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FUZZY SYNTHETIC EVALUATION MODEL BASED ON THE KNOWLEDGE SYSTEM

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Abstract. *Fuzzy measure, as a non-additive metric, is an effective tool for information fusion. Constructing a determining mechanism for fuzzy measure with operability is the key focus in academic and application fields. In this paper, we first take core samples as knowledge carriers on the decision information system. According to changes of knowledge carriers, we second give a knowledge-based metric of attributes importance. Third, combining with Choquet fuzzy integral, we establish a fuzzy synthetic evaluation model (denoted as BIS-FSEM) to deal with correlation indexes. And finally, we analyze the effectiveness of BIS-FSEM and give its implementation steps through a teaching quality evaluation case. Theoretical analysis and computation results show that BIS-FSEM has good operability and interpretability. It enriches the existing data-based metric theories of attributes importance to some extent. So it can be applied in many fields such as information fusion, synthetic evaluation and fuzzy decision-making.*

Keywords: Fuzzy synthetic evaluation, Fuzzy measures, Fuzzy integrals, Core samples set, Decision information system

1. **Introduction.** Synthetic evaluation is a multi-factor decision-making method by giving a comprehensive evaluation to object, and it is the core problem of resource management, complex system optimization and so on. Since the connotation and extension of the evaluation factors are often not precise and they correlate with each other, fuzzy synthetic evaluation is a common used evaluation method. Constructing a fuzzy synthetic evaluation model which can deal with the interaction is concerned widely in academic fields. Many scholars gave many discussions under different backgrounds and also obtained many important research results. Mi et al. [1] did an assessment of environment lodging stress for maize using fuzzy synthetic evaluation and provided a scientific basis for maize variety extension and recommendation and comprehensive management to reduce maize planting risk and loss. Xu et al. [2] proposed a fuzzy synthetic evaluation model aiming at risk assessment of PPP projects in China, which provided a deeper understanding of managing different types of PPP projects. Wang et al. [3] did a fuzzy synthetic evaluation of wetland soil quality degradation of northeast of China with a case study on the Sanjiang Plain. It is of scientific and practical significance for protection and management of soils and for sustainable development of agriculture. Ren et al. [4] provided a method of fuzzy synthetic evaluation of location plan of city distribution center. It

successfully eliminated the fuzziness and uncertainties in determining the location of city distribution center. Sheng et al. [5] gave a fuzzy synthetic evaluation on the quality of different mixed feeds for fattening lambs by using in vitro method. Zou et al. [6] did a risk assessment of concentrating solar power based on fuzzy synthetic evaluation. Yan et al. [7] did a research on the synthetic evaluation of business intelligence system based on BP neural network. Fukami et al. [8] did a quantitative evaluation of eye opening and closure with time variation in routine EEG examinations. Wen [9] put forward a new gray clustering and fuzzy synthetic evaluation method to evaluate the students' scores in a high school of Taiwan successfully. Zuo et al. [10] proposed an application of a hybrid method combining multilevel fuzzy synthetic evaluation with asymmetric fuzzy relation analysis to mapping prospectivity in western China.

At present, the commonly used fuzzy synthetic evaluation methods [11] include (i) the single factors determining type; (ii) the main factors determining type; (iii) the geometric average type; (iv) the weighted average type. However, it is worth noting that these methods are dependent on some weight systems. They are only suitable for some special evaluation problems and all have some shortcomings difficult to overcome. For example, methods (i) and (ii) cannot make good use of the relevant information and the evaluation results excessively depend on a few related values, so the distinguishing ability is lower; methods (iii) and (iv) consider the information of all indexes, but they are too dependent on the weight of each index. When the correlation between the indexes is strong, the evaluation result will distort seriously. AHP-based fuzzy synthetic evaluation considers the category characteristics of the index to a certain degree, but it cannot solve the correlation between indexes. This problem coexists in all fuzzy synthetic evaluations based on weight system. With the development of the fuzzy measure and fuzzy integrals theory, many scholars describe the interaction between the evaluation indexes by using the non-additivity of the fuzzy measure, and they put forward many fuzzy synthetic evaluations based on different fuzzy integrals (such as Sugeno fuzzy integral, Choquet fuzzy integral). These methods can solve information fusion with the interaction in theory, but it is very difficult to determine an appropriate fuzzy measure.

For the determining methods, besides some methods given by some domain experts, the commonly used ones are obtained according to study from some known decision data. Keller et al. [12] studied a method based on confusion matrix. This method was not influenced by the form of integral and could achieve search in short time, but it often could not obtain the optimal solution, so it is only suitable for some problems with special structures. Grabisch and Nicolas [13] presented a method based on quadratic programming to identify fuzzy measures. This method requires the objective function to be differentiable. Although completed theories have formed, they are not universal. Hu [14] put forward a genetic algorithm based method to determine fuzzy measures, in which operation process is simple and easy to understand, while the algorithm design depends on the type of training data. Keller and Osborn [15] proposed a training algorithm based on gradient descent method, which required all the fuzzy density values are considered at each iteration with a problem with many independent training algorithms.

