MODELING AND OPTIMIZATION OF CLOSED-LOOP SUPPLY CHAIN CONSIDERING ORDER OR NEXT ARRIVAL OF GOODS

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ABSTRACT. This paper aims to develop a closed loop supply chain model that integrates facilities for both forward and reverse logistics. In forward logistics, when the number of products cannot meet product demand, another production batch must be produced to address the insufficient supply. In reverse logistics, however, where the timeframe for recovery and the amount of recovered materials is uncertain, it is possible to wait indefinitely for the next batch production. Since these uncertainties of reverse logistics are very inefficient, this study proposes a reverse logistics optimization method to minimize the order volume or the next arrival of goods using a system called Just-in-Time delivery in the reverse logistics. Through optimization algorithms using the priority-based genetic algorithm and the modified hybrid genetic algorithm, optimal costs are determined. Based on the case study of a bottle distilling and sales company in Busan, Korea, a modified model for reusable reverse logistics of empty bottles is developed, and the effectiveness of the proposed method is verified.

Keywords: Closed-loop supply chain, Order or next arrival of goods, Modified hybrid genetic algorithm (mHGA)

1. Introduction. There are two major types of supply chains within any production and distribution system: forward and reverse supply chains (also called forward and reverse logistics) [1]. Reverse logistics are different from traditional forward logistics, in which new materials or parts are produced and sold to customers. In reverse logistics, it is difficult not only to predict when and how many products will arrive at the manufacturer due to unpredictable periods of consumer usage and varying conditions of recovered products, but in addition, recovery routes are complex because there are numerous recovery centers. Moreover, even though recovered products are environmentally friendly, the market for recovered products is currently not large because of stereotypes by customers who regard recovered products as used goods. Furthermore, both the construction and operation of reverse logistics systems cost more than those of traditional forward logistics.

However, due to increasing environmental regulations, reverse logistics have been bolstered based on several factors. First, positive economic effects have resulted from the reduction of costs for raw materials in manufacturing processes. Second, consumer propensity has shifted to environmentally friendly products. Third, business strategies have deliberately aimed to improve the public image of big corporations, in part by adopting environmentally responsible policies and processes. Accordingly, establishment of appropriate reverse logistics systems can contribute to reduced inventory carrying costs, transportation costs, and waste disposal costs, as well as increased potential sales due to increased customer satisfaction. In addition, the importance of designing an optimal
closed-loop supply chain model has gradually increased as interest in supply chain management has expanded toward integrating existing methods of forward logistics, consumer product supply, and reverse logistics for the handling and recovery of returns. The increased interest in integration is due to general increased interest in the environment, customer service, and recycling on the part of corporations [2].

In forward logistics, when the number of products cannot meet demand, the insufficient supply is replenished after the next batch production. In reverse logistics, however, where the timeframe for recovery and the amount of recovered materials are uncertain, it is possible to wait indefinitely for the next production of goods. Many researchers proposed the integrated model with forward and reverse logistics that considered the saving of costs from integrating retailer/return center. However, the model in consideration of uncertainty of amount or occurrence time of the recovery product was not proposed in the previous paper.

Therefore, in order to optimize reverse logistics despite the uncertainties about the amount and arrival timing of recovered products, it is necessary to construct a model that considers not only transportation costs, but also dates and processing decisions. Specifically, processing decisions are about whether it is more efficient and beneficial for the supply chain to wait for the arrival of unknown amounts of end-of-life products to return centers or rather to order the necessary parts for manufacture.

Accordingly, this paper proposes a model of a closed-loop supply chain that integrates forward and reverse logistics. When output cannot meet demand in forward logistics, the next arrival of goods is determined to maintain efficiency. Further, when reverse logistics output cannot meet demand, either a decision to order or the next arrival of goods must be implemented. Importantly, this paper considers a strategy to reduce the open costs of multiple facilities through the integration of retailers and return centers in a multiple planning horizon, without changing the maximum facility capacities.

This paper is organized as follows. We conducted literature review in Section 2. In Section 3, we define problems with mathematical formulation, assumptions, and notations of our model. In Section 4, we describe the priority-based genetic algorithm, and the modified hybrid genetic algorithm. Results of simulation for numerical examples and case study are discussed in Section 5. Finally, we discuss the conclusions in the final section.

