h_{12}\dot{x}^T(t)Z_2\dot{x}(t) \\ &\quad - \int_{t-h_2}^{t-h_1} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_2\dot{x}(s)ds + h_{23}\dot{x}^T(t)Z_3\dot{x}(t) - \int_{t-h_3}^{t-h_2} \dot{x}^T(s)e^{\alpha_1(s-t)}Z_3\dot{x}(s)ds \\ &\quad + \alpha_1 x^T(t)P_1 x(t) \\ &\leq \xi^T(t)\left[\Delta + c_0 N Z_1^{-1} N^T + c_1 S(Z_1+Z_2)^{-1}S^T + c_1 M Z_2^{-1} M^T\right]\xi(t) \end{aligned} \quad (12)$$

where $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-d_1(t)) & x^T(t-h_1) & x^T(t-h_2) \end{bmatrix}^T$, c_0 and c_1 are defined in Lemma 3.1. By using Schur complement and (9) we know $\Delta + c_0 N Z_1^{-1} N^T + c_1 S(Z_1 + Z_2)^{-1} S^T + c_1 M Z_2^{-1} M^T < 0$. Thus, it follows from (12) that $\dot{V}_1(t) + \alpha_1 V_1(t) \leq 0$. Integrating this inequality gives $V_1(t) \leq e^{-\alpha_1(t-t_0)}V_1(t_0)$. The proof is completed.

Consider the following delay system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-d_2(t)), \\ x(\theta) &= \phi(\theta), \quad \theta \in [-h_3, 0], \end{aligned} \quad (13)$$

where A, A_d are constant matrices defined in system (7); $d_2(t)$ satisfies $h_1 < h_2 \leq d_2(t) \leq h_3$. Choose the Lyapunov functional candidate of the following form

$$V_2(t) = V^8(t) + V^9(t) + V^{10}(t) + V^{11}(t) + V^{12}(t) + V^{13}(t) + V^{14}(t), \quad (14)$$

where
$V^8(t) = x^T(t)P_2 x(t)$,

$V^9(t) = \int_{t-h_1}^{t} x^T(s)e^{\alpha_2(t-s)}Q_5x(s)ds + \int_{t-h_2}^{t} x^T(s)e^{\alpha_2(t-s)}Q_6x(s)ds$,
$V^{10}(t) = \int_{t-d_2(t)}^{t} x^T(s)e^{\alpha_2(t-s)}Q_7x(s)ds$,
$V^{11}(t) = \int_{t-h_3}^{t-h_2} x^T(s)e^{\alpha_2(t-s)}Q_8x(s)ds$,
$V^{12}(t) = \int_{-h_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_2(t-s)}Z_4\dot{x}(s)dsd\theta$,
$V^{13}(t) = \int_{-h_2}^{-h_1}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_2(t-s)}Z_5\dot{x}(s)dsd\theta$,
$V^{14}(t) = \int_{-h_3}^{-h_2}\int_{t+\theta}^{t} \dot{x}^T(s)e^{\alpha_2(t-s)}Z_6\dot{x}(s)dsd\theta$, $h_3 > h_2 > h_1 \geq 0$, and P_2, Q_i, $i = 5, 6, 7, 8$ and Z_j, $j = 4, 5, 6$ are positive definite matrices to be determined.

Lemma 3.2. *For given constants $\alpha_2 > 0$, $h_3 > h_2 > h_1 \geq 0$ and $d < 1$, if there exist matrices $P_2 > 0$, $Q_i > 0$ $(i = 5, 6, 7, 8)$, $Z_j > 0$ $(j = 4, 5, 6)$, and any matrices L_i, R_i, Y_i and T_i $(i = 1, 2)$ with appropriate dimensions such that*

$$\begin{bmatrix} \Gamma & c_2L & c_3R & c_4Y & c_4T \\ * & -c_2Z_4 & 0 & 0 & 0 \\ * & * & -c_3Z_5 & 0 & 0 \\ * & * & * & -c_4Z_6 & 0 \\ * & * & * & * & -c_4Z_6 \end{bmatrix} < 0, \tag{15}$$