From the analysis above, fuzzy synthetic evaluation based on the fuzzy integrals can describe the interaction between the evaluation factors effectively. However, the methods to determine fuzzy measures still lack systematic operation mechanism, and it is the bottleneck restricting the application. With the development of the information technology, more and more data have been produced and stored from many fields such as transportation, electric power, production process control. Although these data has uncertainty (i.e., noise, incompleteness), much valid knowledge may be hidden. And the knowledge reflects the relationship of each attribute in information system to a certain degree. If we take the dependent relationship between knowledge and attributes set as a measure basis of attribute correlation, then we can establish some importance metric methods of attribute using proper data mining methods. On this basis, we further establish information fusion methods and fuzzy synthetic evaluation methods. Considering that the correlation metric of attributes cannot be determined by analytic methods, it is very useful and applicable to discuss the correlation metric of attributes based on data sets. In this paper, for the multi-attributes decision problem, according to the influence on decision system by condition attributes, we first propose the concept of the core samples set for the knowledge description of the decision information system. Second, we give an attribute correlation metric method, by using the change of the core samples set as a strategy of measuring the importance of condition attributes sets. Third, we establish a fuzzy synthetic evaluation model which can deal with correlation indexes (BIS-FSEM), combining with Choquet fuzzy integrals. Finally, we analyze the characteristics and effectiveness of this method and BIS-FSEM through a case-based example.

2. **Preliminaries.** Fuzzy measure essentially widens the classic metric, and it made up for the deficiencies in dealing with the non-additive phenomenon. In 1974, Sugeno first proposed the concept of fuzzy measure. Thereafter, many researchers discussed its structure characteristic, and the theoretical system was formed.

Definition 2.1. ([16]) *Let X be a nonempty set, $\mathscr{B}$ be a nonempty class of subsets of X, $\mu : \mathscr{B} \to [0, \infty]$ and satisfies: 1) When $\emptyset \in \mathscr{B}$, $\mu(\emptyset) = 0$; 2) For $A, B \in \mathscr{B}$, and $A \subset B$ imply $\mu(A) \leq \mu(B)$; 3) For $\{A_n\}_{n=1}^{\infty} \subset \mathscr{B}$, $A_1 \subset A_2 \subset \cdots \subset A_n \subset \cdots$, and $\bigcup_{n=1}^{\infty} A_n \in \mathscr{B}$, $\lim\limits_{n\to\infty} \mu(A_n) = \mu(\bigcup_{n=1}^{\infty} A_n)$; 4) For $\{A_n\}_{n=1}^{\infty} \subset \mathscr{B}$, $A_1 \supset A_2 \supset \cdots \supset A_n \supset \cdots$, and $\bigcap_{n=1}^{\infty} A_n \in \mathscr{B}$, $\lim\limits_{n\to\infty} \mu(A_n) = \mu(\bigcap_{n=1}^{\infty} A_n)$. Then μ is called a fuzzy measure on $(X, \mathscr{B})$, and $(X, \mathscr{B}, \mu)$ is called a fuzzy measure space. Especially, when $\mu(x) = 1$, μ is said a normalized fuzzy measure.*

The continuity is naturally satisfied if X is finite universe, so we only need to consider monotonicity when constructing a fuzzy measure on finite domain.

The non-additivity of the fuzzy measures can be intuitively explained as the correlation among elements, and subadditive (superadditive) means that the union of two sup-parts plays a negative (positive) role. Hence, fuzzy measures and fuzzy integrals theories lay a theoretical basis for different information fusion problem. Now we give the concept of Choquet fuzzy integrals.

Definition 2.2. ([16]) *Let $(X, \mathscr{B}, \mu)$ be a fuzzy measure space, $f(x)$ be a non-negative measurable function on $(X, \mathscr{B})$ (that is $N_\alpha(f) = \{x | x \in X, f(x) > \alpha\} \in \mathscr{B}$ for any $\alpha \in [0, +\infty)$), $B \in \mathscr{B}$. Then*

$$(c) \int_B f d\mu = \int_0^{+\infty} \mu(N_\alpha(f) \cap B) d\alpha$$

is called the Choquet fuzzy integral of $f(x)$ on B.

According to the definition above, it is easy to see, when $X = \{x_1, x_2, \cdots, x_n\}$, if we rearrange $f(x_1), f(x_2), \cdots, f(x_n)$ to $f(x_1^*), f(x_2^*), \cdots, f(x_n^*)$, then

$$(c) \int_X f d\mu = \sum_{k=1}^{n} [f(x_k^*) - f(x_{k-1}^*)] \cdot \mu(A_k).$$

Here, $f(x_0^*) = 0$, $A_k = \{x_k^*, x_{k+1}^*, \cdots, x_n^*\}$, $k = 1, 2, \cdots, n$. Fuzzy integrals have many good properties (see details in [16]).

3. **An Attribute Importance Metric Method Based on the Core Samples Set.** Multi-attribute decision making is a part of many fields such as resources allocation, performance evaluation, production process control, expert system. Many decision attributes do not have definite extension. Decision attribute often interacts with each other, and the interaction degree varies with the time and region. Constructing the correlation metric of attributes under special environment is the key for multi-attribute decision making. With the rapid development of information technology (i.e., computer, network, communication), a large amount of data have been accumulated, and many effective knowledge discovery and information processing methods also have been formed. i.e., decision tree-based machine learning methods, support vector machine-based statistics learning methods. If we regard the accumulated information and the relative knowledge as the data describing the feature of attributes, and through the change of which we further establish the correlation metric and importance metric, then we can get a new way to solve complex decision making problems. In this section, we mainly discuss the importance metric of attributes in decision information system. For convenience, in the following, let (U, C, d, F_C) be a decision information system, where i) U is a nonempty finite set of all samples, ii) $C = \{C_1, C_2, \cdots, C_s\}$ is the condition attributes set, $V(C_i) = \{c_{i1}, c_{i2}, \cdots, c_{im}\}$ is the range of C_i, iii) d is the decision attribute and $V(d) = \{d_1, d_2, \cdots, d_n\}$ is the range of d, iv) $F_C = \{f_1, f_2, \cdots, f_s, f_d\}$ is the information function (f_d is a mapping from U to $V(d)$ and f_i is a mapping from U to $V(C_i)$, $i = 1, 2, \cdots, s$). In the following:

1) when $B \subset C$, $F_B = \{f_d, f_k | C_k \in B\}$, we call that (U, C, d, F_B) is a subsystem of (U, C, d, F_C);

2) let $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}) = \{u \in U | f_1(u) = c_{1i_1}, f_2(u) = c_{2i_2}, \cdots, f_s(u) = c_{si_s}\}$ be the R_C-equivalence class when the value of C_k is c_{ki_k}, $k = 1, 2, \cdots, s$;

3) let $(d = d_j) = \{u \in U | f_d(u) = d_j\}$ be the R_d-equivalence class when the value of d is d_j;

4) let $[u]_R$ be the R-equivalence class of u for the equivalence relation R on U, and $U/R = \{[u]_R | u \in U\}$.

Definition 3.1. *For the decision information system (U, C, d, F_C), $B \subset C$, $B \neq \emptyset$.*

1) If there exists a $(d = d_j)$ such that $\{u \in U | f_k(u) = c_{ki_k}, C_k \in B\} \neq \emptyset$ and $\{u \in U | f_k(u) = c_{ki_k}, C_k \in B\} \subset (d = d_j)$, then we call

$$\text{If } C_k = c_{ki_k},\ C_k \in B,\ \text{ Then } d = d_j \tag{1}$$

an "IF-THEN" knowledge of (U, C, d, F_C);

2) If $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}) \neq \emptyset$ and there exists a $(d = d_j)$ such that

$$(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}) \subset (d = d_j), \tag{2}$$

then $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}; d = d_j)$ is called an elementary knowledge factor of (U, C, d, F_C), $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s})$ is called the support samples set of $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}; d = d_j)$, and

$$U_C = \bigcup_{(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s};\ d = d_j) \in \mathscr{K}} (C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}) \tag{3}$$

is called the core samples set of (U, C, d, F_C), and the samples in U_C are called the core samples, where $\mathscr{K}$ is the family of all elementary knowledge factors.

To find "IF-THEN" knowledge from the information system is the core of data mining. It is the most common knowledge of the management and decision-making process. It is easy to see, if (U, C, d, F_C) is regarded as a knowledge system, then U_C is the core of supporting "IF-THEN" knowledge of (U, C, d, F_C). And it also corresponds to the low

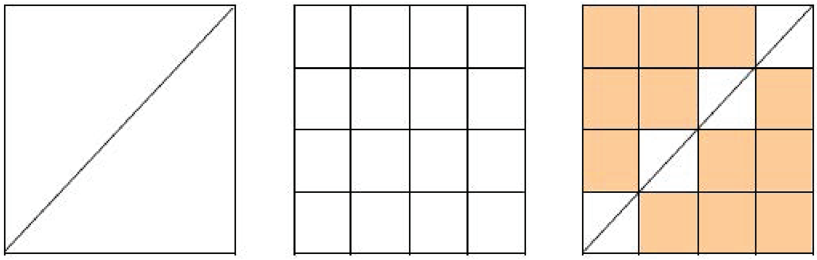

Figure 1. The geometry paraphrase of the core samples set

approximation set of decision class in rough set theory. Its intuitive explanation is shown as Figure 1: (a) means the decision attribute class; (b) means the conditional attribute class; the black area in (c) means the core samples set.

Theorem 3.1. *Suppose U_C be the core samples set of (U, C, d, F_C), then: 1) $U_C = U$ if and only if (U, C, d, F_C) is a consistent decision information system; (For any given condition attribute class $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s})$, there exists a decision attribute class $d = d_j$ such that $(C_1 = c_{1i_1}, C_2 = c_{2i_2}, \cdots, C_s = c_{si_s}) \subset (d = d_j)$). 2) $U_C = \emptyset$ if and only if there does not exist "IF-THEN" knowledge in (U, C, d, F_C).*

Theorem 3.2. *Let (U, C, d, F_B) be the subsystem of (U, C, d, F_C), then $U_B \subset U_C$. Here, U_B and U_C are the core samples sets of (U, C, d, F_B) and (U, C, d, F_C) separately.*

The core samples set is the supporting body of knowledge in information system, and it reflects the influence on information system of each attribute. So the core samples set can be used as the metric basis of attribute importance. Let (U, C, d, F_C) be a decision information system, $\mathscr{P}(C)$ be the power set of $C, B \in \mathscr{P}(C)$,

$$\mu(B) = \begin{cases} |U_B|/|U_C|, & |U_C| \neq 0, \\ 1, & |U_C| = 0, \quad B \neq \emptyset, \\ 0, & B = \emptyset, \end{cases} \tag{4}$$

here, $|A|$ is the number of elements in A, U_B and U_C are the core samples sets of (U, C, d, F_B) and (U, C, d, F_C) respectively. Then we have the following conclusions by Theorem 3.2:

1) $0 \leq \mu(B) \leq 1$ for any $B \in \mathscr{P}(C)$, and $\mu(\emptyset) = 0$, $\mu(C) = 1$;

2) $\mu(A) \leq \mu(B)$ for any $A, B \in \mathscr{P}(C)$ with $A \subset B$.