2. Literature Review.

2.1. Integrated forward and reverse supply chains. A major issue in reverse distribution is the integration of forward and reverse supply chains. Captured information about returns should be integrated with forward supply chain information to achieve optimum planning and cost reduction. Subsequently, entire support networks can be designed in such a way to efficiently service both forward and reverse logistics processes [3]. A pivotal study by Fleischmann et al. [4] proposed the closed-loop supply chain network, which integrates reverse logistics into existing forward logistics supply chains and performs functions of distribution and recovery. Later, Fleischmann et al. [5] revealed a design problem with the reverse logistics network in the closed-loop supply chain (CLSC). The study considered both forward and reverse flows, introducing a model network for the simultaneous optimization of distribution and recovery. An MILP formulation has been proposed that constitutes an extension of the traditional problem of warehouse location. Extending the model of Fleischmann et al. [5], Salema et al. [6] proposed a generalized model for the design of reverse logistics. When suspending the logistics between dismantlers and plants, however, the models of both Fleischmann et al. [5] and Salema et al. [6] do not consider the supplier, and they do not include relationships between forward and reverse flows. Ko
and Evans [7] have proposed theoretical research that simultaneously optimizes forward and reverse networks. In this research, they have integrated forward and reverse networks into a mixed integer nonlinear model and have proposed heuristic methods based on genetic algorithms. Lu and Bostel [8] have defined the problem of forward and reverse logistic integration for remanufacturing facilities and propose the Lagrangian relaxation-based algorithm. A study by Min et al. [9] presented a nonlinear integer program that sorts the multi-echelon, multi-commodity closed-loop network design problem involving product returns. The models therein, however, do not consider temporal consolidation issues in a multiple planning horizon. A study by Min et al. [10] proposed a mixed-integer nonlinear programming model and a genetic algorithm for its solution that could potentially solve reverse logistics problems involving the consolidation of returned products in a CLSC. Shu et al. [11] presented a multi-objective linear programming model to optimize operations of green supply chains with the integration of forward and reverse logistics, including decisions pertaining to shipment and inventory. Factors such as used-product return ratios of raw materials and corresponding subsidies from government organizations for reverse logistics was considered in the model formulation. The authors also proposed a real world case study regarding a notebook computer manufacturer based in Taiwan. Lee and Lee [12] proposed the integrated model with forward and reverse logistics that considered the saving of costs from integrating retailer/returning center in a multiple planning horizon. They determined the optimal delivery routes, and discuss its results on open and integrated facilities through a simulation. Based on the case study of a distilling and sale company in Busan, Korea, the new model of closed-loop supply chain of bottles was built and the effectiveness of the proposed method was verified.

2.2. Inventory management. Inventory levels are important variables for decisions in real-life applications due to their strong influence on the efficiency and responsiveness of logistics networks. Inventory management with the element of product recovery differs from traditional inventory management in essentially two aspects. First, product returns represent an exogenous inbound material flow, causing a loss of monotonicity in stock levels of serviceable products that serve customer demands. Second, two alternative supply sources are available for replenishing the serviceable inventory. One source is to procure externally or to produce internally, while the other source is remanufacturing activity [13]. A current study by Chung et al. [14] analyzed a closed-loop supply chain inventory system. In addition to traditional forward material flows, the model examined the return of used products to a reconditioning facility where they are stored, remanufactured, and finally shipped back to retailers for retail sale. A study by Yang et al. [15] analyzed the utilization of sequential and global optimization considering a closed-loop supply chain inventory system with multiple manufacturing cycles and remanufacturing cycles.

3. Problem Definition.

3.1. Model description. This paper focuses on a single-product closed-loop supply chain which includes the following discrete operations: supply, manufacture, distribution, sales, returns, and disposal. In the forward logistics model, products from the manufacturer are delivered to the retailer through distribution centers (DCs). In current systems, it is standard to wait for the next arrival of goods when the amount of product delivered is less than the demand for the product; conversely, a surplus level of inventory is experienced when the amount of product delivered exceeds the demand. The reverse logistics model for reusable recovered products is considered the Just-in-Time (JIT) model. The model takes into account delivery costs from return centers to the manufacturer when deciding whether it is more beneficial to wait for the arrival of end-of-life products or to
order the necessary parts for manufacture when the amount of end-of-life products at the
return centers by customers is less than the manufacturer demand.

