SCENARIO-BASED PATH SELECTION IN UNCERTAIN EMERGENCY TRANSPORTATION NETWORKS

JUNHU RUAN\textsuperscript{1}, XUPING WANG\textsuperscript{1,}\textsuperscript{*}, YAN SHI\textsuperscript{1,2} AND ZILAI SUN\textsuperscript{1}

\textsuperscript{1}Institute of Systems Engineering
Dalian University of Technology
No. 2, Linggong Road, Dalian 116023, P. R. China
\textsuperscript{*}Corresponding author: wxp@dlut.edu.cn; ruanjunhu@mail.dlut.edu.cn

\textsuperscript{2}Graduate School of Science and Technology
Tokai University
9-9-1, Toroku, Kumamoto 862-8652, Japan
yshi@ktmail.tokai-u.jp

Received May 2012; revised September 2012

\textbf{Abstract.} The transportation networks often become uncertain due to the occurrence and development of disasters. In order to select proper paths under uncertain information, we propose a novel scenario-based approach for emergency path selection. The scenario factors are firstly analyzed and combined to describe the uncertainty of each path section between two adjacent intersections in emergency transportation networks. Then, according to the fuzzy properties of path sections, the membership transformation algorithm M(1,2,3) is applied in fuzzy evaluation on the scenarios of path sections, which can determine the satisfaction degrees of each path section. Combining the highest satisfaction of each section under various scenarios, an optimal multi-attribute vector is constructed to build the scenario-based optimization model for emergency path selection. Finally, a numerical example is shown to illustrate the solution of this approach, whose results show that this approach can produce the paths with maximized overall satisfaction degree in a given confidence and that the satisfaction degree will increase as the given confidence decreases.

\textbf{Keywords:} Large-scale disasters, Emergency path selection, Scenario analysis, M(1,2,3), Scenario-based optimization

1. \textbf{Introduction.} In recent years, large-scale disasters such as earthquakes, tsunamis and rainstorms occur frequently, which have been disturbing people’s normal life and bringing about serious impacts on social stability and development. The suddenness, urgency and uncertainty of these disasters make emergency response decisions face great challenges [1,2]. Timely transportation of relief supplies right after disasters takes an important part in emergency response, directly affecting the efficiency and effectiveness of disaster relief [3]. However, emergency path selection is much different from conventional path selection which mainly aims to searching the shortest path from feasible road networks. In emergency response, disasters may make damages on roads and make some of them unsafe; meanwhile the massive relief supplies transportation often exceeds the limited maximum road capacity and may block some key roads. In a word, there are more uncertainties in emergency transportation, which makes conventional path selection approaches ineffective. Thus, how to select a proper path for transporting relief supplies and wounded victims safely is one of the key issues in emergency transportation, which has attracted some researchers’ attention.
Opasanon and Miller-Hooks [4] developed three adaptive path selection solutions which can be applied in selection of routes for hazardous materials transport and emergency response operations. Zografos and Androutsopoulos [5] presented a decision support system for assessing alternative distribution routes in terms of travel time, risk and evacuation implications while coordinating the emergency response deployment decisions with the hazardous materials routes. Yuan and Wang [6] presented a single-objective path selection model whose objective is to minimize total travel time along a path, and a multi-objective path selection model which considered the chaos, panic and congestions in time of disaster, aiming at minimizing both the total travel time and the path complexity. Liu and Zhao [7] studied the emergency materials distribution problem in anti-bioterrorism system, modeling it as a multiple traveling salesman problem with time windows, and designing a hybrid genetic algorithm to solve the model. Hu [8] took the system of container multimodal transportation emergency relief as an affinity network, and proposed an integer linear programming model to build the path selection for container supply chain in the context of emergency relief.

The above studies produced effective solutions for selecting emergency paths from different aspects, and the consensus can be drawn that emergency path selection is not a conventional shortest path problem due to its characteristics of uncertainty and being with multiple decision-making attributes. Emergency decision-makers would consider various uncertain factors to select relief supplies transportation paths, and the conditions of these factors are often not unchanged due to the updated scenarios such as the occurrence of secondary disasters, the traffic flow in the emergency road networks, and the transportation time of each road section. However, how to represent these uncertain decision-making factors is an open issue needing more effective methods, which is also the precondition of selecting proper emergency paths. Scenario analysis method can effectively solve the decision-making issues under uncertain environment [9]. In this study, we take the combinations of multiple attributes as various scenarios, and then develop a scenario-based emergency path selection model. To sum up, three main contributions are made in our study. (1) The uncertain attributes are fuzzified and assembled to represent the decision-making scenarios of emergency path selection. (2) A scenario-based optimization model is built which aims at selecting the path with maximized overall satisfaction degree in a given confidence. (3) An integrated algorithm of depth-first search and Monte Carlo simulation is designed to solve the built model. These contributions have two main differences from the extant studies. One is that our developed approach considers more factors than the conventional shortest path methods which aim at producing the optimal path with shortest geographical distance. The other is that we build a fuzzy multiple attribute vector to represent the decision-making scenarios of emergency path selection and embed it into a scenario-based optimization model.

