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BILATERAL CONTROL SYSTEM USING NONLINEAR FLEXIBLE SLAVE ARM WITH NETWORK DELAY

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Abstract. *This research focuses on stability analysis of a bilateral control system using a nonlinear single-link flexible slave arm with constant communication network delay. In our study, a rigid master arm and a single-link flexible slave arm comprise a bilateral control system where control is performed over a communication network with constant delay. A flexible arm was modeled by a linearized system in our previous research; however, it is described by a nonlinear system in this paper. The stability of the proposed bilateral control system is analyzed by using the Lyapunov stability theorem. The reaction torque for the rigid master arm and the control torque for the flexible slave arm are generated by a proportional-derivative (PD) controller and a proportional-derivative-strain (PDS) controller, respectively. These controllers are derived from the candidate Lyapunov function, which is defined as the total energy. Several numerical simulations demonstrate the high performance of our bilateral control system.*
Keywords: Flexible arm, Bilateral control, Time delay, Master-slave arms, Lyapunov function, Nonlinear system

1. **Introduction.** When manipulators are introduced into general environments such as houses and offices, to support people to work, we must consider their safety operations. To achieve the above goal, manipulators should be made from light-weight and suitable materials. Such manipulators called flexible manipulators have light-weight and low rigidity of arms. It is well known that this type of manipulators is difficult accurately to control. Therefore, it is a fundamental technology for introducing manipulators into general environments that flexible manipulators are accurately controlled. Even if flexible manipulators collide with obstacles or human operators, objects collided by flexible manipulators are relatively safe due to the mechanical flexibility. Furthermore, if teleoperation are carried out through the communication network, it is possible that human skills are provided to somewhere at a great distance. In past researches, results of studies have been shown with respect to each field such as flexible manipulators, teleoperation or networked systems. In our research, the bilateral control system of flexible master-slave arms with communication networks is considered. Therefore, it is considered that this study integrates the above fields.

In recent years, many researchers have studied robotic teleoperation systems. If robotic teleoperation is designed to be used in every environment, it is reasonable to assume that control should be realized over existing communication networks. Hence, the teleoperation of robotic systems through communication networks such as LAN, WAN and wireless LAN is key to the successful implementation of such technology in the modern world. Teleoperation systems should be considered as feedback systems, including the communication networks through which data flows (reference data, control input and observation data between the master manipulator and the slave manipulator). Shingin and Ohta discussed disturbance rejection with information constraints for a first-order system and obtained a quantitative representation of the trade-off between available information and achievable performance [1]. Zhang et al. analyzed several fundamental issues in network control systems and showed that the stability of a network control system can be characterized by using hybrid system stability analysis techniques [2]. However, these studies have focused only on the effects of the communication network.

Bilateral control of master-slave manipulators through communication networks is one of the main problems in teleoperation. A number of researchers have investigated solutions for such kinds of bilateral control problems. However, the bilateral control systems for which these problems are solved are almost always rigid master-slave manipulators. For instance, Sipahi et al. proved the passivity of bilateral control of rigid master-slave manipulators [3]. Spong, Niemeyer et al. showed the stability of teleoperation systems with time delay based on their passivity [4, 5]. Namerikawa presented the stability of bilateral systems with the time varying delay on the basis of the Lyapunov theorem [6]. Nurung and Nilkhamhang proved the asymptotic stability of bilateral systems with time delay, which is controlled by using sliding-mode control designed on the basis of the Lyapunov function [7]. However, bilateral systems in these studies have been constructed using rigid master and slave arms.

There are very few studies on flexible slave arms. Flexible arms have many advantages: they are lightweight, consume little energy and provide collision safety because of their mechanical flexibility. Thus, employing flexible manipulators as slave manipulators can be used for developing more useful bilateral control systems. However, flexible manipulators are also associated with some problems, such as increased complexity of the system and undesirable vibration. Matsuno et al. proposed a control method for the contact force and the tip position for flexible manipulators and proved its stability by using proportional-derivative-strain (PDS) control [8, 9]. Banavar and Dominic presented the synthesis of a linear-quadratic-Gaussian/H_∞ controller with noncolocated sensing for single-link flexible manipulators [10]. Ge et al. attempted to improve the position accuracy through PD control with nonlinear strain feedback for single-link flexible manipulators [11], and Saito et al. proposed nonlinear vibration control of flexible arms using piezo actuators with hysteresis [12]. Also, one of the authors has reported a using Kalman filter method for risk-sensitive stabilization of a parallel-structured single-link flexible arm [13]. Hoshino, Mori et al. proved the passivity of master-slave manipulators with flexible slave arms controlled by symmetric bilateral controllers [14, 15]. However, communication networks are not considered in these studies. In our previous work, a linearized model of flexible master-slave arms (FMSA) was considered taking communication networks into account. Furthermore, in the case of constant time delay, the stability of our bilateral system was analyzed by using the Lyapunov theorem.

In this paper, a bilateral control system for FMSA systems with time delay (shown in Figure 1) is investigated. A flexible slave arm is modeled as a nonlinear system. To analyze the stability of our bilateral control system, the candidate Lyapunov function is constructed based on the total energy which includes constant time delay. The reaction torque for the master arm and the control torque for the slave arm are generated by the PD and PDS controllers, respectively, by taking the time derivative of the candidate Lyapunov function. The ranges of the control gains are constrained in such a way that the candidate Lyapunov function becomes the actual Lyapunov function. Furthermore, the performance of our bilateral control system is demonstrated through several numerical simulations.