A closed-loop supply chain that integrates forward and reverse logistics is proposed. Importantly, the model considers a strategy to reduce the open costs of multiple facilities through the integration of retailers and return centers in a multiple planning horizon, without changing the maximum facility capacities. To construct the closed-loop supply chain model, $I$ manufacturers, $J$ distribution centers (DCs), $K$ retailers, $L$ customers, $K^3$ return centers, and one disposal center are assumed.

Figure 1 illustrates the forward and reverse flows of the model, including cost savings achieved by integrating retailers and return centers.

3.2. Notations. The parameters, decision variables, objective functions, and restrictions in this closed-loop supply chain model are as follows:

- **Indices**
  - $i$ quad index of manufacturer ($i = 1, 2, \ldots, I$)
  - $j$ index of distribution center ($j = 1, 2, \ldots, J$)
  - $k$ index of retailer ($k = 1, 2, \ldots, K$)
  - $l$ index of customer ($l = 1, 2, \ldots, L$)
  - $k^3$ index of return center ($k^3 = 1, 2, \ldots, K^3$)
  - $t$ index of time period ($t = 1, 2, \ldots, T$)

- **Parameters**
  - $I$ number of manufacturers
  - $J$ number of distribution centers
  - $K$ number of retailers
  - $L$ number of customers
  - $K^3$ number of return centers
  - $N$ disposal center
  - $S$ supplier
  - $T$ planning horizons
\[ a_i \text{ capacity of manufacturer } i \]
\[ b_j \text{ capacity of distribution center } j \]
\[ u_k \text{ capacity of retailer } k \]
\[ u_{kt} \text{ capacity of return center } k^t \]
\[ d_i(t) \text{ demand of manufacturer } i \]
\[ c_{ij}^1 \text{ unit cost of transportation from manufacturer } i \text{ to distribution center } j \]
\[ c_{jk}^2 \text{ unit cost of transportation from distribution center } j \text{ to retailer } k \]
\[ c_{kt}^3 \text{ unit cost of transportation from return center } k^t \text{ to manufacturer } i \]
\[ c_{kl}^4 \text{ unit cost of transportation from return center } k^l \text{ to customer } l \]
\[ c_{SN}^9 \text{ unit cost of transportation from return center } k^S \text{ to disposal center } N \]
\[ c_{j}^0 \text{ open cost of distribution center } j \]
\[ c_{k}^0 \text{ open cost of retailer } k \]
\[ c_{k^t}^0 \text{ open cost of return center } k^t \]
\[ c_{j}^{H} \text{ unit holding cost of inventory per period at distribution center } j \]
\[ c_{k}^{H} \text{ unit holding cost of inventory per period at retailer } k \]
\[ c_{k^t}^{H} \text{ unit holding cost of inventory per period at return center } k^t \]
\[ r_N \text{ disposal rate} \]
\[ r_k(t) \text{ the amount of end-of-life product recovered at return center } k \]
\[ w_i \text{ the order upper limit for manufacturer } i \]

- Decision variables
  \[ x_{ij}^1(t) \text{ amount shipped from manufacturer } i \text{ to distribution center } j \text{ in period } t \]
  \[ x_{jk}^2(t) \text{ amount shipped from distribution center } j \text{ to retailer } k \text{ in period } t \]
  \[ x_{kl}^3(t) \text{ amount shipped from retailer } k \text{ to customer } l \text{ in period } t \]
  \[ x_{k^t}^4(t) \text{ amount shipped from return center } k^t \text{ to manufacturer } i \text{ in period } t \]
  \[ x_{SN}^9(t) \text{ the order amount of manufacturer } i \text{ in period } t \]
  \[ y_{j}^1(t) \text{ inventory amount at distribution center } j \text{ in period } t \]
  \[ y_{k}^2(t) \text{ inventory amount at retailer } k \text{ in period } t \]
  \[ y_{k^t}^3(t) \text{ inventory amount at return center } k^t \text{ in period } t \]
  \[ w_i(t) \text{ binary variable, equals 1 when the demand at manufacturer } i \text{ is met and 0 when order is necessary} \]
  \[ z_{j}^1(t) \text{ binary variable, equals 1 when distribution center } j \text{ is open, otherwise 0} \]
  \[ z_{k}^2(t) \text{ binary variable, equals 1 when retailer } k \text{ is open, otherwise 0} \]
  \[ z_{k^t}^3(t) \text{ binary variable, equals 1 when return center } k^t \text{ is open, otherwise 0} \]