The remainder of this paper is organized as follows. Section 2 constructs a scenario representation approach to describe the uncertain multiple attributes of each path section. In Section 3, a scenario-based emergency path selection model is built. Section 4 illustrates an integrated algorithm for solving the built model, with a numerical example. Conclusions are finally drawn in Section 5, with recommendations in future studies.

2. Scenario Representations of Emergency Path Selection. Scenario analysis method, which considers the key factors and their mutual influences of complicated systems to express the possible future development [9], can effectively solve the decision-making problems under uncertain environment. Inspired by the application of scenario analysis method [10,11], we consider the uncertainties of the occurrence and development of disasters to build the decision-making scenarios of emergency path selection.
Table 1. Scenario factors of emergency path selection

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Scenario factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>The key factors of decision-makers selecting emergency paths</td>
<td></td>
</tr>
<tr>
<td>$f_1$: Road smoothness</td>
<td>$f_{11}$: Road quality</td>
</tr>
<tr>
<td></td>
<td>$f_{12}$: Road width</td>
</tr>
<tr>
<td></td>
<td>$f_{13}$: Traffic density</td>
</tr>
<tr>
<td>$f_2$: Road safety</td>
<td>$f_{21}$: Geographic conditions</td>
</tr>
<tr>
<td></td>
<td>$f_{22}$: Weather conditions</td>
</tr>
<tr>
<td></td>
<td>$f_{23}$: Humane conditions</td>
</tr>
<tr>
<td>$f_3$: Transportation time</td>
<td>$f_{31}$: Road alignment</td>
</tr>
<tr>
<td></td>
<td>$f_{32}$: Road facilities</td>
</tr>
<tr>
<td></td>
<td>$f_{33}$: Road length</td>
</tr>
<tr>
<td>$f_4$: Communication quality</td>
<td>$f_{41}$: Communication equipment</td>
</tr>
<tr>
<td></td>
<td>$f_{42}$: Communication environment</td>
</tr>
</tbody>
</table>

Figure 1. Scenario representations of emergency transportation paths

Different from conventional business logistics, the emergency transportation possesses its own characteristics. By analyzing the key factors of decision-makers selecting emergency paths, we summarize these factors into four attributes: road smoothness, road safety, transportation time, and communication quality. The detailed scenario factors in each attribute are shown in Table 1.

Assume that the emergency transportation network is represented by $G(N, Arc)$ where $N$ denotes the set of road nodes and $Arc$ denotes the set of road sections among nodes. Due to the influence of the occurrence and development of disasters, road sections may be in different statuses. We represent the status of each road section $arc_i$ with the series of the initial scenario, emergency scenarios and factor scenarios to describe the uncertainties of emergency transportation network, shown as Figure 1.

After the occurrence of disasters, the status of each road section $arc_i$ in the transportation network is transformed from the initial scenario to the emergency scenario. Assume that $(ES, P_1)$ is the set of emergency scenarios where $ES = \{es_1, es_2, \ldots, es_M\}$, $M = |ES|$, $es_m$ ($m = 1, 2, \ldots, M$) is one emergency scenario in $ES$, $P_1$ is the set of
probabilities of emergency scenarios, and \( p(es_m) \in P_1 \) is the probability of occurrence of the emergency scenario \( es_m \) under the initial scenario that guarantees \( \sum_{m=1}^{M} p(es_m) = 1 \).

Each emergency scenario \( es_m \) is composed of different factor scenarios. Assume that \( (IS_{/m}, P_{/m}) \) is the set of factor scenarios under the \( m \)-th emergency scenario where \( IS_{/m} = \{is_{1/m}, is_{2/m}, \ldots, is_{S/m}\} \), and \( S_m = |IS_{/m}|. is_{S/m} \) is one factor scenario in \( IS_{/m}, j = 1, 2, \ldots, S_m \), \( P_{/m} \) is the set of probabilities of factor scenarios, and \( p(is_{S/m}/es_m) \in P_{/m} \) is the probability of occurrence of the factor scenario \( is_{S/m} \) under \( m \)-th emergency scenario that guarantees \( \sum_{S=1}^{S_m} p(is_{S/m}/es_m) = 1 \).

Moreover, for each factor scenario \( is_{S/m} \), we use a fuzzy multi-attribute vector \( \alpha = [f_1, f_2, f_3, f_4] \) to denote the status of each path section under factor scenarios (\( f_1 \): Road smoothness, \( f_2 \): Road safety, \( f_3 \): Transportation time, and \( f_4 \): Communication quality). \( \alpha_{S/m}(f_1, f_2, f_3, f_4) \) represents the impacts of emergency scenarios on the multiple attributes in factor scenarios. Meanwhile, each attribute consists of multiple factors, shown as Table 1 and Figure 1. We set four evaluation levels for each factors: {very good, good, general, bad}. According to the evaluation value of each factor, we can finally judge the statuses of each attribute, each factor scenario, and each emergency scenario by fuzzy evaluation methods.