then, along the trajectory of the system (13), we have

$$V_2(t) \leq e^{\alpha_2(t-t_0)}V_2(t_0),$$

where

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & R_1 & Y_1 - L_1 - R_1 & -T_1 \\ * & \Gamma_{22} & R_2 & Y_2 - L_2 - R_2 & -T_2 \\ * & * & -e^{\alpha_2 h_1}Q_5 & 0 & 0 \\ * & * & * & -e^{\alpha_2 h_2}(Q_6 - Q_8) & 0 \\ * & * & * & * & -e^{\alpha_2 h_3}Q_8 \end{bmatrix},$$

$\Gamma_{11} = P_2A + A^TP_2 - \alpha_2P_2 + \sum_{i=5}^{7} Q_i + A^T\Lambda_2A + L_1 + L_1^T$,
$\Gamma_{12} = P_2A_d + A^T\Lambda_2A_d + L_2^T - Y_1 + T_1$,
$\Gamma_{22} = A_d^T\Lambda_2A_d - (1-d)e^{\alpha_2h_2}Q_7 + T_2 + T_2^T - Y_2 - Y_2^T$,
$\Lambda_2 = h_2Z_4 + h_{12}Z_5 + h_{23}Z_6$,

$$L = \begin{bmatrix} L_1 \\ L_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, R = \begin{bmatrix} R_1 \\ R_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, Y = \begin{bmatrix} Y_1 \\ Y_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$c_2 = \frac{1}{\alpha_2}(1 - e^{-\alpha_2h_2})$, $c_3 = \frac{1}{\alpha_2}(e^{-\alpha_2h_1} - e^{-\alpha_2h_2})$,
$c_4 = \frac{1}{\alpha_2}(e^{-\alpha_2h_2} - e^{-\alpha_2h_3})$, $h_{12} = h_2 - h_1$, $h_{23} = h_3 - h_2$.

Proof: It is noted that the following equations are true:

$$-\int_{t-h_3}^{t-h_2} \dot{x}^T(s)e^{\alpha_2(t-s)}Z_6\dot{x}(s)ds = -\int_{t-d_2(t)}^{t-h_2} \dot{x}^T(s)e^{\alpha_2(t-s)}Z_6\dot{x}(s)ds - \int_{t-h_3}^{t-d_2(t)} \dot{x}^T(s)e^{\alpha_2(t-s)}Z_6\dot{x}(s)ds. \tag{16}$$

Then, by simple calculation, along the trajectory of system (13), it holds that

$$\dot{V}_2(t) - \alpha_2V_2(t) \leq \zeta^T(t)[\Gamma + c_2LZ_4^{-1}L^T + c_3RZ_5^{-1}R^T + c_4YZ_6^{-1}Y^T + c_4TZ_6^{-1}T^T]\zeta(t) \tag{17}$$

where $\zeta(t) = \begin{bmatrix} x^T(t) & x^T(t-d_2(t)) & x^T(t-h_1) & x^T(t-h_2) & x^T(t-h_3) \end{bmatrix}^T$, c_2, c_3 and c_4 are defined in Lemma 3.2. By using Schur complement and (15) we know $\Gamma + c_2LZ_4^{-1}L^T +$

$c_3RZ_5^{-1}R^T+c_4YZ_6^{-1}Y^T+c_4TZ_6^{-1}T^T<0$. Thus, it follows from (17) that $\dot{V}_2(t)-\alpha_2V_2(t)\le 0$. Integrating this inequality gives $V_2(t)\le e^{\alpha_2(t-t_0)}V_2(t_0)$. The proof is completed.