3.3. Mathematical formulation.

3.3.1. Objective function. The objective function is to minimize the transportation costs of forward/reverse flows and open costs according to the integration of retailer/return centers and the order costs or inventory costs, using JIT delivery according to the waiting period for the next arrival of goods according to the following equation

\[
 f = \alpha_1 f_1 + \alpha_2 f_2 \\
 = \alpha_1 \sum_{t=1}^{T} \left( \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij}^1 x_{ij}^1(t) + \sum_{j=1}^{J} \sum_{k=1}^{K} c_{jk}^2 x_{jk}^2(t) + \sum_{k=1}^{K} \sum_{l=1}^{L} c_{kl}^3 x_{kl}^3(t) + \sum_{k^t=1}^{K^t} \sum_{t=1}^{T} c_{k^t}^4 x_{k^t}^4(t) + \sum_{j=1}^{J} \sum_{k^t=1}^{K^t} c_{j}^0 z_{j}^1(t) + \sum_{k=1}^{K} \sum_{k^t=1}^{K^t} c_{k}^0 z_{k}^2(t) + \sum_{k^t=1}^{K^t} c_{k^t}^0 z_{k^t}(1 - z_{k}(t)) \right)
\]
\[
+ \alpha_2 \sum_{t=1}^{T} \left[ \sum_{j=1}^{J} \sum_{k=1}^{K} c_{j}^{H} y_{j}^{H}(t) + \sum_{k=1}^{K} \sum_{l=1}^{L} c_{k}^{H} y_{k}^{H}(t) \right. \\
+ \sum_{i=1}^{I} c_{Si} x_{Si}(t) + \sum_{k'=1}^{K'} \sum_{i=1}^{I} c_{k'}^{H} y_{k'}^{H}(t) + \sum_{k'=1}^{K'} c_{k'N} x_{k'N}(t) \left] \right. \\
\rightarrow \min
\]

\begin{align*}
\alpha_1, \alpha_2 & : \text{weight} \\
\text{where the objective function is to minimize total logistics costs, including the amount of} \\
dependent forward and reverse transportation costs (the first, second, third and fourth terms in the equation), the open costs of DCs, retailers and return centers (the fifth, sixth and seventh terms), inventory and order costs (the eighth, ninth, tenth and 11th terms), \\
and disposal costs (the final term). Further, the amount recovered from all return centers \\
is totaled when deciding to wait for the next arrival of goods or to order. \\
\end{align*}

3.3.2. Constraints.
- Open costs:

\begin{align*}
1 - z_{j}(t - 1) &= z_{j}(t) \quad \forall j \in J, t \in T \\
1 - z_{k}(t - 1) &= z_{k}(t) \quad \forall k \in K, t \in T \\
1 - z_{k'}(t - 1) &= z_{k'}(t) \quad \forall k' \in K', t \in T
\end{align*}

- Inventories of DCs, retailers, and return centers:

\begin{align*}
y_{j}(t) &= \sum_{i=1}^{I} x_{ij}^{1}(t) + y_{j}(t - 1) \quad \forall j \in J, t \in T \\
y_{k}(t) + y_{k'}(t) &= \sum_{j=1}^{J} x_{jk}(t) + y_{k}(t - 1) + \sum_{k=1}^{K} x_{k}^{2}(t) - (x_{kN}(t) + x_{k'N}(t)) + y_{k}(t - 1) \quad \forall k \in K, k' \in K', t \in T
\end{align*}

- Capacity constraints

\begin{align*}
\sum_{j=1}^{J} x_{ij}^{1}(t) &\leq a_{i} \quad \forall i \in I, t \in T \\
\sum_{k=1}^{K} x_{jk}^{2}(t) + y_{j}(t - 1) &\leq b_{j} z_{j}(t) \quad \forall j \in J, t \in T
\end{align*}

- Recovered amount of end-of-life products

\begin{align*}
\sum_{l=1}^{L} x_{kl}(t) + y_{k}(t - 1) + \sum_{k'=1}^{I} x_{k'N}(t) + y_{k'}(t - 1) &\leq u_{k} z_{k}(t) + u_{k'} z_{k'}(t) \quad \forall k \in K, k' \in K', t \in T
\end{align*}

- Demand constraints

\begin{align*}
x_{kN}(t) + x_{Si}(t) + \sum_{i=1}^{I} x_{ij}^{1}(t) &= d_{i}(t) \quad \forall i \in I, k' \in K', t \in T
\end{align*}

- Restriction: not able to wait for the arrival of goods and relapse note at the same time in return center

\begin{align*}
x_{Si}(t) &\leq u_{i}(1 - w_{i}(t)) \quad \forall i \in I, t \in T
\end{align*}
4. Optimization of the Closed-Loop Supply Chain with the Genetic Algorithm.