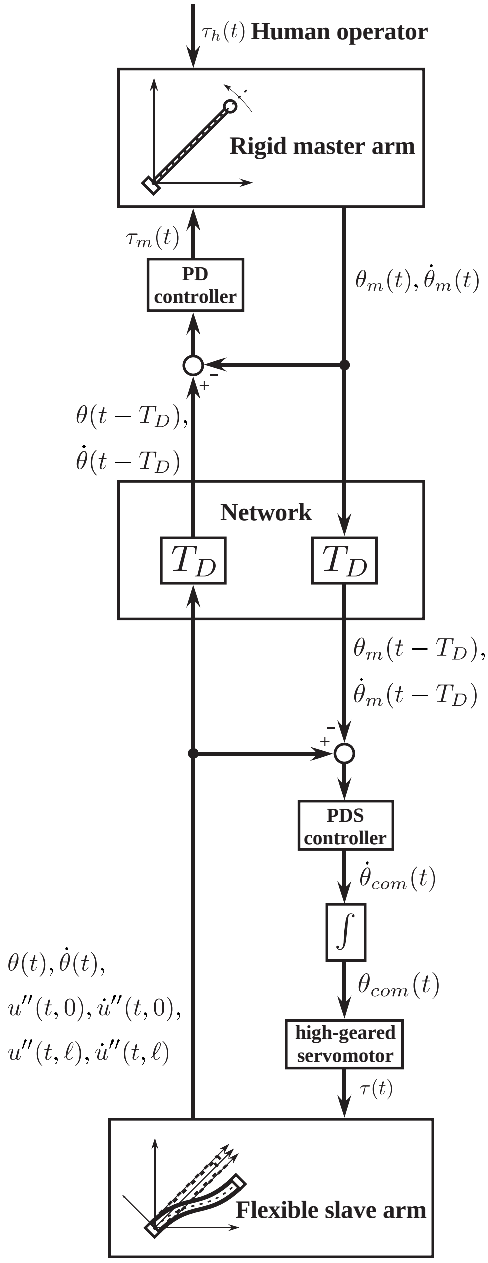


FIGURE 1. Block diagram of a flexible master-slave arm system

2. **Modeling of Flexible Master-Slave Arms.** Schematic drawings of a rigid master arm and a parallel-structured single-link flexible slave arm are shown in Figures 2 and 3(a), respectively. Both arms rotate in the horizontal plane. Figure 3(b) shows a simplified model of a parallel-structured flexible slave arm which is the same as the model investigated in the literature [13]. In our previous work, all nonlinear terms were ignored so that the flexible slave arm was modeled by a linear system. In this paper, the flexible slave arm is treated as a nonlinear system. Since both arms rotate in the horizontal plane, the effects of gravity can be ignored. Moreover, we assume that the flexible slave arm is not affected by any environmental forces.

2.1. **Dynamics of the master arm.** The master arm is constructed of a uniform rigid rod and a hub with an electric motor (shown in Figure 2). Let $O_mX_mY_m$ denote an inertial Cartesian coordinate system. The master arm is rotated by a servomotor attached at O_m.

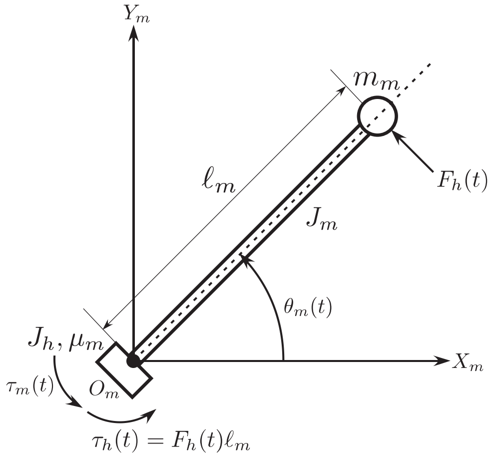
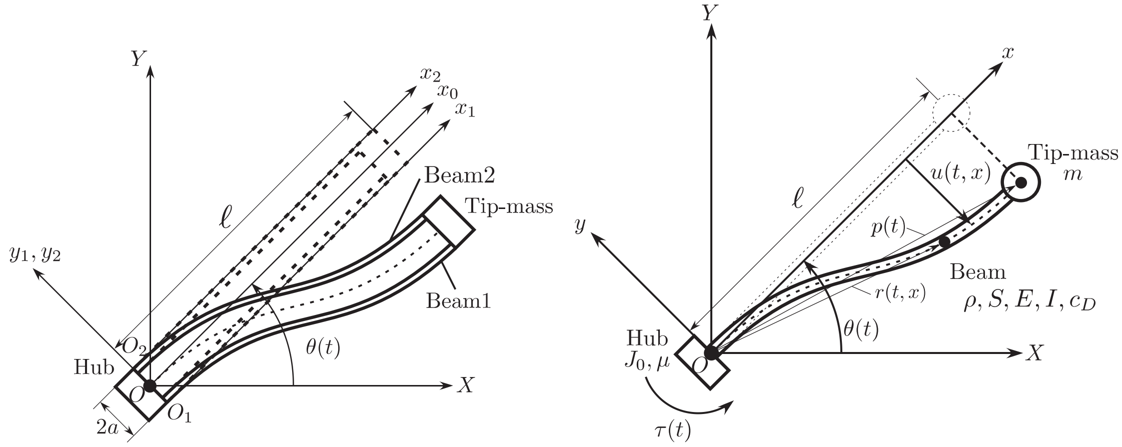


FIGURE 2. Schematic drawing of a rigid master arm



(a) Parallel-structured single-link arm (b) Single-link arm (simplified model)

FIGURE 3. Schematic drawing of a flexible slave arm

The reaction torque $\tau_m(t)$ is generated in accordance with a PD-based control law. The master arm has a length ℓ_m and a mass m_m. Let J_m and J_h denote the moments of inertia of the master arm and the rotor of the master motor, respectively. μ_m is the coefficient of friction of the master motor shaft, $\theta_m(t)$ is the angle of rotation of the master motor, and $\tau_h(t)$ is the torque generated by the force $F_h(t)$ applied by a human operator. We express the dynamics of the master arm as

$$\left(J_h + J_m + \frac{1}{4}m_m\ell_m^2\right)\ddot{\theta}_m(t) + \mu_m\dot{\theta}_m(t) = \tau_h(t) + \tau_m(t). \tag{1}$$

2.2. **Dynamics of the flexible slave arm.** In this paper, a parallel-structured single-link flexible arm is employed as the slave arm (shown in Figure 3(a)). The flexible slave arm is constructed by a pair of uniform Euler-Bernoulli beams. One end of each beam is clamped to a hub unit, and the remaining ends are clamped to a tip mass. The derivation of an accurate mathematical model for a parallel-structured single-link flexible arm yields a system of highly complex nonlinear differential equations. However, the displacements of both beams are almost the same and the centrifugal force is assumed to be sufficiently small. Thus, the mathematical models for both beams are equivalent. Therefore, for the sake of simplicity, the flexible slave arm is regarded as an approximated model (shown in Figure 3(b)). In addition, we consider a simplified structural model of the flexible slave arm consisting of a single beam under the same boundary conditions as the original system approximated as the beams of a parallel-structured single-link flexible arm.

Let OXY denote an inertial Cartesian coordinate system and Oxy, a rotating coordinate system. $\theta(t)$ is defined as the angle between the OX and Ox axes, and $u(t,x)$ is taken as the transverse displacement of the flexible beam from the x axis. The uniform mass density ρ, the cross section S and the uniform flexible rigidity EI are the physical parameters of the beam, where E and I are Young's modulus and the second moment of cross-sectional area, respectively. J_0 is the moment of inertia of the slave motor shaft, μ is the coefficient of friction of the slave motor shaft, c_D is the coefficient of Kelvin-Voigt type damping, and m is the mass of the tip mass. In this paper, the tip mass is assumed to be a point mass; and therefore, its moment of inertia is zero. Because the flexible slave arm is treated as an Euler-Bernoulli beam, the rotary inertia and shear deformation are negligibly small: $|u(t,x)|^2 \ll |x|^2$ is satisfied. Furthermore, the tip mass is taken as non-rigid, and the rotational velocity is sufficiently small: $\left|\dot{\theta}(t)\right|^2 \ll 1$.