Remark 3.1. *For system (7) with delay $h_1\le d_1(t)\le h_2$, the terms $V^4(t)$ and $V^7(t)$ in (8), which contain the information of h_3 and are called large-delay-integral terms (LDITs), do not appear in the traditional construction of Lyapunov functional candidates for the study of delay systems [2-6, 11-14], since the information of delay bound h_3 is not needed in these papers. The use of LDTIs will be considered later in this paper (Remark 3.4). Note that the adding for these terms does not bring conservativeness of Lemma 3.1 because the maximum bound of h_2 can also be obtained by letting $h_{23}=0$ in LMI (9). In addition, LDITs $V^{11}(t)$ and $V^{14}(t)$ in (14) are necessary in that they are used to introduce the delay bound h_3 in LMI (15) of Lemma 3.2.*

Theorem 3.1. *For given constants $\alpha_1>0$, $\alpha_2>0$, $h_3>h_2>h_1\ge 0$ and $d<1$, if there exist matrices $P_i>0$, $i=1,2$, $Q_j>0$, $j=1,2,3,4,5,6,7,8$, $Z_l>0$, $l=1,2,3,4,5,6$, and any matrices N_i, M_i and S_i, L_i, R_i, Y_i and T_i, $i=1,2$, with appropriate dimensions such that LMIs (9) and (15) hold, then under the switching signal $(S1)$, system (3) is exponentially stable and the state decay estimation is given as*

$$\|x(t)\|\le\sqrt{\frac{V_1(t_0)e^ce^{\alpha\eta}}{\rho}}e^{-0.5(\alpha^*-\alpha)(t-t_0)},\tag{18}$$

where switching signal $(S1)$ satisfies the following two conditions:
$Cond_1$. the length rate of LDP satisfies $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}\le\frac{\alpha_1-\alpha^*}{\alpha_2+\alpha^*}$, $\alpha^*\in(0,\alpha_1)$, $\forall k\in\bar{N}$,
$Cond_2$. the frequency of LDP satisfies $F_f(p_k,p_{k+1})\le\frac{\alpha}{ln(\mu^2\mu_1)}$, $\alpha\in(0,\alpha^*)$,
where $\mu\ge 1$ satisfies

$$\begin{aligned}&P_l\le\mu P_h,\ \ \forall l,h\in\{1,2\},\\&Q_i\le\mu Q_j,\ \ \forall\{i,j\}\ or\ \{j,i\}\in\{\{1,5\},\{2,6\},\{3,7\},\{4,8\}\},\\&Z_m\le\mu Z_n,\ \ \forall\{m,n\}\ or\ \{n,m\}\in\{\{1,4\},\{2,5\},\{3,6\}\},\end{aligned}\tag{19}$$

and

$$\mu_1=e^{(\alpha_1+\alpha_2)h_3},\quad c=(\alpha_2+\alpha^*)\frac{\alpha_1-\alpha^*}{\alpha_1+\alpha_2}\eta\tag{20}$$

$$\rho=\min\{\lambda_{\min}(P_1),\lambda_{\min}(P_2)\}.\tag{21}$$

Proof: Construct piecewise Lyapunov functional candidate as follows:

$$V(t)=V_\sigma(t)=\begin{cases}V_1(t) & t\in[t_{2j},t_{2j+1}),\\ V_2(t) & t\in[t_{2j+1},t_{2j+2}),\quad j\in\bar{N},\end{cases}\tag{22}$$

where $V_1(t)$ and $V_2(t)$ are defined in (8) and (14), respectively. From (19) and (20), it is easy to see that

$$V_1(t)\le\mu V_2(t),\quad V_2(t)\le\mu\mu_1V_1(t).\tag{23}$$

Considering the piecewise Lyapunov functional candidate (22), we have from Lemma 3.1 and Lemma 3.2

$$V(t)\le\begin{cases}e^{-\alpha_1(t-t_{2j})}V(t_{2j}) & t\in[t_{2j},t_{2j+1})\\ e^{\alpha_2(t-t_{2j+1})}V(t_{2j+1}) & t\in[t_{2j+1},t_{2j+2}),\ j\in\bar{N}.\end{cases}\tag{24}$$