The genetic algorithm (GA), which is one type of multi-point searching method, is known to be effective. The chromosome representation design is one of the critical factors in the optimization approach using the genetic algorithm. In this paper, the priority-based genetic algorithm (priGA) proposed by Gen et al. [16] is used to design the chromosome. The approach for optimization of the closed-loop supply chain according to priority-based genetic representation is adopted.

4.1. Priority based genetic representation.

4.1.1. Encoding method. A chromosome with a length that is determined by plant (I) and DC (J) is generated. The value of each gene represents the priority, and an initial value allocated by the priority starts from the sum total of genes, (I + J). Then, the priority with a value of one less than that of the randomly selected gene is allocated, until every gene has a priority value. Figure 2 is an example of two chromosomes with plants I = 5 and DCs J = 3, in which the capacities of the plants and DCs, as well as the associated transportation costs, are shown.

4.1.2. Decoding method. In the case of the first chromosome, the lowest plant cost for the DC with the highest priority is DC 1. The shipments (400) are determined based on maximum deliveries (700, 400). Since the maximum capacity of 400 units is satisfied in DC 1, the priority is reset to 0, and plant 1 with the second highest priority is selected. Moreover, DC 4, with the second lowest transportation costs, is selected for plant 3. Shipments (450) are decided by maximum deliveries (580, 450). Table 1 shows the trace table of the calculation process.

4.2. Optimization according to the priGA (priority-based genetic algorithm). Priority-based genetic representation adopted for chromosome representation is used to show the gene position nodes, and the value is used to show the node priority. Figure 3 shows the priGA procedure.
Figure 2. Example representation of a chromosome in a genetic algorithm

<table>
<thead>
<tr>
<th>iteration</th>
<th>$v(ij)$</th>
<th>$a$</th>
<th>$b$</th>
<th>$l$</th>
<th>$j$</th>
<th>$x_{ij}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[2 5 3</td>
<td>7 4 1 6]</td>
<td>[700 450 580]</td>
<td>[400 530 350 450]</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>[2 5 3</td>
<td>0 4 1 6]</td>
<td>[300 450 580]</td>
<td>[0 530 350 450]</td>
<td>3</td>
<td>4</td>
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<tr>
<td>2</td>
<td>[2 5 3</td>
<td>0 4 1 0]</td>
<td>[300 450 130]</td>
<td>[0 530 350 0]</td>
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4.3. Optimization according to the mhGA (modified hybrid genetic algorithm). Genetic strategy parameters in genetic algorithms are generally fixed, and evolution is processed based on value. However, over time, this method could result in the loss of variety regarding evolving processes due to the consistent repetition of crossovers and mutations with the same probability. This study predicts improvement to solutions, as well as the reduction of calculation time for simulation, if GA parameters are adjusted according to the unique group situation. Increasing the mutation rate, a parameter of genetic operators, expands search space. Conversely, decreasing the mutation rate improves the accuracy of local searches. Search speed and solution accuracy can be improved by appropriately adjusting the mutation rate. In addition to the function of priGA, mhGA improves the search ability of GA by appropriately adjusting the parameters in each generation using a fuzzy logic controller (FLC) [17] and creating a suitable situation through optimal solution searches. A forgetting factor is newly adopted into the increase-decrease amount in the updating equation of crossover and mutation rates.

The FLC operates as follows:
Step 1. Changes between the average fitness of the current generation and that of the previous generation are obtained using the following equation.

\[
\Delta f_{\text{avg}}(t) = \overline{f}_{\text{avg}}(t) - \overline{f}_{\text{avg}}(t-1)
\]

Step 2. The control operation corresponding to the average fitness of the current generation and the average fitness of the previous generation (both increases and decreases) are determined based on Table 2.

1. If \( \varepsilon \leq \Delta f_{\text{avg}}(t-1) \leq \gamma \) and \( \varepsilon \leq \Delta f_{\text{avg}}(t) \leq \gamma \), then increase \( p_C \) and \( p_M \) for next generation.