Combined with the discussions in Section 2, we know that (3) is a normalized fuzzy measure on $(C, P(C))$, and we call it the ***attribute correlation metric*** based on knowledge in (U, C, d, F), and denoted as $(C, P(C), \mu) \in (U, C, d, F_C)$.

It is easy to see, (4) is a metric describing the support degree to (U, C, d, F_C) by condition attributes set B. It reflects the synthetic importance of attributes of B, and it can embody the mutual support among attributes to some degree. It can be used in many issues such as synthetic evaluation, information fusion, pattern recognition and multi-attribute decision making.

Remark 3.1. *It is worth noting that metric model (4) has some limitations. If there does not exist "IF-THEN" knowledge in (U, C, d, F_C), (4) cannot effectively reflect the interdependence among attributes. There are many reasons for causing no "IF-THEN" knowledge such as noise, inconsistence information system, incomplete data. Therefore, when conditional attributes set B is small, it is possible to get that $\mu(B) = 0$. That is,*

$\mu(B)$ has worse discrimination ability. And the fact $\mu(B) = 0$ cannot be easily interpreted as that B is not important, it is only a proof that we may not be able to make decision by the attributes value of B. For this problem, we can perfect (4) through the following strategies: 1) weaken the requirement for the knowledge precision by some threshold, and based on this, enlarge the knowledge carriers; 2) take $|B|/|C|$, $|U_B|/|U_C|$ as the secondary index, principal index of importance metric of B, respectively, and construct the importance metric model $\mu(B) = S(|B|/|C|, |U_B|/|U_C|)$ through a synthesis operator $S(u,v)$ (i.e., $S(u,v) = au + bv$, $a, b \in [0,1]$, $a + b = 1$). We will further discuss it in future.

4. **A Fuzzy Synthetic Evaluation Model Based on the Knowledge System.** Synthetic evaluation is the key focus in many fields such as resource management, complex system optimization. Because the connotation of the evaluation factors often cannot be precisely defined, the fuzzy synthetic evaluation is a common used method, and its implementation steps are as follows:

Step 1: Build the factors set $X = \{u_1, u_2, \cdots, u_s\}$.

Step 2: Build the evaluation set $Y = \{v_1, v_2, \cdots, v_m\}$.

Step 3: Determine the single factor evaluation matrix R by fuzzy statistics method, expert scoring method, etc. Here $r_{ij} \in [0,1]$ means the degree of the evaluation objects to each v_j on each u_i, $i = 1, 2, \cdots, s$, $j = 1, 2, \cdots, m$.

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1m} \\ r_{21} & r_{22} & \cdots & r_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ r_{s1} & r_{s2} & \cdots & r_{sm} \end{bmatrix} \triangleq (R_1, R_2, \cdots, R_m).$$

Step 4: Choose a suitable fuzzy synthetic function $S(x_1, x_2, \cdots, x_s)$ satisfying the following conditions: 1) $S(x_1, x_2, \cdots, x_s) : [0,1]^s \to [0,1]$; 2) $S(x, x, \cdots, x) = x$; 3) It is monotone nondecreasing on each variable x_i; 4) It is continuous on each variable x_i.

Step 5: Synthesize each column of R for a value $b_j = S(R_j) = S(r_{j1}, r_{j2}, \cdots, r_{js})$ by synthetic function, then we get the fuzzy synthetic evaluation results $B = (b_1, b_2, \cdots, b_m)$, where b_j means the degree of the evaluation objects to each evaluation set v_j in synthetic sense, $j = 1, 2, \cdots, m$. That is

$$B = S \circ R = (S(R_1), S(R_2), \cdots, S(R_m)). \tag{5}$$

It is easy to see, the selection of a fuzzy synthetic function is very important for fuzzy synthetic evaluation. At present, the commonly used synthetic modes include the weighted average type (6), the geometric average type (7), the single factors determining type (8), the main factors determining type (9).

$$S(x_1, x_2, \cdots, x_s) = \sum_{i=1}^{s} \omega_i x_i, \ 0 \le \omega_i \le 1 \text{ and} \sum_{i=1}^{s} \omega_i = 1, \tag{6}$$

$$S(x_1, x_2, \cdots, x_s) = \prod_{i=1}^{s} (x_i)^{\omega_i}, \ 0 \le \omega_i \le 1 \text{ and} \sum_{i=1}^{s} \omega_i = 1, \tag{7}$$

$$S(x_1, x_2, \cdots, x_s) = \max_{1 \le i \le s} \min(\omega_i, \ x_i), \ 0 \le \omega_i \le 1 \text{ and} \max_{1 \le i \le s} \omega_i = 1, \tag{8}$$

$$S(x_1, x_2, \cdots, x_s) = \max_{1 \le i \le s} T(\omega_i, \ x_i), \ 0 \le \omega_i \le 1 \text{ and} \max_{1 \le i \le s} \omega_i = 1. \tag{9}$$

Here, T is a mapping (we call it a t-norm) from $[0,1]^2$ to $[0,1]$ and satisfies: 1) $T(a,b) = T(b,a)$; 2) $T(T(a,b),c) = T(a,T(b,c))$; 3) $T(a,c) \leq T(b,d)$ for $a \leq b, c \leq d$; 4) $T(1,a) = a$.