Without loss of generality, we assume that $t\in[t_{2l+1},t_{2l+2})\subseteq[p_m,p_{m+1})$, where $l\ge 1$, $m\ge 1$. Based on (23) and (24), along the trajectory of system (3), the piecewise Lyapunov functional candidate (22) satisfies

$$\begin{aligned}V(t) &\le V_2(t_{2l+1})e^{\alpha_2(t-t_{2l+1})}\\ &\le \mu\mu_1 V_1(t_{2l+1})e^{\alpha_2(t-t_{2l+1})}\\ &\le \mu\mu_1 V_1(t_{2l})e^{-\alpha_1(t_{2l+1}-t_{2l})}e^{\alpha_2(t-t_{2l+1})}\\ &= \mu\mu_1 V_1(t_{2l})e^{-\alpha_1 T^-(t_{2l},t)+\alpha_2 T^+(t_{2l},t)}\\ &\le \cdots\\ &\le \mu^{N_\sigma(t_0,t)}\mu_1^{N_f(t_0,t)}V_1(t_0)e^{-\alpha_1 T^-(t_0,t)+\alpha_2 T^+(t_0,t)}.\end{aligned} \tag{25}$$

Next, we will show

$$-\alpha_1 T^-(t_0,t)+\alpha_2 T^+(t_0,t) \le -\alpha^*(t-t_0)+c, \tag{26}$$

and

$$\mu^{N_\sigma(t_0,t)}\mu_1^{N_f(t_0,t)} \le e^{\alpha(t-t_0)+\alpha\eta}, \tag{27}$$

where $c = \frac{(\alpha_2+\alpha^*)(\alpha_1-\alpha^*)}{\alpha_1+\alpha_2}\eta$ is defined in Theorem 3.1.

It holds that

$$\begin{aligned}&-\alpha_1 T^-(t_0,t)+\alpha_2 T^+(t_0,t) - \alpha^*(t-t_0) - (\alpha_1-\alpha^*)T^-(t_0,t)+(\alpha^*+\alpha_2)T^+(t_0,t)\\ =& -\alpha^*(t-t_0)-(\alpha_1-\alpha^*)[T^-(p_0,p_m)+T^-(p_m,t)]\\ &+(\alpha^*+\alpha_2)[T^+(p_0,p_m)+T^+(p_m,t)]\\ \le& -\alpha^*(t-t_0)+[-(\alpha_1-\alpha^*)T^-(p_0,p_m)+(\alpha^*+\alpha_2)T^+(p_0,p_m)]\\ &+(\alpha^*+\alpha_2)T^+(p_m,t).\end{aligned} \tag{28}$$

From $Cond_1$, it holds that

$$\begin{aligned}&-(\alpha_1-\alpha^*)T^-(p_0,p_m)+(\alpha^*+\alpha_2)T^+(p_0,p_m)\\ =&\sum_{q=0}^{m-1}[-(\alpha_1-\alpha^*)T^-(p_q,p_{q+1})+(\alpha^*+\alpha_2)T^+(p_q,p_{q+1})] \le 0,\end{aligned} \tag{29}$$

and

$$T^+(p_m,p_{m+1}) \le \frac{\alpha_1-\alpha^*}{\alpha_2+\alpha_1}\eta_m \le \frac{\alpha_1-\alpha^*}{\alpha_2+\alpha_1}\eta. \tag{30}$$

Applying (29) and (30) to (28) leads to (26). Next, we will show inequality (27). From $Cond_2$ and Definition 2.2, it holds that

$$N_f(p_k,p_{k+1})\ln(\mu^2\mu_1) \le \alpha(p_{k+1}-p_k), \quad \forall k \in \bar{N},$$

and thus

$$N_f(t_0,t)ln(\mu^2\mu_1) = \ln(\mu^2\mu_1)\left[\sum_{k=0}^{m-1}N_f(p_k,p_{k+1})+N_f(p_m,t)\right]$$

$$\le \sum_{k=0}^{m-1}\alpha(p_{k+1}-p_k)+\ln(\mu^2\mu_1)N_f(p_m,t) \le \alpha(p_m-t_0)+\alpha\eta_m \le \alpha(t-t_0)+\alpha\eta. \tag{31}$$

It is clear that from (6)

$$\mu^{N_\sigma(t_0,t)}\mu_1^{N_f(t_0,t)} \le \mu^{2N_f(t_0,t)}\mu_1^{N_f(t_0,t)} = e^{N_f(t_0,t)\ln(\mu^2\mu_1)}. \tag{32}$$

Applying (31) to (32) leads to (27).