2. If \( -\gamma \leq \Delta f_{\text{avg}}(t-1) \leq -\varepsilon \) and \( -\gamma \leq \Delta f_{\text{avg}}(t) \leq -\varepsilon \), then decrease \( p_C \) and \( p_M \) for next generation.

3. If \( -\varepsilon \leq \Delta f_{\text{avg}}(t-1) \leq \varepsilon \) and \( -\varepsilon \leq \Delta f_{\text{avg}}(t) \leq \varepsilon \), then rapidly increase \( p_C \) and \( p_M \) for next generation.

\( \varepsilon \): a given real number in the proximity of zero

\( \gamma \): a given maximum value of a fuzzy membership function

\(-\gamma\): a given minimum value of a fuzzy membership function

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**Figure 3.** Procedure of the priority-based genetic algorithm

**Table 2.** Fuzzy decision table

<table>
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<th>( \Delta f_{\text{avg}}(t) )</th>
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Table 3. Look-up table

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**Figure 4.** The hybrid genetic algorithm procedure

Step 3. Changes in crossover and mutation rates by the forgetting factor are calculated.
\[
\Delta p_C(t) = r_1 \cdot z(i, j)
\]
\[
\Delta p_M(t) = r_2 \cdot z(i, j)
\]
\( r_1, r_2 \): forgetting factor

Step 4. Crossover and mutation rates of the next generation are updated using the following respective equations.
\[
p_C(t + 1) = p_C(t) + \Delta p_C(t)
\]
\[
p_M(t + 1) = p_M(t) + \Delta p_M(t)
\]

Figure 4 shows the mhGA procedure.

4.4. **Genetic operators.** For genetic operators, weight mapping crossovers (WMX) and exchange mutations, which more easily avoid local solutions through the combination of
various crossovers and mutations, are used. The cutting point is randomly determined using WMX and generates offspring by exchanging the applicable parts of the chromosomes after the cutting point. The exchanged parts are arranged in ascending order. Next, the numbers of genes that become matching pairs are identified and changed by relationship. Figure 5 is an example of a WMX. A swap mutation randomly selects pairs of genes that are exchanged. As shown in the example in Figure 6, gene pairs 2 and 5 are selected and exchanged.

5. Simulation. To evaluate the effectiveness of the proposed model in a closed-loop supply chain, numerical examples and a case study from a bottle distilling and sales company were simulated.

5.1. Numerical examples. Figure 7 represents problem 1, showing two DCs, five retailers, ten customers, six return centers, one manufacturer, one disposal center, and one supplier. The relevant data for test problems, including transportation costs, inventory costs, open costs, purchase costs, disposal costs, manufacturer capacities, DCs, retailers, return centers, and disposal centers, were randomly generated to provide realistic scenarios.

The simulation conditions of the genetic algorithm are set to a population size of 20, a 0.7 initial WMX rate, a 0.3 initial mutation rate, and a maximum generation of 5000.
The crossover and mutation rates were determined based on the results of exploratory experiments. However, the crossover rate and the mutation rate of each discrete generation are adjusted because the FLC includes mhGA.

Total costs include transportation costs from the manufacturer to DCs, transportation costs from DCs to retailers, transportation costs from retailers to customers, transportation costs from customers to return centers, and transportation costs from return centers to manufacturers. Further costs include the open costs of DCs, retailers and return centers, as well as the inventory costs of DCs, retailers and return centers, the purchase costs of suppliers, and disposal costs. The optimum cost is determined by taking into account the reduction in open costs due to the integration of retailers and return centers. In addition, 0.2% of all end-of-life products delivered to return centers are disposed of, and the disposed value is rounded to the nearest whole number.

To evaluate effectiveness, the results of our proposed method are compared with solutions procured using the optimization software LINGO with five patterns including from 161 to 22,502 variables. Table 4 shows the minimum costs and computation time according to LINGO, the minimum costs and computation times according to priGA and hGA, and the relative error of costs where \( \text{Gap} = 100 \frac{(GA - LINGO)}{LINGO} \). The results of each method are compared in Figures 8 and 9.

