International Journal of Innovative Computing, Information and Control
Volume **9**, Number **5**, May **2013**

ICIC International ©2013 ISSN 1349-4198
pp. 1851–1864

OPTIMAL SYNTHESIS OF THE CONTINUOUSLY VARIABLE TRANSMISSION INPUT MECHANISM USING AN EVOLUTIONARY APPROACH

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Received February 2012; revised June 2012

Abstract. *This work aims at the optimal synthesis of an input mechanism of a Continuously Variable Transmission (CVT) system in order to fulfill the motion generation case. The synthesis is carried out using an evolutionary-based approach. That is, an optimization problem to obtain the parameters of such mechanism and the use of an evolutionary-based approach to solve the optimization problem are proposed. The evolutionary algorithm DE which has been successfully applied to mechanical design optimization task is used. Two input mechanisms are considered in order to drive the angular motion of the input motor into the CVT system. In order to evaluate such mechanisms, the kinematic analysis of each one of them was carried out and an objective function also mechanical constraints were proposed. A comparison and evaluation for the two mechanisms is carried out in order to select one of them to produce an optimal performance of the whole mechanical system.*

Keywords: Continuously variable transmission, Four-bar mechanism, Quick-return crank mechanism, Differential evolution, Parametric optimization

1. Introduction.

Currently, a lot of machines are composed of a set of connected linkages in order to transfer energy from an input shaft to other mechanical elements. Therefore, the theoretical positions about the design of these mechanisms are related with some aspects such as: they must be composed of a few and conventional mechanical elements, and they must have a large range of motion and a high positioning resolution, among others [1, 2]. An engineering approach to design these mechanisms considers to assign a set of parameters that describe the system, the best possible combination of values. In this approach, a kinematic analysis is carried out at a first stage in order to determinate the minimum set of parameters also as to establish a set of performance functions and constraints to quantify the system behavior [3]. Once a set of parameters is proposed, the designer can propose several potential solutions. An alternative approach to solve a mechanism design problem is to propose an optimization problem where an optimal parametric design is carried out, in order to solve the original design problem, that is: the mechanisms synthesis [5, 6, 7, 8, 9, 10].

Several optimization algorithms have been used to solve engineering design problems. A first approach to solve an optimization problem are the mathematical programming methods such as sequential quadratic programming (SQP), as they can guarantee the convergence to the global optimum. However, if the application of a traditional method is complex (these methods involve computing the gradient and the Hessian of the objective function and constraints, which imply that continuity of the second order must be ensured [11]) or the computational cost is high if/or the results are good but not as expected, the use of a non-traditional method that is a stochastic method must be considered. Moreover, with the increase in the computing power, the use of stochastic methods has increased in the last decades. Evolutionary algorithms (EAs), genetic algorithms (GAs) or particle swarm optimization (PSO) are some of the most used methods to solve such problems. On the other hand, due to the high complexity of the resulting optimization problems, the stochastic methods present advantages when they are used to solve such optimization problems: (*i*) these methods are population-based methods; therefore, a global minima solution can be reached, although not in all kinds of problem; (*ii*) in order to start the search, additional information is not necessary, i.e., gradients, Hessian matrices, initial search points, etcetera; (*iii*) with these methods, complex problems can be solved, meaning that the optimization problem can include discontinuous physical models; (*iv*) finally, these methods are independent of the problem characteristics; these methods can be used and/or adapted to a large set of problems. Therefore, efforts to develop more powerful and robust evolutionary algorithms, are a trend topic in the research community.

In Mermetas et al. [12], an optimal kinematic design of planar manipulator with a four-bar mechanism is presented. In that work, optimum link measurements of the manipulator that maximize the local mobility index depending on the input link location are founded. Also, design charts for the optimum manipulator design are obtained. In [5], the hybrid multi-objective genetic algorithms (GA) are used for Pareto optimum synthesis of four-bar linkages considering the minimization of two objective functions simultaneously. The obtained Pareto fronts demonstrate that trade-offs between these two objectives can be recognized so that a designer can optimally compromise for the selection of a desired four-bar linkage. In [6], the synthesis of a four-bar linkage in which the coupler point performs approximately rectilinear motion is presented. Very high accuracy for motion along a straight line at a large number of given points is achieved by using the method of variable controlled deviations and by applying a differential evolution (DE) algorithm. Khorshidi et al. [7], present a novel approach to the multi-objective design of four-bar linkages for path-generation purposes, which is carried out. In that work, an optimization problem including three conflicting criteria is proposed. In order to accelerate the search in the highly multimodal solution space, a hybrid Pareto genetic algorithm with built-in adaptive local search is employed. In Acharyya et al. [13], three different evolutionary algorithms such as GA, PSO and DE are applied for synthesis of a four-bar mechanism minimizing the error between desired and obtained coupler curve. A new refinement technique for the generation of the initial population is also introduced. A comparative study regarding the strengths and limitations of those algorithms is done and performance of DE is found to be the best.