The total kinetic energy $T(t)$ and the potential energy $U(t)$ are given by

$$T(t) = T_h(t) + T_b(t) + T_{tm}(t) \tag{2}$$

$$U(t) = \int_0^\ell \frac{1}{2} EI \left\{u''(t,x)\right\}^2 dx \tag{3}$$

$$T_h(t) = \frac{1}{2} J_0 \dot{\theta}^2(t) \tag{4}$$

$$T_b(t) = \int_0^\ell \frac{1}{2}\rho S \left[\left\{x\dot{\theta}(t) + \dot{u}(t,x)\right\}^2 + u^2(t,x)\dot{\theta}^2(t)\right] dx \tag{5}$$

$$T_{tm}(t) = \frac{1}{2} m \left[\left\{\ell\dot{\theta}(t) + \dot{\bar{u}}(t)\right\}^2 + \left\{\bar{u}(t)\dot{\theta}(t)\right\}^2\right], \tag{6}$$

where $\bar{u}(t) := u(t,\ell)$ and the prime denotes a derivative with respect to x. The virtual work $\delta W(t)$ due to nonconservative forces is given by

$$\delta W(t) = -\frac{\partial F_0(t)}{\partial \dot{\theta}}\delta\theta - \int_0^\ell \frac{\partial \hat{F}(t,x)}{\partial \dot{u}''}\delta u'' dx + \tau(t)\delta\theta, \tag{7}$$

where $F_0(t)$ and $\hat{F}(t,x)$ are dissipation functions with respect to the rotation of the slave motor and internal damping of the slave arm is described by

$$F_0(t) = \frac{1}{2}\mu\dot{\theta}^2(t) \tag{8}$$

$$\hat{F}(t,x) = \frac{1}{2} c_D I \left\{\dot{u}''(t,x)\right\}^2. \tag{9}$$

The dynamics of the slave arm can be derived by using the Hamilton's principle [13]. Therefore, the dynamics with respect to the motor shaft and the flexible beam are given by the following equations (see Appendix A):

$$\begin{aligned} &J_0\ddot{\theta}(t) + \mu\dot{\theta}(t) - c_D I\{\dot{u}''(t,0) - \dot{u}''(t,\ell)\} - EI\{u''(t,0) - u''(t,\ell)\} \\ &+ m\bar{u}(t)\{2\dot{\bar{u}}(t)\dot{\theta}(t) + \bar{u}(t)\ddot{\theta}(t)\} + \int_0^\ell \rho S\{2u(t,x)u'(t,x)\dot{\theta}(t) + u^2(t,x)\ddot{\theta}(t)\}dx \\ &+ \int_0^\ell \rho S x u(t,x)\dot{\theta}^2(t)dx + m\ell\bar{u}(t)\dot{\theta}^2(t) = \tau(t) \end{aligned} \tag{10}$$

$$\rho S\ddot{u}(t,x) + c_D I\dot{u}''''(t,x) + EIu''''(t,x) = -\rho Sx\ddot{\theta}(t) - m\{\ell\ddot{\theta}(t) + \ddot{\bar{u}}(t)\}\delta(x-\ell) + \rho Su(t,x)\dot{\theta}^2(t) + m\bar{u}(t)\dot{\theta}^2(t)\delta(x-\ell). \quad (11)$$

The fifth to eighth terms on the left-hand side of (10) and the third and fourth terms on the right-hand side of (11) are nonlinear.

The boundary and initial conditions of (10) and (11) are given by

$$\text{B.C.}: u(t,0) = u'(t,0) = u'(t,\ell) = u'''(t,\ell) = 0 \quad (12)$$

$$\text{I.C.}: u(0,x) = \dot{u}(0,x) = 0, \;\; \theta(0) = \theta_0, \;\; \dot{\theta}(0) = 0. \quad (13)$$

3. **Controller Design.** In this section, we derive the controllers, namely the PD controller for the reaction torque of the master arm $\tau_m(t)$ and the PDS controller for the control torque of the slave arm $\tau(t)$. First, the candidate Lyapunov function $V(t)$ is defined as the sum of the energy of the master arm, the energy of the slave arm and the pseudo-energy. Then, the PD and PDS controllers are derived as $\dot{V}(t) = dV(t)/dt$. Finally, substituting $\tau_m(t)$ and $\tau(t)$ into $\dot{V}(t)$, we obtain $\dot{V}(t)$ to verify the stability.

Let us define the candidate Lyapunov function $V(t)$ as

$$V(t) = a_1T_h(t) + a_2\{T_b(t) + T_{tm}(t) + U(t)\} + a_3T_m(t) + \frac{1}{4}a_4\left[\{\theta_m(t-T_D) - \theta(t)\}^2 + \{\theta(t-T_D) - \theta_m(t)\}^2\right], \quad (14)$$

where a_1, a_2, a_3 and a_4 are positive constants. T_D represents the communication network delay, which is assumed to be positive constant in this paper. The candidate Lyapunov function $V(t)$ defined by (14) holds $V(t) \geq 0$ for all $\theta_m(t)$, $\dot{\theta}_m(t)$, $\theta(t)$, $\dot{\theta}(t)$, $u(t,x)$ and $\dot{u}(t,x)$ in this case, where a_1, a_2, a_3 and $a_4 > 0$. $V(t)$ holds $V(t) = 0$ if and only if $\theta_m(t-T_D) - \theta(t) = 0$, $\theta(t-T_D) - \theta_m(t) = 0$, $\dot{\theta}(t) = \dot{\theta}_m(t) = 0$ and $u(t,x) = \dot{u}(t,x) = 0$. Next, differentiating (14) with respect to t and substituting (1)-(13) into the resultant equation, we obtain

$$\begin{aligned}\dot{V}(t) = \Bigg[&-(a_2-a_1)\left[c_DI\left\{\dot{u}''(t,0) - \dot{u}''(t,\ell)\right\} + EI\left\{u''(t,0) - u''(t,\ell)\right\}\right] \\ &-\frac{1}{2}a_4\left\{\theta_m(t-T_D) - \theta(t)\right\} + a_1\tau(t)\Bigg]\dot{\theta}(t) \\ &+\left[-\frac{1}{2}a_4\left\{\theta(t-T_D) - \theta_m(t)\right\} + a_3\{\tau_h(t) + \tau_m(t)\}\right]\dot{\theta}_m(t) \\ &- a_2\int_0^\ell c_DI\{\dot{u}''(t,x)\}^2dx - a_1\mu\dot{\theta}^2(t) - a_3\mu_m\dot{\theta}_m^2(t) \\ &+ \frac{1}{2}a_4\{\theta_m(t-T_D) - \theta(t)\}\dot{\theta}_m(t-T_D) + \frac{1}{2}a_4\{\theta(t-T_D) - \theta_m(t)\}\dot{\theta}(t-T_D) \\ &+ (a_2-a_1)\Bigg[\int_0^\ell \rho S\{xu(t,x)\dot{\theta}^3(t) + 2u(t,x)\dot{u}(t,x)\dot{\theta}^2(t) + u^2(t,x)\dot{\theta}(t)\ddot{\theta}(t)\}dx \\ &+ m\left\{2\bar{u}(t)\dot{\bar{u}}(t)\dot{\theta}^2(t) + \ell\bar{u}(t)\dot{\theta}^3(t) + \bar{u}^2(t)\dot{\theta}(t)\ddot{\theta}(t)\right\}\Bigg]. \end{aligned} \quad (15)$$