However, it is worth noting that (6) and (7) are only suitable for the case when the evaluation factor is independent with each other, (8) and (9) cannot make full use of information from various aspects. These disadvantages exist in other synthetic modes based on the weight system. Therefore, how to construct an operational fuzzy synthetic model is very important in theory and practice. Considering that the factors set of the fuzzy synthetic evaluation and the condition attribute set of decision-making information system have the same meaning, so if factors set X of the fuzzy synthetic evaluation is interpreted as the condition attributes set C of the decision information system, the evaluation set V is the value of decision attribute set d, then we can construct fuzzy synthetic functions by the existing decision-making information system. In the following, combined with attribute importance metric in Section 3 and Choquet fuzzy integrals, we give a fuzzy synthetic function which can deal with the correlation.

Theorem 4.1. *Suppose $(C, \mathscr{P}(C), \mu)$ be the attribute correlation metric based on the knowledge of (U, C, d, F_C), and $C = \{C_1, C_2, \cdots, C_s\}$. For any given $x_i \in [0,1]$, $i = 1, 2, \cdots, s$, let $\overrightarrow{x} = (x_1, x_2, \cdots, x_s)$, $f_{\overrightarrow{x}}(C_i) = x_i$, then*

$$S(x_1, x_2, \cdots, x_s) = \int_0^{+\infty} \mu(N_\alpha(f_{\overrightarrow{x}}))d\alpha \tag{10}$$

is a fuzzy synthetic function.

Proof: 1) For any given $x = (x_1, x_2, \cdots, x_s) \in [0,1]^s$, since $f_x(C_i) = x_i$, $i = 1, 2, \cdots, s$, we know $N_\alpha(f_{\overrightarrow{x}}) = \{C_i | C_i \in C, f_{\overrightarrow{x}}(C_i) \geq \alpha\} = \emptyset$ for $\alpha > 1$. From this and the regularity of $(C, \mathscr{P}(C), \mu)$, we know $\mu(N_\alpha(f_{\overrightarrow{x}})) = 0$ is constant for any $\alpha > 1$, $0 < \mu(N_\alpha(f_{\overrightarrow{x}})) \leq 1$ is constant for any $0 \leq \alpha \leq 1$, that is, $S(x_1, x_2, \cdots, x_s) \in [0,1]$.

2) If $x_1 = x_2 = \cdots = x_s = x \in [0,1]$, then $\mu(N_\alpha(f_{\overrightarrow{x}})) = 1$ is constant for any $0 \leq \alpha \leq x$, $\mu(N_\alpha(f_{\overrightarrow{x}})) = 0$ is constant for any $\alpha > x$, that is, $S(x, x, \cdots, x) = \int_0^x 1 d\alpha = x$.

3) If satisfying $x = (x_1, x_2, \cdots, x_s)$, $y = (y_1, y_2, \cdots, y_s)$, satisfying $0 \leq x_1 < y_1 \leq 1$, $x_i = y_i \in [0,1]$, $i = 2, 3, \cdots, s$, then we have $N_\alpha(f_{\overrightarrow{x}}) \subset N_\alpha(f_{\overrightarrow{y}})$, $\mu(N_\alpha(f_{\overrightarrow{x}})) \leq \mu(N_\alpha(f_{\overrightarrow{y}}))$ for any $0 < \alpha < 1$. From this and the properties of the integrals, we have $S(x_1, x_2, \cdots, x_s) \leq S(y_1, y_2, \cdots, y_s)$. We also can prove $S(x_1, x_2, \cdots, x_s)$ is monotone nondecreasing on other variables similarly.

4) We can prove $S(x_1, x_2, \cdots, x_s)$ is continuous on each variable by the properties of the integrals.

It is easy to see by the proof process of Theorem 4.1, (10) is essentially a Choquet fuzzy integral on fuzzy measure space $(C, \mathscr{P}(C), \mu)$. If the ranking result of $x_1, x_2, \cdots, x_s$ by increasing order is $x_1^*, x_2^*, \cdots, x_s^*$, then we have,

$$S(x_1, x_2, \cdots, x_s) = \sum_{k=1}^{s} (x_k^* - x_{k-1}^*) \cdot \mu(B_k) \tag{11}$$

where $x_0^* = 0$, $B_k = \{C_k^*, C_{k+1}^*, \cdots, C_s^*\}$, C_k^* is the corresponding attribute of x_k^*, $k = 1, 2, \cdots, s$.

For convenience, we call (10) a fuzzy synthetic evaluation model based on the knowledge system (BIS-FSEM), and denoted as

$$\begin{cases} B = S \circ R, \\ S(x_1, x_2, \cdots, x_s) = \int_0^{+\infty} \mu(N_\alpha(f_{\overrightarrow{x}}))d\alpha, \\ (C, \mathscr{P}(C), \mu) \in (U, C, d, F_C). \end{cases} \tag{12}$$

where (U, C, d, F_C) is a decision information system relevant to the synthetic evaluation.

5. **Example Analysis.** A school has the mission to cultivate technology and management professionals for the society. The education level reflects the economic level of a nation or region to some degree. As the core, the teaching quality of the teacher determines the social status of a school. Therefore, strengthening the construction of teaching staff and improving the teaching quality are always the eternal topics of school and the relative department.