This work aims at the optimal synthesis of an input mechanism of a Continuously Variable Transmission (CVT) system in order to fulfill the motion generation case. The synthesis is carried out using an evolutionary approach. That is, an optimization problem to obtain the parameters of such mechanism and the use of an evolutionary-based approach to solve the optimization problem are proposed. The evolutionary algorithm DE which has been successfully applied to mechanical design optimization task is used. Two input mechanisms are considered in order to drive the angular motion of the input motor into the CVT system: a four-bar mechanism and the quick-return crank mechanism. A comparison and evaluation for the two mechanisms is carried out in order to select one of them to produce the best performance of the whole mechanical system. Also, it is important to remark that the selection of the mechanism is based on the required kinematic by the whole system and the fact that the mechanisms selected must fulfill an easy building.

This paper is organized as follows: the description of the design problem is presented in Section 2. The kinematic analysis for the two proposed mechanisms is developed in Section 3. Theoretical positions about the optimal selection of the input mechanism are exposed in Section 4. The evolutionary-based approach used to solve the optimization problem is explained in Section 5. Numerical simulations of the optimization results for both mechanisms are given in Section 6. The discussion of results is provided in Section 7 and finally, some conclusions are drawn in Section 8.

2. **CVT Description.** Continuously Variable Transmissions (CVTs) are well known mechanisms developed in order to fulfill the necessity for mechanical power transmission. These mechanisms allow to continuously changing the transmission rate in order to operate in the most efficient operating range of the input engine [9]. Therefore, the performance and fuel economy could be improved. Several CVTs were designed taking into account two aspects: they must be composed of few and conventional mechanical elements [1].

In [14], a new CVT configuration based on a crank-slider mechanism in order to use a pedaling motion is proposed. The operational principle of this mechanism is explained as follows: the proposed mechanism converts the input pedalling motion onto the driving wheel rotating input by unilateral transmission. In order to do this, the mechanism uses ratchets that are alternatively engaged inside the links of the output chain to push and pull them. On the other hand, a special mechanism in order to change the effective radius of pulling in the whole mechanical system is included. The transmission ratio changes when the length of the crank is modified.

The above mentioned continuously variable transmission is proposed in order to introduce a gradual, linear change of velocity on pedalling vehicles. A simplified drawing of this mechanism can be observed in Figure 1.

It is important to remark that due to the particular manufacturing of the shaft that couples the input pedalling motion into the rest of the CVT, this configuration is not easy to implement. Therefore, in this work alternative mechanism is proposed in order to replace such shaft.

3. **Mathematical Analysis of the Proposed Mechanisms.** As we can observe in Section 2, in order to obtain an appropriate performance of the CVT system, a mechanism is necessary wherein its velocity oscillates around zero. That is, due to the input motor of the CVT system producing an angular velocity in only one direction and a drive mechanism is needed to generate an oscillating angular motion. The selected mechanism must convert the constant angular velocity of the input motor into an oscillating velocity centered on zero. Moreover, the selected mechanism must produce the most symmetric motion about $\frac{\pi}{2}$, in θ_4.

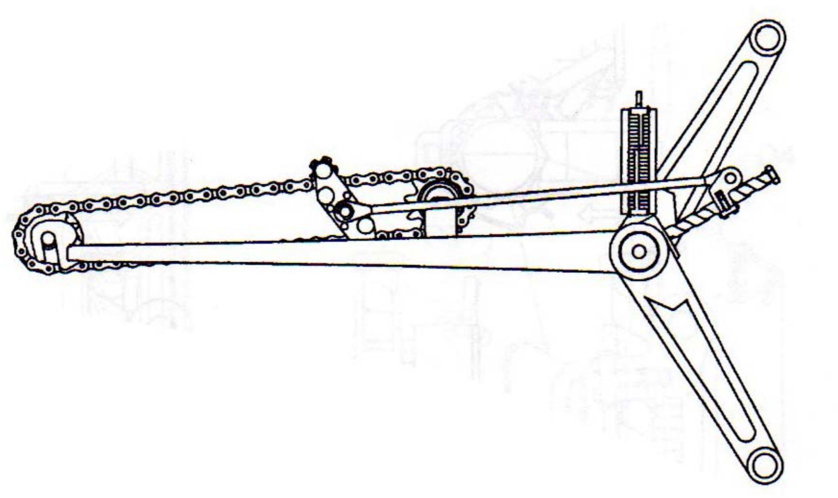

FIGURE 1. Continuously variable transmission

As it was said in Section 1, two mechanisms are proposed in order to fulfill the CVT system kinematic requirements. Such mechanisms are: a four-bar mechanism and a quick-return crank mechanism. The kinematic analysis of each one of the mechanisms proposed is carried out.