Substituting (26) and (27) to (25) leads to

$$V(t) \le V_1(t_0)e^c e^{\alpha\eta}e^{-(\alpha^*-\alpha)(t-t_0)}. \tag{33}$$

Noting that $V(t) \geq \rho\|x(t)\|^2$, we have from (33) that

$$\rho\|x(t)\|^2 \leq V_1(t_0)e^c e^{\alpha\eta} e^{-(\alpha^*-\alpha)(t-t_0)}, \tag{34}$$

where $\rho = \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}$. And thus, it is obtained that

$$\|x(t)\| \leq \sqrt{\frac{V_1(t_0)e^c e^{\alpha\eta}}{\rho}} e^{-0.5(\alpha^*-\alpha)(t-t_0)}. \tag{35}$$

The proof is completed.

For the case without LDP, that is $h_1 \leq d_1(t) \leq h_2$ for $\forall t \in [t_0, +\infty)$, then it holds that $N_\sigma(t_0, t) = N_f(t_0, t) = 0$, $T^+(t_0, t) = 0$, $T^-(t_0, t) = t - t_0$. Therefore, from (25), it can be seen that

$$V(t) \leq V_1(t_0)e^{-\alpha_1(t-t_0)}, \tag{36}$$

then we can get

$$\|x(t)\| \leq \sqrt{\frac{V_1(t_0)}{\lambda_{\min}(P_1)}} e^{-0.5\alpha_1(t-t_0)}. \tag{37}$$

Thus the following corollary is obtained.

Corollary 3.1. *If the delay satisfies $h_1 \leq d_1(t) \leq h_2$ and $\dot{d}_1(t) \leq d \leq 1$ for $\forall t \in [0, +\infty)$, then under the LMI (9), system (1) is exponentially stable.*

Remark 3.2. *If $\alpha_1 \to 0$, then $c_0 \to h_2$ and $c_1 \to h_{12}$, where c_1 and c_2 are defined in Lemma 3.1. Thus, if $\alpha_1 \to 0$, $h_{23} \to 0$ and $Q_4 \to \varepsilon I$, (ε is a sufficiently small positive constant), then LMI (9) is reduced to LMI (5) in* [13]. *Thus, Corollary 3.1 contains Theorem 3.1 in* [13] *as a special case. In addition, it is easy to see for the case of $\dot{d}(t) \leq \beta$, β is an unknown positive constant, Corollary 3.1 also holds if we delete the terms containing Q_3 in LMI (9).*

Remark 3.3. *In Theorem 3.1, $Cond_1$ is used to restrict the length of the LDP in each interval $[p_i, p_{i+1})$; $Cond_2$ is used to restrict the frequency of LDP. Theorem 3.1 shows that under the restriction of these two conditions, exponential stability for system (3) can be guaranteed if some LMIs' conditions are satisfied.*

Remark 3.4. *In the piecewise Lyapunov functional candidates, the LDITs $V^4(t)$ and $V^7(t)$ defined in $V_1(t)$ in (8) are important because the adding of these terms makes the corresponding integral terms in $V_1(t)$ in (8) and $V_2(t)$ in (14) have the same integral interval, for example, the term $V^i(t)$ correspond to $V^{7+i}(t)$, $i = 2, 3, 4, 5, 6, 7$, respectively, and thus makes the ratio between $V_1(t)$ and $V_2(t)$ at each switching point, reflected by (23), easy to calculate.*

Remark 3.5. *If delay condition (2) is replaced as $h_1 \leq d(t) \leq h_3$, $\dot{d}(t) \leq d$, where d is an unknown positive constant, then Theorem 3.1 also holds if the terms containing Q_3 in LMI (9) and terms containing Q_7 in LMI (15) are removed.*