In this paper, the PDS and PD controllers apply to the control torque for the flexible slave arm $\tau(t)$ and the reaction torque for the rigid master arm $\tau_m(t)$, respectively. The controllers are designed as follows:

$$\tau_m(t) = \frac{1}{a_3}\left\{\frac{a_4}{2}\{\theta(t-T_D) - \theta_m(t)\} - a_6\dot{\theta}_m(t)\right\} \tag{16}$$

$$\begin{aligned}\tau(t) = \frac{1}{a_1}\Bigg\{&\frac{a_4}{2}\{\theta_m(t-T_D) - \theta(t)\} - a_5\dot{\theta}(t) \\ &+ (a_2 - a_1)\Big[c_D I\{\dot{u}''(t,0) - \dot{u}''(t,\ell)\} + EI\{u''(t,0) - u''(t,\ell)\}\Big]\Bigg\},\end{aligned} \tag{17}$$

where a_5 and a_6 are positive constants. Then, substituting (16) and (17) into (15), we arrive at

$$\begin{aligned}\dot{V}(t) = &- a_2\int_0^\ell c_D I\{\dot{u}''(t,x)\}^2 dx - a_1\mu\dot{\theta}^2(t) \\ &- a_3\mu_m\dot{\theta}_m^2(t) - a_5\dot{\theta}^2(t) - a_6\dot{\theta}_m^2(t) + a_3\tau_h(t)\dot{\theta}_m(t) \\ &+ \frac{1}{2}a_4\{\theta_m(t-T_D) - \theta(t)\}\dot{\theta}_m(t-T_D) + \frac{1}{2}a_4\{\theta(t-T_D) - \theta_m(t)\}\dot{\theta}(t-T_D) \\ &+ (a_2 - a_1)\Bigg[\int_0^\ell \rho S\{xu(t,x)\dot{\theta}^3(t) + 2u(t,x)\dot{u}(t,x)\dot{\theta}^2(t) + u^2(t,x)\dot{\theta}(t)\ddot{\theta}(t)\}dx \\ &+ m\left\{2\bar{u}(t)\dot{\bar{u}}(t)\dot{\theta}^2(t) + \ell\bar{u}(t)\dot{\theta}^3(t) + \bar{u}^2(t)\dot{\theta}(t)\ddot{\theta}(t)\right\}\Bigg].\end{aligned} \tag{18}$$

As seen in this equation, the first to fifth terms on the right-hand side are trivial negative semidefinite and become zero if $\dot{u}''(t,x) = 0$, $\dot{\theta}(t) = 0$ and $\dot{\theta}_m(t) = 0$. To check the internal stability, we assume that the operational torque is zero, i.e., $\tau_h(t) \equiv 0$. The seventh and eighth terms on the right-hand side of (18) are negative semidefinite, which has already been proved in [16]. However, it is not obvious that the ninth term on the right-hand side of (18) is negative semidefinite, that is, the nonlinear term is investigated regardless of whether it is negative semidefinite. In the next section, it is proved that the value of the integral of the nonlinear term is negative semidefinite.

4. **Stability Analysis.** The value of the integral of (18) is written as follows for verifying the internal stability of the proposed system:

$$\begin{aligned}\int_0^t \dot{V}(\tau)d\tau = &-a_2\int_0^t\int_0^\ell c_D I\{\dot{u}''(\tau,x)\}^2 dxd\tau \\ &- a_1\mu\int_0^t \dot{\theta}^2(\tau)d\tau - a_3\mu_m\int_0^t \dot{\theta}_m^2(\tau)d\tau - a_5\int_0^t \dot{\theta}^2(\tau)d\tau - a_6\int_0^t \dot{\theta}_m^2(\tau)d\tau \\ &+ \frac{1}{2}a_4\int_0^t \{\theta_m(\tau-T_D) - \theta(\tau)\}\dot{\theta}_m(\tau-T_D)d\tau + \frac{1}{2}a_4\int_0^t \{\theta(\tau-T_D) - \theta_m(\tau)\}\dot{\theta}(\tau-T_D)d\tau \\ &+ (a_2 - a_1)\int_0^t\Bigg[\int_0^\ell \rho S\{xu(\tau,x)\dot{\theta}^3(\tau) + 2u(\tau,x)\dot{u}(\tau,x)\dot{\theta}^2(\tau) + u^2(\tau,x)\dot{\theta}(\tau)\ddot{\theta}(\tau)\}dx \\ &+ m\left\{2\bar{u}(\tau)\dot{\bar{u}}(\tau)\dot{\theta}^2(\tau) + \ell\bar{u}(\tau)\dot{\theta}^3(\tau) + \bar{u}^2(\tau)\dot{\theta}(\tau)\ddot{\theta}(\tau)\right\}\Bigg]d\tau.\end{aligned} \tag{19}$$

In this equation, the first five terms on the right-hand side are clearly negative, and the sixth and the seventh terms on the right-hand side are also negative, which is verified in our previous work [16]. However, it is not obvious whether the eighth term is negative. Therefore, the following function is defined:

$$V_N(t) := \int_0^t \left[\int_0^\ell \rho S \Big\{ xu(\tau,x)\dot{\theta}^3(\tau) + 2u(\tau,x)\dot{u}(\tau,x)\dot{\theta}^2(\tau) + u^2(\tau,x)\dot{\theta}(\tau)\ddot{\theta}(\tau) \Big\} dx + m\left\{ 2\bar{u}(\tau)\dot{\bar{u}}(\tau)\dot{\theta}^2(\tau) + \ell\bar{u}(\tau)\dot{\theta}^3(\tau) + \bar{u}^2(\tau)\dot{\theta}(\tau)\ddot{\theta}(\tau) \right\} \right] d\tau. \quad (20)$$

If $V_N(t)$ is negative semidefinite, the internal stability of our bilateral control system is guaranteed.