3.1. **Four-bar mechanism (FBM).** A schematic drawing of the four-bar mechanism is shown in Figure 2. This four-bar mechanism is composed of a reference bar (r_1), a crank bar (r_2), a connecting rod bar (r_3) and a rocker bar (r_4). Let $\theta_i \in \mathbb{R}\ \forall i = 1 \ldots 4$, the i-th angle between the horizontal axis and the i-th bar and counterclockwise positive direction.

A set of four important angles has to be considered, beginning with θ_1, described by the angle between horizontal axis and r_1, θ_2 is the angle between horizontal axis and r_2, θ_3 is the angle between horizontal axis and r_3 and θ_4 is the angle between horizontal axis and r_4. The angle marked as μ is the transmission angle. This special configuration is called crank-rocker.

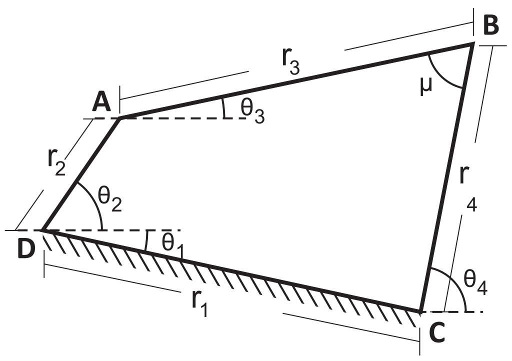

FIGURE 2. Four-bar mechanism diagram

3.1.1. *Analysis of position.* For the mechanism in Figure 2, a loop-closure equation is proposed as follows:

$$\vec{r_1} + \vec{r_4} = \vec{r_2} + \vec{r_3} \tag{1}$$

where each one of the vector is related with each one of the linkages.

On the other hand, if vectors are written in polar form [15], (1) can be expressed as follows:

$$r_1 e^{j\theta_1} + r_4 e^{j\theta_4} = r_2 e^{j\theta_2} + r_3 e^{j\theta_3} \tag{2}$$

Applying Euler's formula into (2), it can be written as:

$$r_1(\cos\theta_1 + j\sin\theta_1) + r_4(\cos\theta_4 + j\sin\theta_4) = r_2(\cos\theta_2 + j\sin\theta_2) + r_3(\cos\theta_3 + j\sin\theta_3) \tag{3}$$

Separating the real and imaginary part of (3):

$$\begin{aligned} r_1\cos\theta_1 + r_4\cos\theta_4 &= r_2\cos\theta_2 + r_3\cos\theta_3 \\ jr_1\sin\theta_1 + jr_4\sin\theta_4 &= jr_2\sin\theta_2 + jr_3\sin\theta_3 \end{aligned} \tag{4}$$

In order to obtain the angular position θ_3, the left side of the equations system (4) is put in terms of θ_4:

$$\begin{aligned} r_4\cos\theta_4 &= r_2\cos\theta_2 + r_3\cos\theta_3 - r_1\cos\theta_1 \\ r_4\sin\theta_4 &= r_2\sin\theta_2 + r_3\sin\theta_3 - r_1\sin\theta_1 \end{aligned} \tag{5}$$

Taking the square of (5) and adding them, the Freudenstein's equation in a compact form [15] is established as follows:

$$A_1\cos\theta_3 + B_1\sin\theta_3 + C_1 = 0 \tag{6}$$

where

$$A_1 = 2r_3\left(r_2\cos\theta_2 - r_1\cos\theta_1\right) \tag{7}$$

$$B_1 = 2r_3\left(r_2\sin\theta_2 - r_1\sin\theta_1\right) \tag{8}$$

$$C_1 = r_1^2 + r_2^2 + r_3^2 - r_4^2 - 2r_1r_2\cos\left(\theta_1 - \theta_2\right) \tag{9}$$

The angle θ_3 can be explicitly found as a function of the parameters A_1, B_1, C_1 and θ_2. Such solution is obtained by expressing $\sin\theta_3$ and $\cos\theta_3$ in terms of $\tan\left(\frac{\theta_3}{2}\right)$ as indicated in (10).

$$\sin\theta_3 = \frac{2\tan\left(\frac{\theta_3}{2}\right)}{1+\tan^2\left(\frac{\theta_3}{2}\right)}, \quad \cos\theta_3 = \frac{1-\tan^2\left(\frac{\theta_3}{2}\right)}{1+\tan^2\left(\frac{\theta_3}{2}\right)} \tag{10}$$

and substituting those in (6), the angular position θ_3 is given by (11).