After getting all the statuses of path sections in emergency scenarios, we can use optimization methods to search one satisfactory path connecting the starting point with the end point in the uncertain emergency transportation network.

3. The Developed Approach for Emergency Path Selection in Uncertain Transportation Networks. As mentioned above, the developed approach for emergency path selection in this study can be taken as a two-stage method. Stage I uses fuzzy multi-attribute vectors to describe the uncertainty of each path section and evaluates the scenarios of all path sections in uncertain emergency transportation networks. Stage II builds an optimization model to find the paths with maximized overall satisfaction degree in a given confidence. In this section, we firstly apply the membership transformation algorithm-M(1,2,3) to determine the satisfaction degrees of each path section in Stage I. Based on the evaluation results in Stage I, we combine the highest satisfaction of each section under various scenarios to construct an optimal multi-attribute vector and further establish the optimization model for emergency path selection with scenarios embedded in Stage II.

3.1. Membership transformation algorithm-M(1,2,3). The core of fuzzy evaluation is the membership degree transformation from index membership to object membership. For the several common transformation methods, redundant data in index membership degree is also used to compute object membership degree. In our previous study [12], we proposed a novel membership transformation algorithm which can eliminate the redundant data in index membership for object evaluation by defining distinguishable weight and extract valid values to compute object membership. In this study, we use the algorithm to calculate the fuzzy evaluation vector of each path section under different scenarios.

Assume that there are \( m \) indexes which have impacts on evaluation object \( f \), and the importance weights \( \lambda_j(f) \) of the \( j \)-th index (\( j = 1, 2, \ldots, m \)) to \( f \) is given, which satisfies:

\[
0 \leq \lambda_j(f) \leq 1, \quad \sum_{j=1}^{m} \lambda_j(f) = 1
\]
Each index is classified into $p$ levels. $C_k$ represents the $k$-th level and $C_k$ is superior to $C_{k+1}$. The membership $\mu_{jk}(f)$ of the $j$-th index belonging to $C_k$ is given, where $k = 1, 2, \ldots, p$ and $j = 1, 2, \ldots, m$, and $\mu_{jk}(f)$ satisfies:

$$0 \leq \mu_{jk}(f) \leq 1, \quad \sum_{k=1}^{p} \mu_{jk}(f) = 1 \quad (j = 1, 2, \ldots, m)$$  \hspace{1cm} (2)

Then, what is the membership $\mu_k(f)$ of object $f$ belonging to $C_k$? The membership degree transformation is used to answer this question. The membership transformation new algorithm we previously proposed can be briefly overviewed as follows.

A real number $\alpha_j(f)$ can be quantitatively described by the entropy $H_j(f)$ which satisfies the following equations:

$$H_j(f) = -\sum_{k=1}^{p} \mu_{jk}(f) \cdot \log \mu_{jk}(f)$$  \hspace{1cm} (3)

$$v_j(f) = 1 - \frac{1}{\log p} H_j(f)$$  \hspace{1cm} (4)

$$\alpha_j(f) = \frac{v_j(f)}{\sum_{j=1}^{m} v_j(f)} \quad (j = 1, 2, \ldots, m)$$  \hspace{1cm} (5)

**Definition 3.1.** If $\mu_{jk}(f)$ ($k = 1, 2, \ldots, p, j = 1, 2, \ldots, m$) is the membership of the $j$-th index belonging to $C_k$ and satisfies Equation (2), then, by Equations (3)-(5), $\alpha_j(f)$ is called the distinguishable weight of the $j$-th index with respect to object $f$. Obviously, $\alpha_j(f)$ satisfies:

$$0 \leq \alpha_j(f) \leq 1, \quad \sum_{j=1}^{m} \alpha_j(f) = 1$$  \hspace{1cm} (6)

**Definition 3.2.** If $\mu_{jk}(f)$ ($k = 1, 2, \ldots, p, j = 1, 2, \ldots, m$) is the membership of the $j$-th index belonging to $C_k$ and satisfies Equation (2), and $\alpha_j(f)$ is the distinguishable weight of the $j$-th index with respect to $f$, then

$$\alpha_j(f) \cdot \mu_{jk}(f) \quad (k = 1, 2, \ldots, p)$$  \hspace{1cm} (7)

is called the effective value of the $k$-th class membership of the $j$-th index, or the $k$-th class effective value for short.

**Definition 3.3.** If $\alpha_j(f) \cdot \mu_{jk}(f)$ is the $k$-th class effective value of the $j$-th index, and $\beta_j(f)$ is importance weight of the $j$-th index with respect to $f$, then

$$\beta_j(f) \cdot \alpha_j(f) \cdot \mu_{jk}(f) \quad (k = 1, 2, \ldots, p)$$  \hspace{1cm} (8)

is called comparable effective value of the $k$-th class membership of the $j$-th index, or the $k$-th class comparable value for short.