Evaluation of teaching quality is an important link of teaching administrative department, and it can guarantee teaching quality. Evaluation involves many subjective factors and objective factors, i.e., the basic condition of students, teaching aims, teaching facilities, teaching sessions. Therefore, using an exact numeral to describe teaching quality often cannot objectively reflect the actual condition of a teacher. Fuzzy synthetic evaluation is a common model for teaching quality evaluation. To highly arouse the enthusiasm of the teachers, find and solve the problems existing in teaching process in time, a high school has established its own fuzzy synthetic evaluation system for the evaluation of teaching quality. In this system, professional level (C_1), teaching attitude (C_2), teaching method (C_3) and teaching effect (C_4) is evaluation indexes set, very good, good, common, poor is remarks set, weighted average synthesis function is synthesis operator, and weighted system is $W = (0.25, 0.25, 0.2, 0.3)$. In this system, both teachers and administrative departments find that there exists different correlation among four evaluation indexes. And weighted average synthesis method cannot effectively deal with the interaction between indexes. To better play the role of evaluation and enhance the teacher team, this school decides to perfect the existing teaching quality evaluation system, and the basic guiding ideas are: 1) consider the mutual support and correlation between evaluation indexes as much as possible, and provide basis for the construction of teaching staff; 2) consider the scientific nature and operability of established model, avoid the subjectivity as much as possible.

It is easy to see that the determination of importance metric of each index, which can embody the correlation between indexes, is the key to this problem. Because the teaching quality is a long-term problem concerned by education department and school, there accumulated much related information. Table 1 shows 20 pieces of information extracted at random from the database of the teaching quality evaluation of the school, d is the synthetic evaluation result. Therefore, we can use the fuzzy synthetic evaluation model (11) based on the knowledge system given in Section 4 as the evaluation pattern of the teaching quality. Its implementation steps are stated as follows:

Step 1: Determine attribute importance metric of the evaluation indexes according to (4) (Table 2 is the result of Table 1);

Step 2: Let (10) be a synthetic function, combined with a concrete single factor evaluation matrix R. Then, we can get the fuzzy synthetic evaluation results (5).

From Table 2, the importance of single attribute has a certain relationship with that of attribute group (i.e., monotone non-decreasing), but the relationship is not a simple superposition. For example, the separate importance of C_2, C_3, C_1, C_4 is 0, 0, 0.2, 0.55 respectively, the synthetic importance 0.75 of C_2, C_3 is higher than 0.65 of C_1, C_4, the synthetic importance 0.9 of C_2, C_3, C_4 is higher than 0.8 of C_1, C_2, C_4. The above situation demonstrates that the interaction between attributes is a complex relation, which can be obtained by model (4).

Table 1. Evaluation information system of 20 teachers' teaching quality

Samples	C_1	C_2	C_3	C_4	d
1	very good	very good	very good	very good	very good
2	good	very good	very good	very good	very good
3	common	very good	common	good	common
4	common	common	poor	poor	poor
5	poor	good	good	common	common
6	very good	common	poor	common	common
7	good	common	good	good	good
8	good	poor	good	good	good
9	very good	good	common	good	good
10	good	poor	poor	poor	poor
11	good	good	common	common	common
12	poor	very good	very good	good	common
13	common	very good	very good	good	good
14	good	common	good	common	common
15	good	good	good	good	good
16	poor	good	very good	common	common
17	poor	very good	good	common	common
18	good	common	common	good	common
19	common	good	good	good	good
20	very good	common	poor	poor	poor

Table 2. Attribute importance metric of the evaluation indices

(U, B, d, F_B)	U_B	$\|U_B\|$	$\mu(B)$
$B = \{C_1\}$	$\{5, 12, 16, 17\}$	4	0.2
$B = \{C_2\}$	$\emptyset$	0	0
$B = \{C_3\}$	$\emptyset$	0	0
$B = \{C_4\}$	$\{1, 2, 4, 5, 6, 10, 11, 14, 16, 17, 20\}$	11	0.55
$B = \{C_1, C_2\}$	$\{1, 2, 4, 5, 9, 12, 16, 17, 19\}$	9	0.45
$B = \{C_1, C_3\}$	$\{1, 2, 3, 4, 5, 9, 10, 11, 12, 13, 16, 17, 18, 19\}$	14	0.7
$B = \{C_1, C_4\}$	$\{1, 2, 4, 5, 6, 9, 10, 11, 12, 14, 16, 17, 20\}$	13	0.65
$B = \{C_2, C_3\}$	$\{3, 8, 10, 16, 17, 18\}$	6	0.3
$B = \{C_2, C_4\}$	$\{1, 2, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 19, 20\}$	15	0.75
$B = \{C_3, C_4\}$	$\{1, 2, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 19, 20\}$	15	0.75
$B = \{C_1, C_2, C_3\}$	$\{1, 2, 3, 4, 5, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19\}$	16	0.8
$B = \{C_1, C_2, C_4\}$	$\{1, 2, 4, 5, 6, 8, 9, 10, 11, 12, 14, 15, 16, 17, 19, 20\}$	16	0.8
$B = \{C_1, C_3, C_4\}$	U	20	1
$B = \{C_2, C_3, C_4\}$	$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20\}$	18	0.9
$B = \{C_1, C_2, C_3, C_4\}$	U	20	1