$$\theta_3 = 2\arctan\left[\frac{-B_1 \pm \sqrt{B_1^2 + A_1^2 - C_1^2}}{C_1 - A_1}\right] \tag{11}$$

A similar mathematical procedure to obtain θ_4 must be done. The Freudestein's equation from (1) in compact form is given by (12).

$$D_1\cos\theta_4 + E_1\sin\theta_4 + F_1 = 0 \tag{12}$$

where

$$D_1 = 2r_4\left(r_1\cos\theta_1 - r_2\cos\theta_2\right) \tag{13}$$

$$E_1 = 2r_4\left(r_1\sin\theta_1 - r_2\sin\theta_2\right) \tag{14}$$

$$F_1 = r_1^2 + r_2^2 + r_4^2 - r_3^2 - 2r_1r_2\cos\left(\theta_1 - \theta_2\right) \tag{15}$$

Therefore, the angular position θ_4 is given by (16).

$$\theta_4 = 2\arctan\left[\frac{-E_1 \pm \sqrt{D_1^2 + E_1^2 - F_1^2}}{F_1 - D_1}\right] \tag{16}$$

TABLE 1. Choosing the sign of the radical according with the type of mechanism

Four-bar mechanism configuration	θ_3	θ_4
open	$+\sqrt{\ }$	$-\sqrt{\ }$
crossed	$-\sqrt{\ }$	$+\sqrt{\ }$

Equations (11) and (16) must take the signs of radicals according with the four-bar mechanism used. Table 1 shows the corresponding signs. It is important to remark that in this work, the open configuration was used.

3.1.2. *Analysis of velocity.* In order to carry out the analysis of velocity, (1) is time derivative. It is important to remark that $\theta_1 = constant$, therefore:

$$jr_4\omega_4 e^{j\theta_4} = jr_2\omega_2 e^{j\theta_2} + jr_3\omega_3 e^{j\theta_3} \tag{17}$$

where

$$\omega_2 = \frac{d\theta_2}{dt} \tag{18}$$

$$\omega_3 = \frac{d\theta_3}{dt} \tag{19}$$

$$\omega_4 = \frac{d\theta_4}{dt} \tag{20}$$

Dividing (17) by $je^{j\theta_3}$:

$$r_4\omega_4 e^{j(\theta_4-\theta_3)} = r_2\omega_2 e^{j(\theta_2-\theta_3)} + r_3\omega_3 \tag{21}$$

Applying Euler's formula into (21), it can be written as:

$$r_4\omega_4[\cos(\theta_4-\theta_3) + j\sin(\theta_4-\theta_3)] = r_2\omega_2[\cos(\theta_2-\theta_3) + j\sin(\theta_2-\theta_3)] + r_3\omega_3 \tag{22}$$

Taking the imaginary part of (22)

$$r_4\omega_4\sin(\theta_4-\theta_3) = r_2\omega_2\sin(\theta_2-\theta_3) \tag{23}$$

From (23), the angular velocity ω_4 can be calculated as follows:

$$\omega_4 = \left(\frac{r_2}{r_4}\right)\left[\frac{\sin(\theta_2-\theta_3)}{\sin(\theta_4-\theta_3)}\right]\omega_2 \tag{24}$$

In this mechanism, the input engine is coupled to the shaft `D` and the output motion is joined to the r_4 linkage which is coupled to the shaft `C`. That is, the input angular velocity of the mechanism is the angular velocity of the crank (ω_2) and the output angular velocity, is the angular velocity of the rocker (ω_4) which is established in (24).

Finally, as the magnitude of the input angular velocity of the four-bar mechanism is constant, the angular position θ_2 is given by (25).

$$\theta_2 = \omega_2 t \tag{25}$$

3.2. **Quick-return crank mechanism (QRCM).** Figure 3(a) shows the Quick-Return Crank Mechanism (QRCM).

A schematic drawing of this mechanism is shown in Figure 3(b). This mechanism is composed of a reference bar (r_2), a crank bar (r_1) and a rocker bar (r_3). It is important to remark that the last mechanical element has a variable magnitude due to the kind of joint.

In this mechanism, the input engine is coupled to the shaft of the crank. This mechanical point is the joint of the crank bar and the reference bar. Also the output motion is related to the motion rocker.