**Definition 3.4.** If $\beta_j(f) \cdot \alpha_j(f) \cdot \mu_{jk}(f)$ is the $k$-th class comparable value of the $j$-th index of $k$, ($j = 1, 2, \ldots, m$), then

$$M_k(f) = \sum_{j=1}^{m} \beta_j(f) \cdot \alpha_j(f) \cdot \mu_{jk}(f) \quad (k = 1, 2, \ldots, p)$$  \hspace{1cm} (9)

is named as the $k$th class comparable sum of $f$. 
Definition 3.5. If $M_k(f)$ is the $k$-th class comparable sum of object $f$, and $\mu_k(f)$ is the membership of object $f$ belonging to $C_k$, then

$$\mu_k(f) = \frac{M_k(f)}{\sum_{l=1}^{p} M_l(f)} \quad (k = 1, 2, \ldots, p) \quad (10)$$

Up to now, assume that the index membership of each factor $\mu_{jk}(f)$ and their importance weights $\lambda_j(f)$ are given, by Equations (3)-(10), the membership $\mu_k(f)$ of object $f$ belonging to $C_k$ can be calculated. We call this membership transformation algorithm as M(1,2,3).

In Stage I of the proposed approach, we use the above membership transformation algorithm to calculate the membership vectors of path sections under different scenarios which will be embedded in the emergency path selection optimization model in Stage II.

3.2. A scenario-based optimization model for emergency path selection.

For the attributes of some path section under different emergency scenarios $f_{S/m}^i$, $i = 1, 2, 3, 4$, $S = 1, 2, \ldots, S_m$, we can find their best evaluation values and worst evaluation values according to the calculation results by the algorithm in Subsection 3.1. Then, we use the Euclidean distance to measure the deviation of each attribute value from the best values and define the satisfaction degree of the multi-attribute vector by the relative deviation.

Assume that the best and worst values of the attribute $f_{S/m}^i$ are $f_{\text{max}}^i$ and $f_{\text{min}}^i$ which respectively equal max $f_{S/m}^i$ and min $f_{S/m}^i$, $i = 1, 2, 3, 4$, $S = 1, 2, \ldots, S_m$, $m = 1, 2, \ldots, M$, then an optimal multi-attribute vector $\alpha_{\text{max}} = (f_{\text{max}}^1, f_{\text{max}}^2, f_{\text{max}}^3, f_{\text{max}}^4)$ can be defined.

Definition 3.6. If $\alpha_{S/m}$ denotes the values vector of some attribute $f_{S/m}^i$, $i = 1, 2, 3, 4$ and $\alpha_{\text{max}}$ is the best value of this attribute, then the deviation of $\alpha_{S/m}$ from $\alpha_{\text{max}}$ is equivalent to $d = ||\alpha_{S/m} - \alpha_{\text{max}}|| = [(f_{\text{max}}^1 - f_{S/m}^1)^2 + (f_{\text{max}}^2 - f_{S/m}^2)^2 + (f_{\text{max}}^3 - f_{S/m}^3)^2 + (f_{\text{max}}^4 - f_{S/m}^4)^2]^{1/2}$, and the satisfaction degree of this multi-attribute vector is defined as

$$\theta_{S/m} = 1 - \frac{d(\alpha_{S/m}, \alpha_{\text{max}})}{d(\alpha_{\text{min}}, \alpha_{\text{max}})} \quad (11)$$

where $\theta_{S/m} \in [0, 1]$, $d(\alpha_{S/m}, \alpha_{\text{max}})$ denotes the deviation of $\alpha_{S/m}$ from $\alpha_{\text{max}}$, and $d(\alpha_{\text{min}}, \alpha_{\text{max}})$ is the distance between the best values vector $f_{\text{max}}^i$ and the worst value vector $f_{\text{min}}^i$. For $\theta_{S/m} = 0$ means that the values vector $f_{S/m}$ equals the worst values vector $f_{\text{min}}$; $\theta_{S/m} = 1$ means the values vector $f_{S/m}$ equals the best values vector $f_{\text{max}}$.

According to Equation (11), we can calculate all the satisfaction degrees of path sections in the emergency transportation network. The following part is to develop an optimization model to select the path with maximized overall satisfaction degrees. For a transportation network $G(N, Arc)$, $\vec{x} = \{x_{ij} | (i, j) \in Arc\}$ denotes a path connecting the starting point with the end point, where $x_{ij} = 1$ means arc $(i, j)$ is included in this path and $x_{ij} = 0$ means arc $(i, j)$ is not in this path. Because the length of paths in $G(N, Arc)$ is uncertain, we define that the length of the transportation network is the sum of all the path sections, that is, $D_{\text{max}} = |Arc|$. An ideal path $\vec{x}_{D_{\text{max}}}(1, 2, \ldots, D - 1, D_{\text{max}})$ can be constructed in which $k$th path section takes the best values vector under different scenarios (denoted by $\theta_{\text{max}}^k$, $k \in D_{\text{max}}$). In actual emergency response, the decision-makers often pay main attention to the most unsatisfactory path section, thus the overall satisfaction degree of the ideal path is defined as $\theta_{\text{max}} = \min \{\theta_{\text{max}}^k\}, k = 1, 2, \ldots, D_{\text{max}}$.