According to the discussion above, if one teacher's single factor evaluation matrix is

$$R = \begin{bmatrix} 0.70\ 0.20\ 0.10\ 0.00 \\ 0.35\ 0.55\ 0.10\ 0.00 \\ 0.15\ 0.65\ 0.10\ 0.10 \\ 0.40\ 0.40\ 0.10\ 0.10 \end{bmatrix} \triangleq (R_1, R_2, R_3, R_4), \tag{13}$$

then by using (11) in Section 4 we can get the following:

$S(R_1) = S(0.70, 0.35, 0.15, 0.40) = (0.15-0.00)\times\mu(C)+(0.35-0.15)\times\mu(\{C_1, C_2, C_4\})+ (0.40 - 0.35) \times \mu(\{C_1, C_4\}) + (0.70 - 0.40) \times \mu(\{C_1\}) = 0.15 \times 1 + 0.20 \times 0.8 + 0.05 \times 0.65 + 0.30 \times 0.20 = 0.4025$;

$S(R_2) = S(0.20, 0.55, 0.65, 0.40) = (0.20-0.00)\times\mu(C)+(0.40-0.20)\times\mu(\{C_2, C_3, C_4\})+ (0.55 - 0.40) \times \mu(\{C_2, C_3\}) + (0.65 - 0.55) \times \mu(\{C_3\}) = 0.20 \times 1 + 0.20 \times 0.9 + 0.15 \times 0.3 + 0.10 \times 0 = 0.4250$;

$S(R_3) = S(0.10, 0.10, 0.10, 0.10) = (0.10 - 0.00) \times \mu(C) = 0.10 \times 1 = 0.1000$;

$S(R_4) = S(0.00, 0.00, 0.10, 0.10) = (0.10 - 0.00) \times \mu(\{C_2, C_3\}) = 0.10 \times 0.75 = 0.0750$, that is, the teacher's teaching quality evaluation is $B = S((R_1), S(R_2), S(R_3), S(R_4)) = (0.0425, 0.4250, 0.1000, 0.0750)$, i.e., the degrees of the teacher's teaching quality corresponding to very good, good, common and poor are 0.4025, 0.4250, 0.1000 and 0.0750.

In order to further analyze the characteristics and effectiveness of (12), for 6 different evaluation objects, we use the original model (i.e., (6) is fuzzy synthetic function, $W = (0.25.0.25, 0.2, 0.3)$ is weighted system) and model (12) to obtain evaluation results, respectively. And the results are listed in Table 3.

From Table 3, we can see the results of the two models have obvious differences, and they are presented in the following. 1) The results of two models are similar (i.e., No.1, No.2, No.3), and also they have obvious differences (i.e., No.4, No.5, No.6). 2) When the corresponding evaluation value of each attribute has no big difference, the results of two models are similar (i.e., the 2th component of No.2, the 3th component of No.3). 3) Results for the weighted average type closely depend on weight system (see Table 4),

Table 3. Evaluation results of original model and model (12)

No.	R	Evaluation results	
		Original model	Model (12)
1	$\begin{bmatrix} 0.70 & 0.20 & 0.10 & 0.00 \\ 0.35 & 0.55 & 0.10 & 0.00 \\ 0.15 & 0.65 & 0.10 & 0.10 \\ 0.40 & 0.40 & 0.10 & 0.10 \end{bmatrix}$	(0.4125, 0.4375, 0.1000, 0.0500)	(0.4025, 0.4250, 0.1000, 0.0750)
2	$\begin{bmatrix} 0.10 & 0.65 & 0.25 & 0.00 \\ 0.15 & 0.70 & 0.10 & 0.05 \\ 0.10 & 0.55 & 0.25 & 0.10 \\ 0.10 & 0.60 & 0.20 & 0.10 \end{bmatrix}$	(0.1125, 0.6275, 0.1975, 0.0625)	(0.1000, 0.6125, 0.2350, 0.0825)
3	$\begin{bmatrix} 0.70 & 0.20 & 0.05 & 0.00 \\ 0.55 & 0.35 & 0.10 & 0.00 \\ 0.20 & 0.65 & 0.15 & 0.00 \\ 0.55 & 0.25 & 0.10 & 0.00 \end{bmatrix}$	(0.5600, 0.3425, 0.0975, 0.0000)	(0.4850, 0.2750, 0.0950, 0.0000)
4	$\begin{bmatrix} 0.00 & 0.15 & 0.75 & 0.10 \\ 0.75 & 0.15 & 0.10 & 0.00 \\ 0.65 & 0.30 & 0.10 & 0.00 \\ 0.00 & 0.10 & 0.70 & 0.20 \end{bmatrix}$	(0.3175, 0.1650, 0.4425, 0.0850)	(0.1950, 0.1400, 0.5000, 0.0200)
5	$\begin{bmatrix} 0.00 & 0.00 & 0.30 & 0.70 \\ 0.75 & 0.25 & 0.00 & 0.00 \\ 0.45 & 0.55 & 0.00 & 0.00 \\ 0.00 & 0.10 & 0.20 & 0.70 \end{bmatrix}$	(0.2775, 0.2025, 0.1350, 0.3850)	(0.1350, 0.1350, 0.1500, 0.4550)
6	$\begin{bmatrix} 0.00 & 0.00 & 0.60 & 0.40 \\ 0.05 & 0.35 & 0.15 & 0.00 \\ 0.00 & 0.25 & 0.65 & 0.10 \\ 0.00 & 0.05 & 0.20 & 0.75 \end{bmatrix}$	(0.1250, 0.1525, 0.3775, 0.3450)	(0.0000, 0.1050, 0.4800, 0.4875)