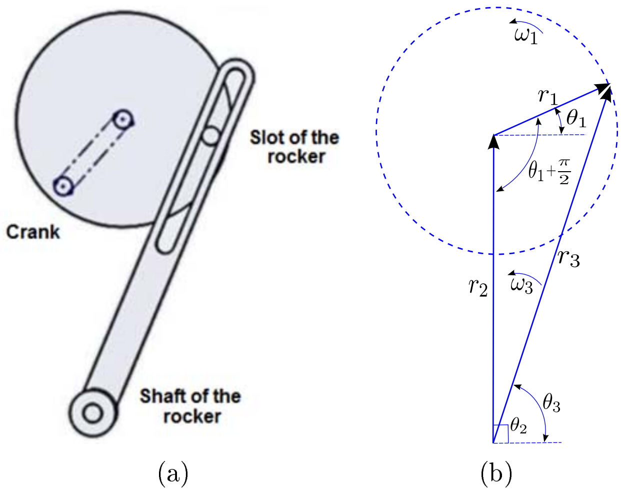

FIGURE 3. Quick-return crank mechanism

In order to carry out the kinematic analysis of this mechanism, the closed-loop equation for the mechanism in Figure 3(b), is given by

$$\vec{r_1} - \vec{r_2} - \vec{r_3} = 0 \tag{26}$$

In polar form (26) can be expressed as follows:

$$r_1 e^{j\theta_1} - r_2 e^{j\theta_2} - r_3 e^{j\theta_3} = 0 \tag{27}$$

Applying Euler's formula into (27) and separating the real and the imaginary part:

$$r_1 \cos\theta_1 - r_2 \cos\theta_2 - r_3 \cos\theta_3 = 0 \tag{28}$$

$$r_1 \sin\theta_1 - r_2 \sin\theta_2 - r_3 \sin\theta_3 = 0 \tag{29}$$

From (28) and taking into account that $\theta_2 = \frac{\pi}{2}$, the angular position θ_3 is given by:

$$\theta_3 = \cos^{-1}\left(\frac{r_1}{r_3}\cos\theta_1\right) \tag{30}$$

On the other hand, as we can observe in Figure 3(b) vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$ form a triangle. Therefore, using the law of cosines the variable magnitude of r_3 can be computed as follows:

$$r_3 = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\left(\theta_1 + \frac{\pi}{2}\right)} \tag{31}$$

The linear velocity V_3 and the angular velocity ω_3 of the rocker bar can be obtained taking the time derivative from (31) and (30) respectively. These mathematical relations are written as follows:

$$V_3 = \frac{r_1 r_2 \cos(\theta_1 + \pi/2)\omega_1}{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 + \pi/2)}} \tag{32}$$

$$\omega_3 = \frac{r_1 \cos(\theta_1) V_3 + r_1 r_3 \sin(\theta_1)\omega_1}{r_3^2 \sin(\theta_3)} \tag{33}$$

4. **Optimal Mechanism Selection.** As discussed previously, an optimization problem is proposed in order to select an input mechanism for the CVT system. Therefore, an objective function, constraints and a set of structural parameters must be proposed to each mechanism. The parameter vector which is a solution of the optimization problem will be an optimal set of structure parameters which minimize the performance criterion and subject to design constraints.

4.1. **Objective function.** Due to the motion that the input mechanism must be symmetric, a mathematical function to the angular motion is proposed. As we can observe, the rocker motion must be symmetric around the vertical axis, therefore the mathematical function is given by:

$$f_d(t) = \frac{\pi}{2} + A\sin(\omega t) \tag{34}$$

Since the motion of the rocker must be closest to the function established in (34), the objective function of the optimization problem is proposed as:

$$\phi = (f_d - \theta_m)^2 \tag{35}$$

where $m = 3$ or $m = 4$ depends of the proposed mechanism.

4.2. **Parameters and constraints.** As the aim is to obtain a set of optimal dimensions for the mechanism, a set of parameters and constraints are proposed for each of them.

4.2.1. *Four-bar mechanism.* The four-bar mechanism is one of the most mechanical systems studied. As is known the angular displacement of the rocker is related with the magnitude of the four bars of the mechanism (r_1, r_2, r_3 and r_4), also with the angular displacement between the input and the rocker shaft (θ_1 in Figure 2). On the other hand, the Grashof's law provides that: for a plane four-bar linkage, the sum of the lengths shortest and longest links cannot be greater than the sum of the lengths of the two remaining links, if a continuous relative rotation between two elements is desired [2]. Denoting as s and l the shortest and the longest links of the four-bar mechanism and as p and q the other two links, Grashof's law is established as:

$$s + l \leq p + q \tag{36}$$

For this case, Grashof's law is given by:

$$r_2 + r_3 \leq r_1 + r_4 \tag{37}$$

In addition, to ensure that the solution method produces a Grashof mechanism, it must be fulfilled:

$$r_1 \leq r_3 \tag{38}$$

$$r_4 \leq r_3 \tag{39}$$

One of the design considerations used in the four-bar mechanism was the maximum size of the links. Due to the available space, the length was determined between 0.05m and 0.5m, so that the restrictions are established as:

$$0.05 \leq r_1 \leq 0.5 \tag{40}$$

$$0.05 \leq r_2 \leq 0.5 \tag{41}$$

$$0.05 \leq r_3 \leq 0.5 \tag{42}$$

$$0.05 \leq r_4 \leq 0.5 \tag{43}$$

On the other hand, the angle between the horizontal axis and the reference bar (r_1) is limited between 45° and −45°, therefore:

$$-45^\circ \leq \theta_1 \leq 45^\circ \tag{44}$$

Finally, the optimization problem is given as follows:

$$\min_{x \in R^5} \quad \Sigma(f_d - \theta_4)^2 \tag{45}$$

subject to:

$$g_1(x) = x_2 + x_3 - x_1 - x_4 \leq 0 \quad (46)$$
$$g_2(x) = x_1 - x_3 \leq 0 \quad (47)$$
$$g_3(x) = x_4 - x_3 \leq 0 \quad (48)$$
$$g_4(x) = x_1 - 0.5 \leq 0 \quad (49)$$
$$g_5(x) = 0.05 - x_1 \leq 0 \quad (50)$$
$$g_6(x) = x_2 - 0.5 \leq 0 \quad (51)$$
$$g_7(x) = 0.05 - x_2 \leq 0 \quad (52)$$
$$g_8(x) = x_3 - 0.5 \leq 0 \quad (53)$$
$$g_9(x) = 0.05 - x_3 \leq 0 \quad (54)$$
$$g_{10}(x) = x_4 - 0.5 \leq 0 \quad (55)$$
$$g_{11}(x) = 0.05 - x_4 \leq 0 \quad (56)$$
$$g_{12}(x) = x_5 - \frac{\pi}{4} \leq 0 \quad (57)$$
$$g_{13}(x) = -\frac{\pi}{4} - x_5 \leq 0 \quad (58)$$

where

$$\vec{x} = (x_1, x_2, x_3, x_4, x_5)^T = (r_1, r_2, r_3, r_4, \theta_1)^T \quad (59)$$

4.2.2. *Quick-return crank mechanism.* It is important to remark that in the QRCM, the mechanical element so called reference bar (r_2), establishes the size of the whole mechanism. As the first mechanism, one of the design considerations used in QRCM was the maximum size of the links. Due to the available space, the length was determined between 0.1m and 0.5m, and the ratio between crank bar and the reference bar is established as 2 : 1, so that the restrictions are established as:

$$0.1 \leq p_1 \leq 0.5 \quad (60)$$
$$2p_1 \leq p_2 \leq 0.5 \quad (61)$$

On the other hand, the angle θ_2 is established with a constant value of $\frac{\pi}{2}$.
Therefore, the optimization problem is given as follows:

$$\min_{p \in R^2} \quad \Sigma(f_d - \theta_3)^2 \quad (62)$$

subject to:

$$g_1(p) = 0.1 - p_1 \leq 0 \quad (63)$$
$$g_2(p) = p_1 - 0.5 \leq 0 \quad (64)$$
$$g_3(p) = 0.2 - p_2 \leq 0 \quad (65)$$
$$g_4(p) = p_2 - 0.5 \leq 0 \quad (66)$$
$$g_5(p) = 2p_1 - p_2 \leq 0 \quad (67)$$

where

$$\vec{p} = (p_1, p_2)^T = (r_1, r_2)^T \quad (68)$$

5. **Evolutionary Optimization.** DE is a population-based EA, where NP vectors are used:

$$x_{i,G} \quad \text{where} \quad i = 1, 2, 3, \ldots, NP \tag{69}$$

where subscripts i and G are the number of the individual in the population and the generation of such individual. The parameter NP is constant between generations and the first generation is randomly created. As a rule-of-thumb, a uniform probability distribution is assumed in all random decisions. The main idea of a DE algorithm is to propose a new way to generate vectors. The ED generates these new vectors by adding the weighted difference between two individuals of the population and a third individual. If the fitness of the resulting vector is less than the fitness of a selected population individual, the new vector replaces the vector with which it was compared.

We use the standard version of the DE algorithm [11] called *DE/rand/1/bin* and its algorithm is presented in Figure 4. The "CR" parameter controls the influence of the parent in the generation of the offspring. High values mean less influence of the parent. The parameter "F" weighs the influence of two of the three individuals selected at random to generate the offspring.

```
Begin
    G=0
    Create a random initial population x⃗^i_G ∀i, i = 1,...,NP
    Evaluate f(x⃗^i_G) ∀i, i = 1,...,NP
    For G=1 to MAX_GENERATIONS Do
        For i=1 to NP Do
            Select randomly r1 ≠ r2 ≠ r3 :
            j_rand = randint(1, D)
            For j=1 to D Do
                If (rand_j[0,1) < CR or j = j_rand) Then
                    u^i_{j,G+1} = x^{r3}_{j,G} + F(x^{r1}_{j,G} − x^{r2}_{j,G})
                Else
                    u^i_{j,G+1} = x^i_{j,G}
                End If
            End For
            If (f(u⃗^i_{G+1}) ≤ f(x⃗^i_G)) Then
                x⃗^i_{G+1} = u⃗^i_{G+1}
            Else
                x⃗^i_{G+1} = x⃗^i_G
            End If
        End For
        G = G + 1
    End For
End
```

FIGURE 4. DE algorithm. randint(min,max) is a function that returns an integer number between min and max. rand$[0, 1)$ is a function that returns a real number between 0 and 1. Both are based on a uniform probability distribution. "NP", "MAX_GENERATIONS", "CR" and "F" are user-defined parameters.