As Figure 2 shows, any change of each path section’s scenario can be embedded in the ideal path. We just replace the value of the corresponding path section in the ideal path with the value of the changed section. Each time a path is chosen, the unchosen path
sections are replaced by the ideal sections, which can make sure the length of the path unchanged and thus make the satisfaction degrees of different paths comparable.

Inspired by the idea of α key paths proposed by Liu et al. [13], we use $\sigma$ to represent the confidence level that decision-makers expect, and define $\min_{(i,j) \in \text{Arc}} (\theta_{ijS/m}, \theta_{ij\text{max}}) x_{ij}/\theta_{\text{max}}$ as the overall satisfaction degree of path $\vec{x}$ where $\theta_{ijS/m}$ denotes the satisfaction degree of the chosen path section and $\theta_{ij\text{max}}$ denotes the satisfaction degree of the unchosen path section whose values are taken as the ideal values of this section. Let $\Pr\{\min_{(i,j) \in \text{Arc}} (\theta_{ijS/m}, \theta_{ij\text{max}}) x_{ij}/\theta_{\text{max}}\}$ represent the probability of occurrence of selecting the path $\vec{x}$ under different emergency scenarios. By this, we can embed the scenarios of each path section and their impacts into the emergency path selection, and finally build the scenario-based optimization model:

$$\max \overline{\theta}$$

s.t.

$$\Pr\left\{ \frac{\min_{(i,j) \in \text{Arc}} (\theta_{ijS/m}, \theta_{ij\text{max}}) x_{ij}}{\theta_{\text{max}}} \geq \overline{\theta} \right\} \geq \sigma, \quad S = 1, 2, \ldots, S_m, \ m = 1, 2, \ldots, M$$

$$\sum_{(i,j) \in \text{Arc}} x_{ij} - \sum_{(j,i) \in \text{Arc}} x_{ji} = \begin{cases} 1, \quad i = 1 \\ 0, \quad i = 2, 3, \ldots, n - 1 \\ -1, \quad i = n \end{cases}$$

$$x_{ij} \in \{0, 1\}, \ \forall (i, j) \in \text{Arc}$$

The optimization objective (12) is to maximize the overall satisfaction degree of the chosen path $\overline{\theta}$ with the given confidence level $\sigma$. Constraint (13) makes sure the probability of occurrence of each scenario is satisfied where $\theta_{ijS/m}$ is calculated by Equation (11) and $\Pr\{\min_{(i,j) \in \text{Arc}} \theta_{ijS/m} x_{ij}\} = \min_{(i,j) \in \text{Arc}} \min_{S \in [0, S_m], m \in [0, M]} \theta_{ijS/m} p(S/m, eS/m) x_{ij}$. Constraint (14) makes sure the starting point and the end point are always selected and all the path nodes in the transportation network have opportunities to be selected. Constraint (15) defines the values of the decision variable $x_{ij}$, where $x_{ij} = 1$ means arc $(i, j)$ is included in this path and $x_{ij} = 0$ means arc $(i, j)$ is not in this path.

4. The Designed Solution and an Illustrated Example. In this section, we design the solution for the developed approach in Section 3, illustrated with an example.
Figure 3. Emergency transportation network in the example

Figure 3 shows a part of some transportation network, where node 1 is the starting point and node 5 is the end point. As we can see, there are five different paths connecting node 1 with node 5. According to conventional shortest path principle, path 1-3-5 is the best selection. However, in the emergency response, the status of each path section may be influenced by various uncertain factors as shown in Section 2. Decision-makers need to consider these factors comprehensively to select the path with maximized satisfaction degree. For simplicity, here we assume that there are two emergency scenarios and each emergency scenario produces one factor scenario. The factors’ importance weights and membership vectors of each path section’s factor scenario are given as Table 2 shows. In Table 2, the values in brackets behind corresponding factors are their importance weights; the vectors below each path section are various factors’ membership vectors with respect to four evaluation levels: {very good, good, general, bad}. In this paper, we assume that these original data are given.

4.1. Calculating the satisfaction degrees of attributes using M(1,2,3). According to the importance weights and membership vectors in Table 2, we can calculate the satisfaction degrees of the four attributes: road smoothness, road safety, transportation time, and communication quality for each path section under the two emergency scenarios.