The second term on the right-hand side of this equation becomes

$$\begin{aligned} \int_0^t\!\!\int_0^\ell 2u(\tau,x)\dot{u}(\tau,x)\dot{\theta}^2(\tau)dxd\tau &= \int_0^t\!\!\int_0^\ell \left\{ \frac{d}{d\tau}u^2(\tau,x) \right\} \dot{\theta}^2(\tau)dxd\tau \\ &= \int_0^\ell u^2(t,x)\dot{\theta}^2 dx - \int_0^t\!\!\int_0^\ell 2u^2(\tau,x)\dot{\theta}(\tau)\ddot{\theta}(\tau)d\tau dx, \end{aligned} \quad (21)$$

and the fourth term on the right-hand side of (20) becomes

$$\int_0^t 2\bar{u}(\tau)\dot{\bar{u}}(\tau)\dot{\theta}^2(\tau)d\tau = \int_0^t \left\{ \frac{d}{d\tau}\bar{u}^2(\tau) \right\} \dot{\theta}^2(\tau)d\tau = \bar{u}^2(t) - \int_0^t 2\bar{u}^2(\tau)\dot{\theta}(\tau)\ddot{\theta}(\tau)d\tau. \quad (22)$$

Substituting (21) and (22) into (20), we obtain

$$\begin{aligned} V_N(t) = &\rho S \left[\int_0^t\!\!\int_0^\ell xu(\tau,x)\dot{\theta}^3(\tau)dxd\tau + \int_0^\ell u^2(t,x)\dot{\theta}^2(t)dx - \int_0^t\!\!\int_0^\ell u^2(\tau,x)\dot{\theta}(\tau)\ddot{\theta}(\tau)\}dxd\tau \right] \\ &+ m\left[\bar{u}^2(t) + \int_0^t \ell\bar{u}(\tau)\dot{\theta}^3(\tau)d\tau - \int_0^t \bar{u}^2(\tau)\dot{\theta}(\tau)\ddot{\theta}(\tau)d\tau \right]. \end{aligned} \quad (23)$$

Next, under the assumptions of $|u(t,x)|^2 \ll |x|^2$ and $\left|\dot{\theta}(t)\right|^2 \ll 1$, (23) can be approximated as

$$\begin{aligned} V_N(t) &\cong \rho S \int_0^t\!\!\int_0^\ell xu(\tau,x)\dot{\theta}^3(\tau)dxd\tau + m\ell\int_0^t \bar{u}(\tau)\dot{\theta}^3(\tau)d\tau \\ &\ll \rho S \int_0^t\!\!\int_0^\ell |x|^2 \left|\dot{\theta}^3(\tau)\right| dxd\tau + m\ell\int_0^t \ell\left|\dot{\theta}^3(\tau)\right| d\tau \\ &= \frac{1}{3}\rho S\ell^3 \int_0^t \left|\dot{\theta}^3(\tau)\right| d\tau + m\ell^2\int_0^t \left|\dot{\theta}^3(\tau)\right| d\tau \cong 0. \end{aligned} \quad (24)$$

Therefore, $V_N(t)$ is negative semidefinite. As a result, (19) satisfies the following inequality:

$$\int_0^t \dot{V}(\tau)d\tau \le 0. \quad (25)$$

Consequently, our bilateral control is asymptotically stable.

5. **Passivity Analysis.** To consider the passivity of our bilateral control system, (18) is rewritten as

$$\dot{V}(t) = a_3\tau_h(t)\dot{\theta}_m(t) - \Psi(t), \quad (26)$$

where $\Psi(t)$ is defined as

$$\begin{aligned}\Psi(t) :=& a_2 \int_0^\ell c_D I\{\dot{u}''(t,x)\}^2 dx + a_1\mu\dot{\theta}^2(t) + a_3\mu_m\dot{\theta}_m^2(t) + a_5\dot{\theta}^2(t) + a_6\dot{\theta}_m^2(t)\\ &- \frac{1}{2}a_4\{\theta_m(t-T)-\theta(t)\}\dot{\theta}_m(t-T) - \frac{1}{2}a_4\{\theta(t-T)-\theta_m(t)\}\dot{\theta}(t-T)\\ &- (a_2-a_1)\left[\int_0^\ell \rho S\{xu(\tau,x)\dot{\theta}^3(\tau) - 2u(\tau,x)\dot{u}(\tau,x)\dot{\theta}^2(\tau) + u^2(\tau,x)\dot{\theta}(\tau)\ddot{\theta}(\tau)\}dx\right.\\ &\left.- m\left\{2\bar{u}(\tau)\dot{\bar{u}}(\tau)\dot{\theta}^2(\tau) + \ell\bar{u}(\tau)\dot{\theta}^3(\tau) + \bar{u}^2(\tau)\dot{\theta}(\tau)\ddot{\theta}(\tau)\right\}\right]. \qquad (27)\end{aligned}$$

Then, the integral in (26) is evaluated as

$$\int_0^t \dot{V}(\tau)d\tau = \int_0^t a_3\tau_h(\tau)\dot{\theta}_m(\tau)d\tau - \int_0^t \Psi(t)d\tau. \qquad (28)$$

Because $V_N(t)$ is negative semidefinite, which is shown in (24), the second term on the right-hand side of (28) becomes a positively valued function. Therefore, the following inequality is obtained:

$$\int_0^t \dot{V}(\tau)d\tau \leq \int_0^t a_3\tau_h(\tau)\dot{\theta}_m(\tau)d\tau. \qquad (29)$$

Then, performing the integration on the left-hand side of (29), we obtain

$$V(t) - V(0) \leq \int_0^t a_3\tau_h(\tau)\dot{\theta}_m(\tau)d\tau, \qquad (30)$$

which is the dissipation inequality. This means that our bilateral control system satisfies the passivity condition.

6. **Numerical Simulations.** The flexible slave arm is assumed to be made of phosphor bronze. Its length and cross-sectional area (= thickness × width) are $\ell = 0.3$[m] and $S = 1.0 \times 10^{-3}$[m] $\times$ 4.0×10^{-2}[m], respectively. The other physical parameters of the master and slave arms are listed in Tables 1 and 2.

The state space model developed on the basis of the modal expansion method is employed for the numerical simulations. The number of modes N of our system was taken

Table 1. Physical parameters of the master arm

Symbol	Value	
J_h	0.70	[kg · m^2]
μ_m	3.03×10^{-2}	[kg · m^2 · s]
m_m	0.57	[kg]
ℓ_m	0.30	[m]
J_m	0.7172	[kg · m^2]

Table 2. Physical parameters of the slave arm

Symbol	Value	
J_0	0.70	[kg · m^2]
μ	3.03×10^{-2}	[kg · m^2 · s]
ℓ	0.30	[m]
ρ	8.8×10^3	[kg/m^3]
S	8.0×10^{-5}	[m^2]
E	1.1×10^{11}	[Pa]
I	2.67×10^{-11}	[m^4]
c_D	4.82×10^4	[N · s/m^2]
m	0.245	[kg]

TABLE 3. Resonance frequency [Hz]

Mode	Frequency [Hz]
1	20
2	109
3	269
4	501
5	804
6	1.178×10^3
7	1.623×10^3
8	2.140×10^3
9	2.728×10^3
10	3.387×10^3

as 10. Hence, the displacement $u(t,x)$ is expressed as follows:

$$u(t,x) \cong \sum_{k=1}^{N} u_k(t)\phi_k(x), \tag{31}$$

where $u_k(t)$ $(k = 1, 2, \cdots, N)$ is the modal displacement. $\phi_k(x)$ $(k = 1, 2, \cdots, N)$ is the eigenfunction corresponding to each eigenvalue λ_k $(k = 1, 2, \cdots, N)$. The eigenfunction $\phi_k(x)$ is the solution of the eigenvalue problem $\mathscr{A}\phi_k(x) = \lambda_k\phi_k(x)$, $\mathscr{A} := \{(EI)/(\rho S)\}d^4/dx^4$, which is conditioned by $\phi_k(0) = \phi_k'(0) = \phi_k'(\ell) = \phi_k'''(\ell) = 0$. By using (31), the left-hand side of (11) can be rewritten as the mode equation for $u_k(t)$ as follows:

$$(\text{Left-hand side of (11)}) = \ddot{u}_k(t) + \frac{c_D}{E}\lambda_k\dot{u}_k(t) + \lambda_k u_k(t). \tag{32}$$