TABLE 4. Results of different weighted average synthetic evaluation

No.	R	Weight systems		
		(0.25.0.25, 0.2, 0.3)	(0.15. 0.25, 0.30, 0.30)	(0.35.0.15, 0.25, 0.25)
1	$\begin{bmatrix} 0.70 & 0.20 & 0.10 & 0.00 \\ 0.35 & 0.55 & 0.10 & 0.00 \\ 0.15 & 0.65 & 0.10 & 0.10 \\ 0.40 & 0.40 & 0.10 & 0.10 \end{bmatrix}$	(0.4125, 0.4375, 0.1000, 0.0500)	(0.3575, 0.4825, 0.1000, 0.0600)	(0.4350, 0.4150, 0.1000, 0.0500)
2	$\begin{bmatrix} 0.10 & 0.65 & 0.25 & 0.00 \\ 0.15 & 0.70 & 0.10 & 0.05 \\ 0.10 & 0.55 & 0.25 & 0.10 \\ 0.10 & 0.60 & 0.20 & 0.10 \end{bmatrix}$	(0.1125, 0.6275, 0.1975, 0.0625)	(0.1125, 0.6175, 0.1975, 0.0725)	(0.1075, 0.6200, 0.2150, 0.5750)
3	$\begin{bmatrix} 0.70 & 0.20 & 0.05 & 0.00 \\ 0.55 & 0.35 & 0.10 & 0.00 \\ 0.20 & 0.65 & 0.15 & 0.00 \\ 0.55 & 0.25 & 0.10 & 0.00 \end{bmatrix}$	(0.5600, 0.3425, 0.0975, 0.0000)	(0.5050, 0.3875, 0.1075, 0.0000)	(0.5575, 0.3475, 0.0950, 0.0000)
4	$\begin{bmatrix} 0.00 & 0.15 & 0.75 & 0.10 \\ 0.75 & 0.15 & 0.10 & 0.00 \\ 0.65 & 0.30 & 0.10 & 0.00 \\ 0.00 & 0.10 & 0.70 & 0.20 \end{bmatrix}$	(0.3175, 0.1650, 0.4425, 0.0850)	(0.3825, 0.1800, 0.3775, 0.0750)	(0.2750, 0.1750, 04775, 0.0850)
5	$\begin{bmatrix} 0.00 & 0.00 & 0.30 & 0.70 \\ 0.75 & 0.25 & 0.00 & 0.00 \\ 0.45 & 0.55 & 0.00 & 0.00 \\ 0.00 & 0.10 & 0.20 & 0.70 \end{bmatrix}$	(0.2775, 0.2025, 0.1350, 0.3850)	(0.3225, 0.2575, 0.1050, 0.3150)	(0.2250, 0.2000, 0.1550, 0.4200)
6	$\begin{bmatrix} 0.00 & 0.00 & 0.60 & 0.40 \\ 0.05 & 0.35 & 0.15 & 0.00 \\ 0.00 & 0.25 & 0.65 & 0.10 \\ 0.00 & 0.05 & 0.20 & 0.75 \end{bmatrix}$	(0.1250, 0.1525, 0.3775, 0.3450)	(0.1250, 0.1775, 0.3825, 0.3150)	(0.0750, 0.1275, 0.4450, 0.3525)

but the results under any weight system are not as the same as those of model (12). It shows that the weighted average synthetic evaluation does not have objectivity. 4) The results of model (12) closely depend on the importance of attributes group with interaction. Only when the evaluation values of attributes group with big importance are all big, the corresponding synthesis value is big (i.e., the 3th component of No.6). Single or a small amount of evaluation value of attribute does not decide the synthesis value (the 1th component of No.6, the 1th component of No.4). 5) Model (12) just depends on the past data information. Although these data may have various kinds of noise, we can construct the importance metric of attributes group by some measures, such as, weakening the knowledge precision, adjusting the representation of knowledge carrier. Therefore, model (12) has a certain generality and objectivity. It not only has dynamic self-organizing characteristic, but also can solve the theory deficiencies and subjectivity of weighted system evaluation model. Also model (12) conforms to the basic requirements for the teaching evaluation system.

6. **Conclusion.** In this paper, we analyze the characteristic of knowledge of the decision information system, and propose the concept of the core samples set, further establish an attribute importance metric method. Furthermore, combined with Choquet fuzzy integrals, we establish a fuzzy synthetic evaluation model (BIS-FSEM) which can deal with correlation indexes. Then we analyze the effectiveness of this model through a cased-based example. Theoretical analysis and computation results show that the attribute importance metric method has strong interpretability. It can effectively induce the correlation

among attributes from the existing knowledge. Hence, these discussions enrich the fuzzy measure theory to a certain degree and make up for the existing fuzzy evaluation methods. So it can be applied in many problems such as information fusion, synthetic evaluation, fuzzy decision. It is worth noting that the attribute correlation metric method in this paper is only suitable for the information system with discrete attribute value. This deficiency restricts the application of BIS-FSEM to a certain extent. We will construct a general BIS-FSEM by using decision information system with multiple decision attribute values or continuous attribute value in the future work.

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