In addition, the results of each method are compared in Figures 8 and 9. In Figure 8, the computation time of LINGO is shorter than mhGA in variable 161, and the computation time of LINGO increases rapidly compared with that of mhGA from variable 1032 onward. However, the minimum costs of LINGO and mhGA are almost the same.
## Table 4. Comparisons of LINGO, priGA and mhGA

<table>
<thead>
<tr>
<th>Problem no.</th>
<th>Number of var.</th>
<th>LINGO</th>
<th>mhGA</th>
<th>priGA</th>
<th>Gap (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time(s)</td>
<td>Cost</td>
<td>Time(s)</td>
<td>Cost</td>
<td>Time(s)</td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>470,823</td>
<td>0.12</td>
<td>470,823</td>
<td>0.12</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6,408,750</td>
<td>1.08</td>
<td>6,425,841</td>
<td>1.20</td>
</tr>
<tr>
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<td>86,516,547</td>
<td>2.92</td>
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<tr>
<td>4</td>
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<td>239,542,608</td>
<td>3.42</td>
<td>246,304,705</td>
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</tr>
<tr>
<td>5</td>
<td>N/A</td>
<td>N/A</td>
<td>6.24</td>
<td>986,886,324</td>
<td>8.62</td>
</tr>
</tbody>
</table>

![Graph showing comparison of computation times for LINGO, priGA and mhGA](image1)

**Figure 8.** Comparison of computation times for LINGO, priGA and mhGA

![Graph showing comparison of costs for LINGO, priGA and mhGA](image2)

**Figure 9.** Comparison of costs for LINGO, priGA and mhGA
As depicted in Figure 10, the convergence of mhGA is faster than priGA. Also, the simulation shows that the proposed mhGA is effective to find the best solution and to pick up the convergence speed.

5.2. Case study of the closed-loop supply chain in a bottle distilling and sales company. The empty bottles collected in retailers are delivered to a manufacturing plant through recovery centers of three kinds: shopping center, farming store and recycle dealer. The recovery center is assumed to 188 minimum administrative areas. Figure 11 shows the location of 188 retailers, 59 returning centers, and 1 plant of this company. The signs in Figure 11 show plant (P), returning center (RC), and retailer (R), respectively.
The problem of optimizing the closed-loop supply chain of a bottle distilling and sales company in Busan, Korea was simulated with a case study of the company’s real data. In the flow of forward logistics, bottles produced in the plant are delivered to two kinds of DCs (shopping centers and farm stores) and are sold through retailers (supermarkets). In the reverse flow, the empty bottles collected by retailers are delivered to three kinds of manufacturing plants through recovery centers (shopping centers, farm stores, and recycle dealers). After processing, the recovered materials are reused in a forward flow.

The type of bottle produced and collected is a Soju bottle, and the bottles are collected once daily. The integrated DC/recovery centers are located in a minimum of 188 administrative areas, although there are a total of 6,233 retailers (stores and supermarkets) in the city. On the other hand, it assumes the truckload quantity to be the same, 120 boxes per one truck and 30 empty bottles per one box to be loaded. In the closed-loop supply chain, bottles produced at plants are transported to retailers through DCs. At the retailers, products and end-of-life products are treated simultaneously.

The minimum costs of priGA and mhGA are 1,018,752,451 and 986,886,324, respectively. The total value of variables in this case study is 22,502, and it is impossible for the traditional optimization software LINGO to calculate solutions in a practical amount of time. However, priGA and mhGA are able to more quickly calculate solutions (CPU: Intel Pentium4 3.20 [GHz], RAM: 1 [GB]), in 8.62 [sec] and 6.24 [sec], respectively. Table 5 shows that closed-loop supply chain costs are reduced in integrated facilities that use both reverse and forward logistics.

6. Conclusions. We have extended the closed-loop supply chain model that describes not only the saving of costs from integrating retailer/return center, but also the decision of ordering or waiting for the next arrival of goods. The modified hybrid genetic algorithm (mhGA) proposes an improved search-ability of best solution, and the convergence speed. The sub-optimal costs and delivery routes are determined with the proposed mhGA. The effectiveness of the proposed method is verified with a case study of a bottle distilling and sales company in Busan, Korea.

This paper makes two contributions to the literature. First, the first adoption of the forgetting factor in GA is theoretically novel in this field. Second, the advanced performance of the proposed model (mgGA) is clarified in the updating equation of crossover and mutation rates with a practical case example. The proposed model can be applied to
reduce the costs of a closed-loop supply chain management with the integrated facilities for reverse and forward logistics.

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REFERENCES