6. **Numerical Simulation.** In this work, 10 independent runs with the same parameters for each of the mechanisms were performed. Population number $NP = 100$, $MAX\_GENERATIONS = 350$; parameters F and CR were randomly generated. Also, the values for the function given by (34) were: $A = \frac{\pi}{8}$ and $\omega = 6.9\ \frac{\text{rad}}{\text{s}}$.

The optimization process was carried out and (70) and (71) present the optimal parameter vectors of each mechanism, respectively.

$$x^* = (0.5, 0.05, 0.5, 0.1312, -0.1366)^T \tag{70}$$

$$p^* = (0.1306, 0.3581)^T \tag{71}$$

The behavior of the mechanisms and the desired profile (DP) is shown in Figure 5. As we can observe, both mechanisms have an oscillating behavior and can provide the desire motion to the CVT system. This is the reason why both proposals are possible solutions.

7. **Discussion of Results.** Based on the performance observed on numerical simulations by both analyzed mechanisms, we highlight the following issues:

- The FBM presents a better behavior than the QRCM, that is, the rocker motion of the first mechanism is closest to the desired profile. Also, the speed profile of the FBM presents a most symmetric motion around zero, as shown in Figure 6.
- When a large dimensional ratio between the rocker and the crank is proposed on the QRCM, the angular displacement of the rocker (peak to peak value of the motion profile) is reduced, with this fact the slider displacement of the CVT system is reduced also.
- In order to obtain the same magnitude on the forward and backward motion sense by the QRCM, the magnitude of the rocker must be greater than the magnitude of the crank. The magnitude ratio between these mechanical elements must be 10 or greater. This fact produces a big size in the mechanism.

All these issues lead us to conclude that the four-bar mechanism is clearly a better option than the quick-return crank mechanism.

A possible mechanical implementation of this mechanism in the CVT system is shown in Figure 7. There, the four-bar mechanism is coupled into the CVT system as the input mechanism. The changes of the transmission ratio are produced by means of a lead screw