This equation describes a second-order system. Hence, the damping ratio of each mode ζ_k is represented as

$$\zeta_k = \frac{c_D}{2E}\sqrt{\lambda_k}. \tag{33}$$

Therefore, the resonance angular frequency ω_k is given by

$$\omega_k = \omega_{0k}\sqrt{1-\zeta_k^2} \tag{34}$$

where $\omega_{0k}(:= \sqrt{\lambda_k})$ is the natural angular frequency of each mode. Table 3 indicates the resonance frequency of each mode, which is calculated by (34).

The numerical simulations are performed by using Matlab and Simulink. Instead of a human operator, we use a virtual operator controlled by the PD controller for driving the master arm in the numerical simulations. The virtual operator generates an operation torque $\tau_h(t)$ as follows:

$$\tau_h(t) = K_P\{\theta_h(t) - \theta_m(t)\} + K_D\left\{\dot{\theta}_h(t) - \dot{\theta}_m(t)\right\}, \tag{35}$$

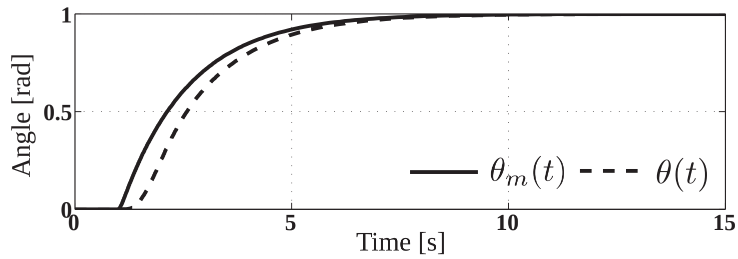
where $\theta_h(t)$ is the desired angle of the master arm. K_P and K_D are the proportional gain and the derivative gain, respectively. Next, we set $K_P = 10$ and $K_D = 15$. In these numerical simulations, $\theta_h(t)$ is taken as

$$\theta_h(t) = u_s(t - t_s), \tag{36}$$

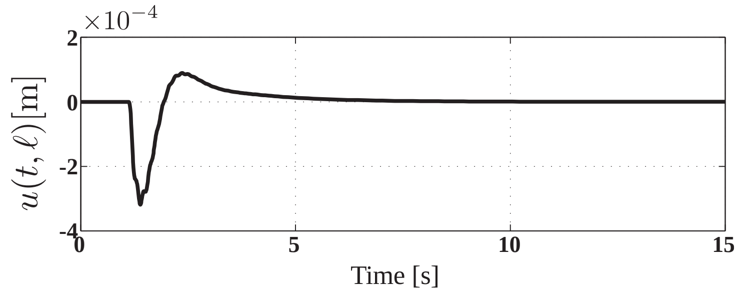
where $u_s(t)$ is the unit step function and t_s represents the time at which the virtual operator commenced operation. The initial conditions of our system are set as follows: $\theta_m(0) = 0$[rad], $\dot{\theta}_m(0) = 0$[rad/s], $\theta(0) = 0$[rad], $\dot{\theta}(0) = 0$[rad/s], $u(0,x) = 0$[m] and

$\dot{u}(0, x) = 0$[m/s]. The delay time of the communication network is set to $T_D = 0.1,\ 0.5$[s], and the starting time of the virtual operator is set to $t_s = 1.0$[s]. Furthermore, the weight coefficients of the Lyapunov function are chosen as $a_1 = 0.1$, $a_2 = 2.1$, $a_3 = 20$, $a_4 = 4.0$, $a_5 = 0.7$ and $a_6 = 20$. The gains of the reaction torque of the master arm $\tau_m(t)$ and the control torque of the slave arm $\tau(t)$ are set by using (16) and (17).

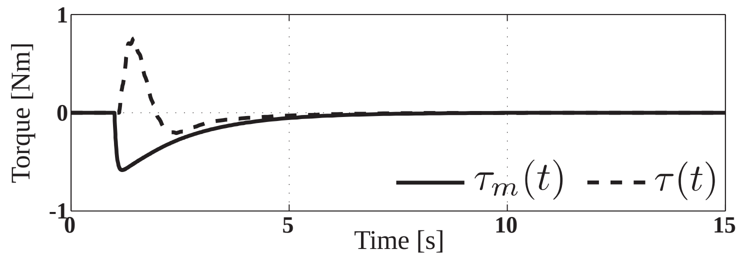
6.1. **Numerical results.** Figures 4 and 5 present the results of the numerical simulations. In these simulations, we assume that the master arm is controlled to follow a step-like trajectory. In Figures 4 and 5, (a) shows the angles of the master arm and the slave arm; (b) shows the displacement $u(t, \ell)$ of the tip of the slave arm; (c) shows the