Remark 3.6. *From Theorem 3.1, it can be seen that the maximum allowed delay bound (MADB) guaranteeing exponential stability of system (3) can be given as $h_3 = h_2 + h_{23}$. Also, $T^+(p_k, p_{k+1})$, which denotes the permitted length of LDP during time interval $[p_k, p_{k+1})$, can be given as $\frac{\alpha_1-\alpha^*}{\alpha_2+\alpha_1}\eta_i$ according to (30). Existing literature [2-6, 11-14] mainly concentrate on the size of h_2, that is the bound of the small delay case, but the important parts h_{23} and $\frac{\alpha_1-\alpha^*}{\alpha_2+\alpha_1}\eta_i$ caused by LDP cannot be dealt with and therefore previous publications have adopted a restrictive assumption to delete these parts.*

Remark 3.7. *In Theorem 3.1, to get MADB, we need to solve matrix inequalities (9), (15) and (19). Usually, we select a positive* μ *such that (19) becomes an LMI. Select a larger* μ *so that LMIs (9), (15) may have a feasible solution, and then stepwise try to choose a smaller* μ *so as to relax* $Cond_2$*.*

Remark 3.8. *To check exponential stability of system (3) with LDP, the following steps are needed:*

(I) Give α_1, α_2, h_1, h_2, h_{23}, μ, *and check whether LMIs (9), (15) and (19) have feasible solutions.*

(II) If (I) holds, choose $\alpha^* \in (0, \alpha_1)$ *to calculate the bound of length rate,* $\frac{\alpha_1-\alpha^*}{\alpha_2+\alpha^*}$.

(III) Choose $\alpha \in (0, \alpha^*)$, *and then calculate the bound of frequency,* $\frac{\alpha}{\ln(\mu^2\mu_1)}$.

(IV) Based on (II) and (III), if switching signal satisfies $Cond_1$ *and* $Cond_2$, *then system (3) is exponentially stable. The allowed* $T^+(p_k, p_{k+1})$ *in each interval* $[p_i, p_{i+1})$ *is given as (30).*

4. **Examples.**

Example 4.1. [13]. *In this section, we use two examples to show the benefits of our result. Consider system (1) with*

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}.$$

According to Remark 3.8, the following steps are needed:

(I) Given $h_1 = 0$, $h_2 = 1.6$, $\alpha_1 = 0.14$, $\alpha_2 = 0.3$, $h_{23} = 8.4$, $\mu = 5$ *and* $d = 0.5$, *it is found that LMIs (9), (15) and (19) have feasible solutions;*

(II) Given $\alpha^* = 0.082 < \alpha_1$, *it holds that* $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})} \le 0.1518$ *according to* $Cond_1$*;*

(III) Given $\alpha = 0.08 < \alpha^*$, *it holds that* $F_f(p_k, p_{k+1}) \le \frac{\alpha}{\ln(\mu^2\mu_1)} = 0.0105$ *according to* $Cond_2$*;*

(IV) Thus, if $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})} \le 0.1518$ *and* $F_f(p_k, p_{k+1}) \le 0.0105$ *for switching signal* $\sigma(t)$, *the considered system is exponentially stable.*

Suppose $p_{i+1} - p_i = \eta$ *for* $\forall i \in \{1, 2, \ldots\}$ *and choose* $\eta = 100s$ *(second). Based on Remark 3.6, it can be seen that* $T^+(p_k, p_{k+1}) \le 13.1818$, *and thus* $N_f(p_k, p_{k+1}) \le \eta \times F_f(p_k, p_{k+1}) \le 1.05$. *It means that the LDP, in which* $d(t)$ *satisfies* $1.6 \le d(t) \le 10$, *can be permitted to occur once in each* $100s$ *and the permitted length of LDP can reach to* 13.1818.