Taking the calculation process of the attribute $f_1$ (road smoothness) of path section 1-2 under emergency scenario $ES_1/IS$ as an example, the detailed steps are as follows.

Step 1: Calculating the distinguishable weights. $f_1$ includes three factors $f_{11}, f_{12}, f_{13}$, and the membership matrix is:

$$U(f_1) = \begin{pmatrix} 0 & 0.8 & 0.2 & 0 \\ 0.8 & 0.2 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \end{pmatrix}$$

According to the $j$-th row $j$ ($j = 1, 2, 3$) of $U(f_1)$, we can calculate the distinguishable weight of $f_{1j}$ using Equations (3)-(5), and then the distinguishable weight vector can be obtained:

$$\alpha(f_1) = (0.348 \ 0.348 \ 0.304)$$

Step 2: Determining the importance weights. In Table 2, the importance weight vector of $f_{11}, f_{12}, f_{13}$ with respect to $f_1$ is given:

$$\beta(f_1) = (0.38 \ 0.22 \ 0.40)$$

Step 3: Calculating the comparable values. According to $\alpha(f_1)$ and $\beta(f_1)$, we can get the $k$-th comparable value of $f_{1j}$ ($j = 1, 2, 3$) using Equations (7) and (8), and then obtain
Table 2. The factors’ importance weights and membership vectors of each path section’s factor scenario

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Factors</th>
<th>Path sections and their membership vectors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1-2</td>
</tr>
<tr>
<td>ES1/ES2</td>
<td>$f_1$</td>
<td>0.080,0.100</td>
</tr>
<tr>
<td></td>
<td>$f_2$</td>
<td>0.080,0.100</td>
</tr>
<tr>
<td></td>
<td>$f_3$</td>
<td>0.300,0.300</td>
</tr>
<tr>
<td></td>
<td>$f_4$</td>
<td>0.300,0.500</td>
</tr>
<tr>
<td></td>
<td>$f_5$</td>
<td>0.300,0.300</td>
</tr>
<tr>
<td></td>
<td>$f_6$</td>
<td>0.200,0.200</td>
</tr>
</tbody>
</table>

the comparable value matrix $N(f_1)$ with respect to $f_1$:

$$N(f_1) = \begin{pmatrix}
0 & 0.106 & 0.026 & 0 \\
0.061 & 0.015 & 0 & 0 \\
0.037 & 0.085 & 0 & 0
\end{pmatrix}$$

**Step 4: Calculating the comparable sum.** According to $N(f_1)$ and Equation (9), we can calculate the $k$-th comparable sum with respect to $f_1$ and obtain the comparable sum vector:

$$M(f_1) = (0.098 \ 0.206 \ 0.026 \ 0)$$

**Step 5: Calculating the membership vector.** According to $M(f_1)$ and Equation (10), we calculate the membership vector $\mu(f_1)$ with respect to $f_1$:

$$\mu(f_1) = (0.296 \ 0.624 \ 0.080 \ 0)$$

**Step 6: Calculating single-attribute satisfaction degree.** Assign the four evaluation levels: very good, good, general and bad with values 0.875, 0.625, 0.375 and 0.125 respectively, then we can calculate the single-attribute satisfaction degree with respect to $f_1$:

$$s(f_1) = (0.296 \ 0.624 \ 0.080 \ 0) \ast (0.875 \ 0.625 \ 0.375 \ 0.125)^T = 0.679$$

In the same process, we can use the M(1,2,3) algorithm to calculate all the attributes’ satisfaction degrees of each path section under the two emergency scenarios, whose results are shown as Table 3. After getting the attributes’ satisfaction degrees of each path section, we can find the best and worst values vectors: $[0.781, 0.739, 0.821, 0.808]$ and $[0.456, 0.404, 0.576, 0.375]$, and then calculate the satisfaction degrees of all the path sections in the ideal path, as shown in the second column of Table 3.
Table 3. Single-attributes’ satisfaction degrees of each path sections under various scenarios