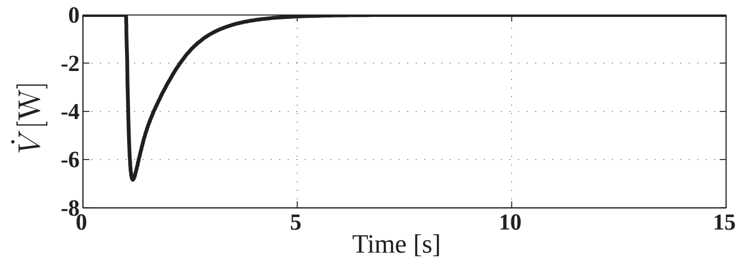
(a) The result of angles



(b) The result of the tip displacement

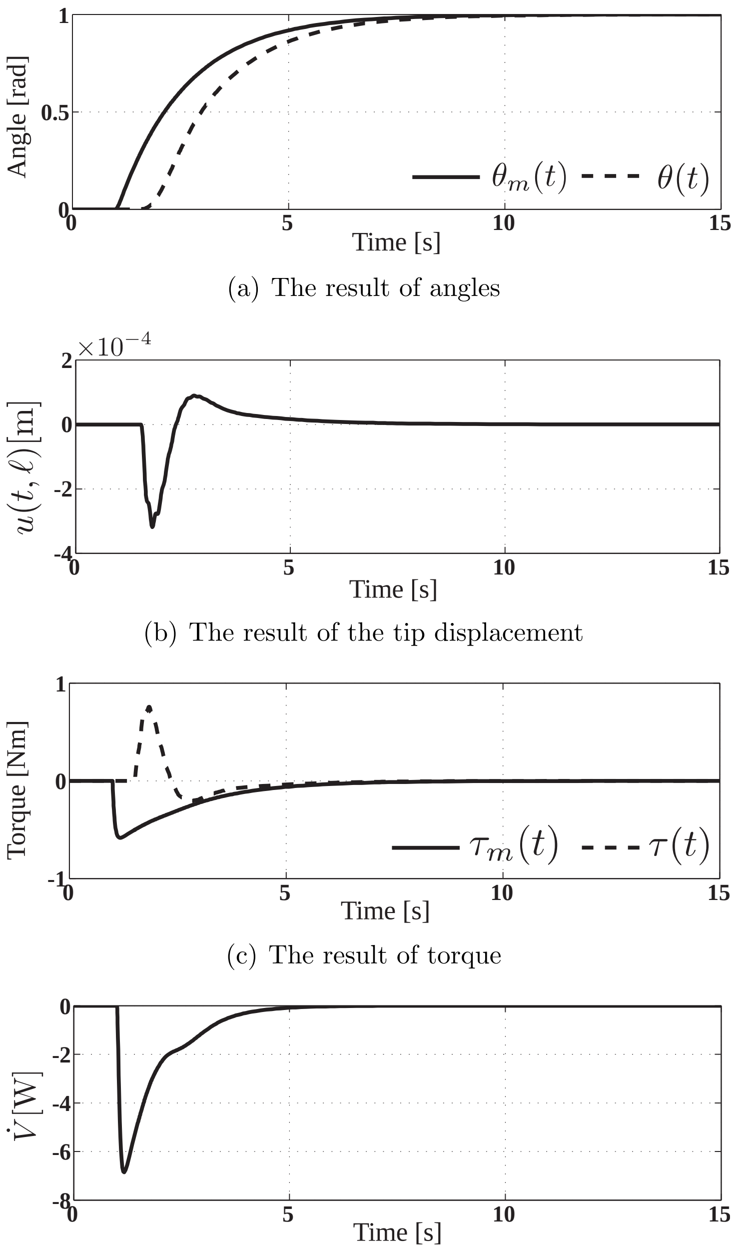


(c) The result of torque



(d) The result of $\dot{V}(t)$

FIGURE 4. Numerical simulation results for the proposed bilateral control system with time delay. (Input: step signal; time delay: $T_D = 0.1$[s].)



(a) The result of angles

(b) The result of the tip displacement

(c) The result of torque

(d) The result of $\dot{V}(t)$

FIGURE 5. Numerical simulation results for the proposed bilateral control system with time delay. (Input: step signal; time delay: $T_D = 0.5$[s].)

reaction torque of the master arm $\tau_m(t)$ and the control torque of the slave arm $\tau(t)$; and (d) shows the time derivative of the Lyapunov function $\dot{V}(t)$. Figure 6 shows the result for the tip's displacement $u(t, \ell)$ with the PD controller.

As seen in Figures 4 and 5, the angle of the flexible slave arm tracks the angle of the rigid master arm with a constant time delay $T_D = 0.1$ or 0.5 [s]. With respect to the displacement of the tip of the flexible slave arm, overshoot can be confirmed at $t = 2$ [s]. However, the vibration of the tip quickly converges to zero due to the effect of the PDS controller. Thus, the controllers as designed in this paper are suitable for application to master-slave arms systems (see the lower middle panel of Figures 4 and 5). Furthermore, in Figure 6, the tip's displacement vibrates during the simulation regardless of the no

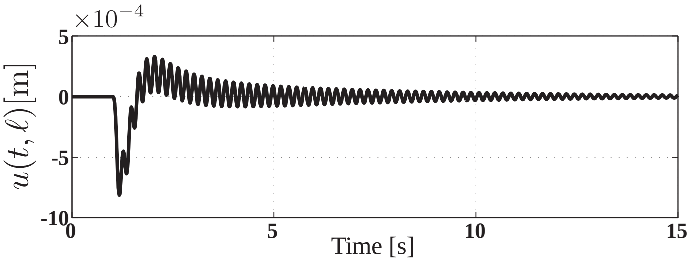


FIGURE 6. The result of the simulation with time delay. (Input: step signal; $T_D = 0$[s]; controller: PD.)

delay. Hence, the effectiveness of the PDS controller is also indicated. However, it is not clear whether the reaction torque of the master arm $\tau_m(t)$ is practical. Also, the time derivative of the Lyapunov function $\dot{V}(t)$ remains negative during the numerical simulations, ensuring the stability of our bilateral control system.

7. **Conclusions.** A bilateral control system for flexible master-slave arms controlled over a communication network was investigated. The flexible arm was modeled as a nonlinear system, and the communication network was assumed to have a constant time delay. The stability of the proposed bilateral control system was proved as follows. First, the candidate Lyapunov function was constructed from the total energy of our FMSA. Then, controllers which generate the reaction torque of the rigid master arm and the control torque of the flexible slave arm were derived by taking the time derivative of the candidate Lyapunov function. Finally, the stability of our bilateral control system was verified by using the reaction and control torques. The range of the gains of these controllers was set such that the candidate Lyapunov function became the actual Lyapunov function. From the viewpoint of the Lyapunov function theory, the stability and passivity of our bilateral control system were proved under some assumptions. The performance of the proposed bilateral system was demonstrated through numerical simulations.

By this research, it was confirmed that the flexible arm was accurately controlled through the communication network having the constant time delay in the numerical simulations. In the future, we should confirm it by experimental studies. As a result, the proposed bilateral control system will be very useful to help people who live in the distance place, when this system will be introduced into general environments.

REFERENCES

[1] H. Shingin and Y. Ohta, Disturbance rejection with information constraints, *Proc. of the 17th International Symposium on Mathematical Theory of Networks and Systems*, pp.2125-2129, 2006.
[2] W. Zhang, M. S. Branicky and S. M. Phillips, Stability of networked control systems, *IEEE Control Systems Magazine*, vol.21, pp.84-99, 2001.
[3] R. Sipahi, S. Niculescu, C. T. Abdallah, W. Michiels and K. Gu, Stability and stabilization of systems with time delay, *IEEE Control Systems Magazine*, vol.31, no.1, pp.38-65, 2011.
[4] R. J. Anderson and M. W. Spong, Bilateral control of teleoperators with time delay, *IEEE Trans. on Automatic Control*, vol.34, no.5, pp.494-501, 1989.
[5] G. Niemeyer and J.-J. E. Slotine, Stable adaptive teleoperation, *IEEE Journal of Oceanic Engineering*, vol.16, no.1, pp.152-162, 1991.
[6] T. Namerikawa, Bilateral control with constant feedback fains for teleoperation with time varying delay, *Proc. of the 48th IEEE Conf. on Decision and Control, 2009 Held Jointly with 2009 the 28th Chinese Control Conference*, Shanghai, China, pp.7527-7532, 2009.
[7] S. Nurung and I. Nilkhamhang, A robust adaptive algorithm for bilateral control system without force sensor with time delay, *Proc. of SICE Annual Conference 2010*, Taipei, pp.678-683, 2010.