For the case of $0 \le d_1(t) \le 1.6$ *and* $d = 0.5$, *the bounds of* $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ *and* $F_f(p_k, p_{k+1})$, *and MADB under different* $d_2(t)$ *can be seen in Table 2. Without considering the case of LDP, MADB* h_2 *obtained by using different methods for* $h_1 = 0$ *and for different* d *can be seen in Table 1. Comparing Table 1 and Table 2, it can be seen that for LDP occurring within a certain frequency, the method proposed in this paper can provide a much larger MADB compared with existing methods. It is also noticed that with MADB increasing, the bounds of* $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ *and* $F_f(p_k, p_{k+1})$ *decrease. For different* $d_2(t)$ *and different* d, *the bounds of* $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ *and* $F_f(p_k, p_{k+1})$, *and MADB with* $0 \le d_1(t) \le 1$, *can be seen in Table 3. It is obvious that the MADB provided in Table 3 are also much larger than those in Table 1.*

Example 4.2. *Consider system (1) with*

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}.$$

TABLE 1. MADB h_2 for $h_1 = 0$ and different d under different methods (Example 4.1)

Methods	[2, 4, 11],	[14]	[13]
MADB for $d = 0.5$	2.0	–	2.04
MADB for $d = 0.9$	1.18	–	1.37
MADB for unknown d	0.99	1.01	1.34

TABLE 2. The bounds of $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ and $F_f(p_k, p_{k+1})$, and MADB under different $d_2(t)$ with $0 \le d_1(t) \le 1.6$ and $d = 0.5$ (Example 4.1)

$d_2(t)$	$1.6 \le d_2(t) \le 10$	$1.6 \le d_2(t) \le 24$	$1.6 \le d_2(t) \le 37$
the bound of $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$	0.1518	0.1408	0.1374
the bound of $F_f(p_k, p_{k+1})$	0.0105	0.0055	0.0038
MADB	10	24	37

TABLE 3. The bounds of $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ and $F_f(p_k, p_{k+1})$, and MADBs under different $d_2(t)$ and d for $0 \le d_1(t) \le 1.6$ (Example 4.1)

	$d = 0.9$	d is unknown
	$1 \le d_2(t) \le 14$	$1 \le d_2(t) \le 6$
the bound of $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$	0.1567	0.2126
the bound of $F_f(p_k, p_{k+1})$	0.0118	0.0298
MADB	14	6

TABLE 4. The bounds of $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ and $F_f(p_k, p_{k+1})$ and MADB for $0.3 \le d_1(t) \le 0.6$ and unknown d (Example 4.2)

$d_2(t)$	$0.6 < d(t) \le 1.2$	$0.6 < d(t) \le 4.1$	$0.6 < d(t) \le 5.7$
the bound of $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$	0.0638	0.0508	0.0376
the bound of $F_f(p_k, p_{k+1})$	0.0503	0.0172	0.0108
MADB	1.2	4.1	5.7

For the case of $0.3 \le d_1(t) \le 0.6$*, the bounds of* $\frac{T^+(p_k,p_{k+1})}{T^-(p_k,p_{k+1})}$ *and* $F_f(p_k, p_{k+1})$*, and MADB for unknown* d *and different* $d_2(t)$ *can be seen in Table 4. It is worth noting that MADB obtained in* [14] *and* [13] *are* 0.91 *and* 0.94*, respectively, while in the method proposed in this paper it can be as high as* 5.7 *under a certain restriction in the length rate and the frequency of LDP. It is clearly shown that the method in this paper can deal with the case of LDP effectively while existing methods reported in the literature cannot.*

5. **Conclusions.** This paper has considered exponential stability problem for a class of time-varying delay systems with LDP. It has addressed how long and how frequent the LDP can be to still ensure that the system is exponentially stable. Such a system with

LDP has been firstly modeled as a switched delay system, which may include an unstable subsystem caused by large delay bound. Based on a novel piecewise Lyapunov functional, exponential stability of the considered system is guaranteed under the restriction of frequency and length rate of LDP. We also have shown in the theory that the obtained result contains existing ones without considering LDP as a special case. Two examples have been given to show the effectiveness of the proposed method.

Acknowledgment. This work was partially supported supported in part by the NSFC under Grants 61174058 and 61004020, and the Program for New Century Excellent Talents in University NCET-09-0257, and the Fundamental Research Funds for the Central Universities DUT11SX07.

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