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Path sections</th>
<th>$\rho_{max}^k$</th>
<th>$ES_1/IS$</th>
<th>$ES_2/IS$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$p_1/(f_1, f_2, f_3, f_4)$</td>
<td>$p_2/(f_1, f_2, f_3, f_4)$</td>
<td>$p_3/(f_1, f_2, f_3, f_4)$</td>
</tr>
<tr>
<td>$M(\bullet, +)$</td>
<td>1-2</td>
<td>0.619</td>
<td>0.2 / (0.680, 0.604, 0.753, 0.636)</td>
<td>0.8 / (0.530, 0.496, 0.684, 0.525)</td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td>0.628</td>
<td>0.4 / (0.602, 0.641, 0.705, 0.700)</td>
<td>0.6 / (0.487, 0.447, 0.620, 0.518)</td>
</tr>
<tr>
<td></td>
<td>2-3</td>
<td>0.789</td>
<td>0.3 / (0.744, 0.740, 0.704, 0.703)</td>
<td>0.7 / (0.541, 0.471, 0.573, 0.514)</td>
</tr>
<tr>
<td></td>
<td>2-4</td>
<td>0.449</td>
<td>0.5 / (0.613, 0.710, 0.648, 0.525)</td>
<td>0.5 / (0.494, 0.400, 0.579, 0.375)</td>
</tr>
<tr>
<td></td>
<td>3-4</td>
<td>0.617</td>
<td>0.7 / (0.675, 0.574, 0.689, 0.711)</td>
<td>0.3 / (0.461, 0.446, 0.574, 0.457)</td>
</tr>
<tr>
<td></td>
<td>3-5</td>
<td>0.695</td>
<td>0.1 / (0.675, 0.670, 0.803, 0.621)</td>
<td>0.9 / (0.511, 0.471, 0.752, 0.503)</td>
</tr>
<tr>
<td></td>
<td>4-5</td>
<td>0.895</td>
<td>0.6 / (0.747, 0.677, 0.807, 0.804)</td>
<td>0.4 / (0.666, 0.543, 0.752, 0.775)</td>
</tr>
<tr>
<td>$M(1,2,3)$</td>
<td>1-2</td>
<td>0.636</td>
<td>0.2 / (0.679, 0.604, 0.759, 0.637)</td>
<td>0.8 / (0.552, 0.463, 0.701, 0.615)</td>
</tr>
<tr>
<td></td>
<td>1-3</td>
<td>0.629</td>
<td>0.4 / (0.604, 0.662, 0.698, 0.700)</td>
<td>0.6 / (0.482, 0.452, 0.624, 0.518)</td>
</tr>
<tr>
<td></td>
<td>2-3</td>
<td>0.790</td>
<td>0.3 / (0.781, 0.739, 0.703, 0.728)</td>
<td>0.7 / (0.559, 0.478, 0.576, 0.457)</td>
</tr>
<tr>
<td></td>
<td>2-4</td>
<td>0.457</td>
<td>0.5 / (0.614, 0.714, 0.651, 0.525)</td>
<td>0.5 / (0.505, 0.404, 0.582, 0.375)</td>
</tr>
<tr>
<td></td>
<td>3-4</td>
<td>0.622</td>
<td>0.7 / (0.682, 0.556, 0.700, 0.715)</td>
<td>0.3 / (0.456, 0.435, 0.596, 0.399)</td>
</tr>
<tr>
<td></td>
<td>3-5</td>
<td>0.707</td>
<td>0.1 / (0.667, 0.694, 0.807, 0.650)</td>
<td>0.9 / (0.503, 0.433, 0.751, 0.533)</td>
</tr>
<tr>
<td></td>
<td>4-5</td>
<td>0.894</td>
<td>0.6 / (0.749, 0.674, 0.821, 0.808)</td>
<td>0.4 / (0.674, 0.565, 0.761, 0.775)</td>
</tr>
</tbody>
</table>

Meanwhile, we also give the results produced by one existing membership transformation method $M(\bullet, +)$ [12], as shown in Table 3. Seen from the compared results, we can find that the results of the $M(1,2,3)$ algorithm are in accord with those of the $M(\bullet, +)$ algorithm, and most the satisfaction degrees of path sections produced by the $M(1,2,3)$ algorithm are higher than those by the $M(\bullet, +)$ algorithm, which test the effectiveness and advantage of the $M(1,2,3)$ algorithm.

4.2. Solving the scenario-based optimization model. We integrate depth-first search and Monte Carlo simulation to design a hybrid algorithm to solve the built scenario-based optimization model for emergency path selection in Subsection 3.2. The main steps of this hybrid algorithm are as follows.

Step 1: Finding all the alternative paths. According to the topology of the emergency transportation network, the depth-first search algorithm is applied to finding all the paths that can connect the starting point with the end point in the transportation network.

Step 2: Simulating the scenarios of path sections. In this step, we randomly select one path from the alternative paths and apply the Monte Carlo to simulating all the scenarios of path sections in this path in terms of the given probabilities of occurrence $p(es_m)$ and $p(is_m)$.

Step 3: Calculating the overall satisfaction degree. According to the simulated results in Step 2 and a given decision-making confidence level $\sigma$, we calculate the overall satisfaction degree of this selected path under various scenarios, that is, $\theta_{S/m}$.

Step 4: Getting the path with maximized satisfaction degree. We repeat Steps 2 and 3 until all the overall satisfaction degrees of alternative paths are attained. Then, we sort the alternative paths in terms of their overall satisfaction degrees and finally get the path with maximized overall satisfaction degree in the given confidence level $\sigma$.

Assume that the decision-making confidence levels $\sigma$ take 0.9, 0.7 and 0.5 respectively. Based on the results in Table 3, we can use the above hybrid algorithm to get the paths with maximized overall satisfaction degrees in these confidence levels and their corresponding overall satisfaction degrees $\bar{\theta}s$, shown as Table 4.