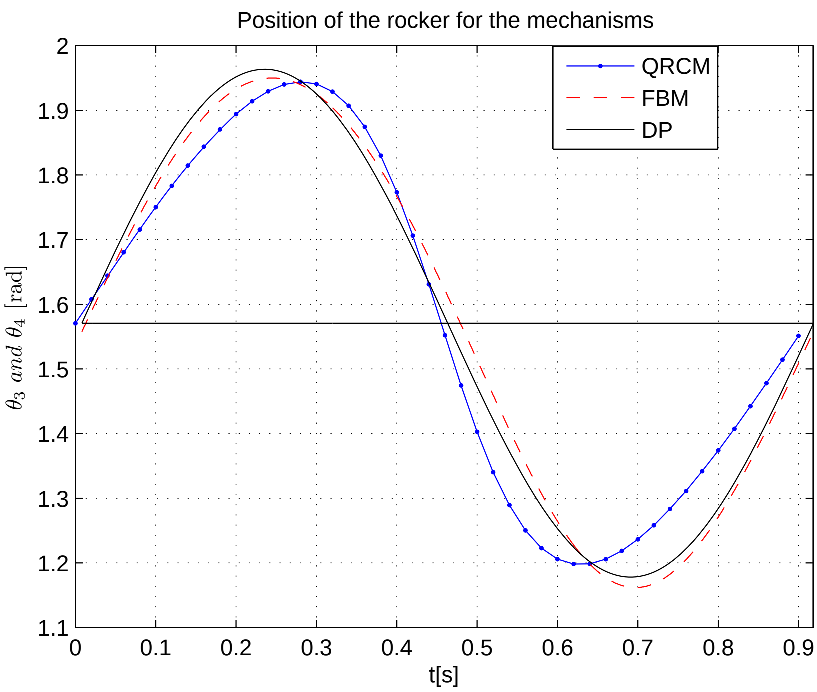

FIGURE 5. Rocker's angular position for the mechanisms

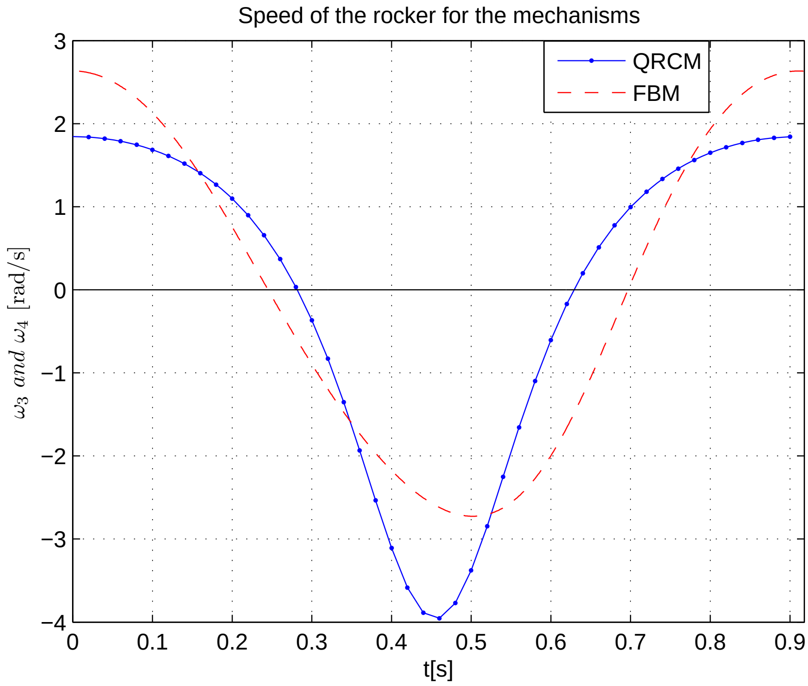

Figure 6. Rocker's angular speed for the mechanisms

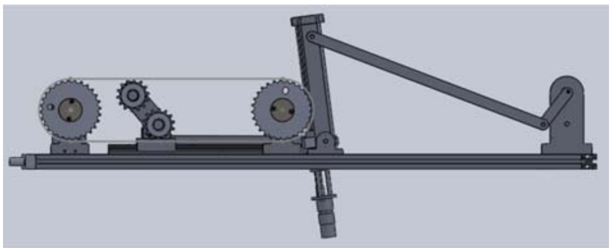

Figure 7. Continuously variable transmission with the four-bar mechanism

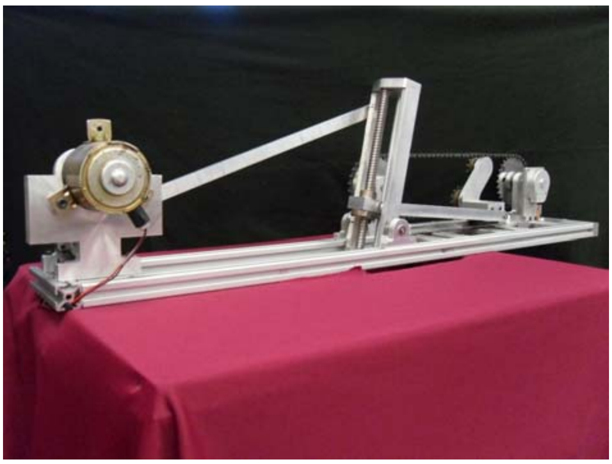

Figure 8. CVT prototype

coupled to a DC motor. This mechanical configuration seems to provide an easier way to build it. In Figure 8, a CVT prototype is shown.

8. **Conclusions.** In this paper, we have presented a constrained optimal selection of one input mechanism in order to fulfill the motion required of a CVT system. In order to compare the behavior of two mechanisms, an optimization problem was proposed to obtain their optimal mechanical parameters, based on the kinematic model developed in this work. Also, an objective function and mechanical constraints were proposed. The optimization problem was solved using an evolutionary-based approach called differential evolution. From the optimization problem results, numerical simulations presented the expected behavior. It can be observed that the structural parameters for each mechanism fulfill the desired profile motion. However, a better behavior of the FBM is achieved. Therefore, this mechanism fulfills the necessity of the CVT motion.

On the other hand, the use of an evolutionary-based approach allows a comparison of the two mechanisms without a preference choice selection. Finally, the use of heuristics in mechanical design problems was successfully applied.

Further research includes proposing the kinematic and dynamic model of the whole CVT system. Finally, authors will propose the parametric optimal design of the CVT system also to establish this as a multiobjective dynamic optimization problem.

Acknowledgment. This work was supported by the Comisión de Operación Y Fomento de Actividades Académicas (COFAA) and the Secretaría de Investigación y Posgrado (SIP), both from the Instituto Politécnico Nacional-México under grants 20111027 and 20121472, and by the National Council of Science and Technology (CONACyT) under Grant 182298. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the paper.

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