Seen from the results in Table 4 and Figures 4-6, the developed approach in this study can select the paths with maximized overall satisfaction degrees which are not in accord with that selected by the conventional shortest path principle, and different confidence
Table 4. Path ranking as confidence levels and overall satisfaction degrees

<table>
<thead>
<tr>
<th>Results by the conventional shortest approach</th>
<th>1-3-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results based on $M(\bullet, +)$ algorithm</td>
<td></td>
</tr>
<tr>
<td>$\sigma = 0.9$</td>
<td>$\tilde{\theta}$ $\sigma = 0.7$ $\tilde{\theta}$ $\sigma = 0.5$ $\tilde{\theta}$</td>
</tr>
<tr>
<td>1-2-3-5</td>
<td>0.637</td>
</tr>
<tr>
<td>1-3-5</td>
<td>0.615</td>
</tr>
<tr>
<td>1-2-4-5</td>
<td>0.537</td>
</tr>
<tr>
<td>1-3-4-5</td>
<td>0.504</td>
</tr>
<tr>
<td>1-2-3-4-5</td>
<td>0.396</td>
</tr>
</tbody>
</table>

| Results based on $M(1,2,3)$ algorithm |       |
| $\sigma = 0.9$                              | $\tilde{\theta}$ $\sigma = 0.7$ $\tilde{\theta}$ $\sigma = 0.5$ $\tilde{\theta}$ |
| 1-3-5                                       | 0.668 | 1-3-4-5 | 0.817 | 1-2-4-5 | 0.904 |
| 1-2-3-5                                     | 0.654 | 1-3-5 | 0.760 | 1-3-4-5 | 0.899 |
| 1-2-4-5                                     | 0.551 | 1-2-3-5 | 0.741 | 1-3-5 | 0.871 |
| 1-2-3-4-5                                   | 0.489 | 1-2-3-4-5 | 0.698 | 1-2-3-4-5 | 0.701 |
| 1-3-4-5                                     | 0.407 | 1-2-4-5 | 0.647 | 1-2-3-5 | 0.647 |

Figure 4. The selected path by the conventional shortest path principle

Figure 5. The selection paths by the developed approach based on $M(\bullet, +)$ with different confidence levels

levels produce different optimal transportation paths. For example, if the confidence level of the $M(1,2,3)$ based approach is taken as 0.9, path 1-3-5 is the optimal; if $\sigma$ is taken as
The selection paths by the developed approach based on M(1,2,3) with different confidence levels

(a) $\sigma = 0.9$

(b) $\sigma = 0.7$

(c) $\sigma = 0.5$

**Figure 6.** The selection paths by the developed approach based on M(1,2,3) with different confidence levels

0.7, path 1-3-4-5 is the optimal. Decision-makers can select the most satisfactory paths under various scenarios according to their expected confidence levels.

Meanwhile, the overall satisfaction degrees will increase as the confidence levels decrease, which implies that decision-makers may select the paths with low confidence levels but high satisfaction degrees if they can bear bigger risks. The developed approach can consider not only the distance between the starting point and the end point but also other key attributes of each path section in the transportation networks which often have important impacts on decision-makers in emergency response. However, how to integrate the values of multiple attributes is a key issue where redundant and noisy information is often mixed. The classical membership transformation algorithms such as the $M(\bullet,+)$ do not recognize the redundant information, which may produce disturbed results. For example, in Figures 5 and 6, the $M(\bullet,+)$ algorithm mistakenly selects the path 1-2-3-5 when $\sigma$ is taken as 0.9.

5. **Conclusions.** The occurrence and development of disasters are always uncertain, which directly influences the efficiency and effectiveness of emergency response. The emergency path selection, which involves various uncertain factors, is not a conventional shortest path problem. Scenario analysis approach provides a good technique for dealing with the uncertainties of emergency transportation networks in response to disasters.

Different from the conventional approaches, we construct scenarios by multiple attitudes and factors to describe the uncertainties of path sections, which can avoid the disadvantages of conventional path selection approaches like the shortest path methods. Moreover, we build a scenario-based optimization model for emergency path selection whose aim is to select the path with maximized overall satisfaction degree in a given confidence. Experimental results show the effectiveness of the developed approach, concluding that this approach can provide emergency decision-makers the most satisfactory paths under various uncertain scenarios according to their expected confidence levels and the overall satisfaction degree will increase as the given confidence decreases.

In this study we provide a novel approach for selecting transportation paths in emergency response. However, we do not consider the development of emergency scenarios here. Meanwhile, the original evaluation values on the attributes of path sections have direct impacts on results, which we also do not pay enough attention to. Further studies are needed to focus on these directions.
Acknowledgment. We gratefully acknowledge the anonymous reviewers for their constructive comments on the manuscript. This research is supported by National Natural Science Foundation of China (Nos. 90924006 and 71171029) and the central university basic scientific research expenses.

REFERENCES