[8] F. Matsuno, T. Asano and Y. Sakawa, Modeling and quasi-static hybrid position/force control of constrained planar two-link flexible manipulators, *IEEE Trans. on Robotics and Automation*, vol.10, no.3, pp.287-297, 1994.

[9] F. Matsuno, T. Ohno and Y. Orlov, PDS (proportional derivative and strain) feedback control of a flexible structure with closed-loop mechanism, *Proc. of the 38th Conf. on Decision and Control*, Phoenix, Arizona, pp.4331-4335, 1999.

[10] R. N. Banavar and P. Dominic, An LQG/H_∞ controller for a flexible manipulator, *IEEE Trans. on Control Systems Technology*, vol.3, no.4, pp.409-416, 1995.

[11] S.-S. Ge, T.-H. Lee and G. Zhu, Improving regulation of a single-link flexible manipulator with strain feedback, *IEEE Trans. on Robotics and Automation*, vol.14, no.1, pp.179-185, 1998.

[12] S. Saito, M. Deng, A. Inoue and C. Jiang, Vibration control of a flexible arm experimental system with hysteresis of piezoelectric actuator, *International Journal of Innovative Computing, Information and Control*, vol.6, no.7, pp.2965-2975, 2010.

[13] Y. Sawada and J. Kondo, Kalman filter based LEQG control of a parallel-structured single-link flexible arm mounted on moving base, *International Journal of Innovative Computing, Information and Control*, vol.6, no.1, pp.29-42, 2010.

[14] Y. Hoshino, Y. Kobayashi and Y. Furuta, Motion and force control of flexible master-slave system (improvement of state estimation during contacting with objects), *Trans. of the Japan Society of Mechanical Engineers, Series C*, vol.74, no.740, pp.91-98, 2008 (in Japanese).

[15] T. Mori, Y. Morita, H. Ukai and H. Kando, A study on passivity of flexible master-slave manipulators, *SICE Annual Conference*, pp.2407-2410, 2004.

[16] M. Yagi, A. Hayashi, Y. Sawada and H. Kimura, Bilateral control of master-slave flexible arm with time delay, *Proc. of SICE the 12th Annual Conference on Control Systems*, Akita, 2012.

Appendix A. **Derivation of the Mathematical Model of the Slave Arm.** The mathematical model of the slave arm is derived by using the Hamilton's principle. The position vectors of the arm and the tip mass are given by

$$r(t,x) = \begin{bmatrix} x\cos\theta(t) - u(t,x)\sin\theta(t) \\ x\sin\theta(t) + u(t,x)\cos\theta(t) \end{bmatrix} \tag{37}$$

$$p(t) = \begin{bmatrix} \ell\cos\theta(t) - \bar{u}(t)\sin\theta(t) \\ \ell\sin\theta(t) + \bar{u}(t)\cos\theta(t) \end{bmatrix}, \tag{38}$$

where $\bar{u}(t) = u(t,\ell)$. The total kinetic energy $T(t)$ and the position energy $U(t)$ are defined as follows:

$$T(t) = T_0(t) + \int_0^\ell \hat{T}(t,x)dx \tag{39}$$

$$T_0(t) = \frac{1}{2}J_0\dot{\theta}^2(t) + \frac{1}{2}m[\dot{p}^T(t)\dot{p}(t)], \quad \hat{T}(t,x) = \frac{1}{2}\rho S[\dot{r}^T(t,x)\dot{r}(t,x)] \tag{40}$$

$$U(t) = \int_0^\ell \hat{U}(t,x)dx, \quad \hat{U}(t,x) = \frac{1}{2}EI\{u''(t,x)\}^2, \tag{41}$$

u, $\dot{u}$, u'', $\dot{u}''$ and $\dot{\theta}$ are selected as the generalized coordinates. The variation of (39)-(41) is written as

$$\delta T(t) = \frac{\partial T_0}{\partial\dot{\theta}}\delta\dot{\theta} + \frac{\partial T_0}{\partial\dot{\bar{u}}}\delta\dot{\bar{u}} + \int_0^\ell \left(\frac{\partial\hat{T}}{\partial\dot{\theta}}\delta\dot{\theta} + \frac{\partial\hat{T}}{\partial\dot{u}}\delta\dot{u}\right)dx \tag{42}$$

$$\delta U(t) = \int_0^\ell \frac{\partial\hat{U}}{\partial u''}\delta u''dx. \tag{43}$$

The virtual work $\delta W(t)$ due to nonconservative forces and the dissipation functions $F_0(t)$ and $\hat{F}(t,x)$ are given by

$$\delta W(t) = -\frac{\partial F_0(t)}{\partial \dot{\theta}}\delta\theta - \int_0^\ell \frac{\partial \hat{F}(t,x)}{\partial \dot{u}''}\delta u'' dx + \tau(t)\delta\theta \tag{44}$$

$$F_0(t) = \frac{1}{2}\mu\dot{\theta}^2(t), \;\; \hat{F}(t,x) = \frac{1}{2}c_D I \left\{\dot{u}''(t,x)\right\}^2. \tag{45}$$

By using the Hamilton's principle:

$$\int_{t_1}^{t_2} \left\{\delta T(t) - \delta U(t) + \delta W(t)\right\} dt = 0, \tag{46}$$

substituting (42)-(45) into (46), we obtain

$$\begin{aligned}
&\int_{t_1}^{t_2}\left\{-\frac{\partial}{\partial t}\left(\frac{\partial T_0}{\partial \dot{\theta}}\right) - \int_0^\ell \frac{\partial}{\partial t}\left(\frac{\partial \hat{T}}{\partial \dot{\theta}}\right) dx - \frac{\partial F_0}{\partial \dot{\theta}} + \tau(t)\right\}\delta\theta dt \\
&+ \int_{t_1}^{t_2}\int_0^\ell \left[-\frac{\partial}{\partial t}\left(\frac{\partial \hat{T}}{\partial \dot{u}}\right) - \frac{\partial^2}{\partial x^2}\left(\frac{\partial \hat{U}}{\partial u''}\right) - \frac{\partial^2}{\partial x^2}\left(\frac{\partial \hat{F}}{\partial \dot{u}''}\right)\right]\delta u dx dt - \int_{t_1}^{t_2} \frac{\partial}{\partial t}\left(\frac{\partial T_0}{\partial \dot{\bar{u}}}\right)\delta \bar{u} dt \\
&+ \int_{t_1}^{t_2}\left[\left\{\frac{\partial}{\partial x}\left(\frac{\partial \hat{U}}{\partial u''}\right) + \frac{\partial}{\partial x}\left(\frac{\partial \hat{F}}{\partial \dot{u}''}\right)\right\}\delta u\right]_{x=0}^{x=\ell} dt - \int_{t_1}^{t_2}\left[\left(\frac{\partial \hat{U}}{\partial u''} + \frac{\partial \hat{F}}{\partial \dot{u}''}\right)\delta u\right]_{x=0}^{x=\ell} dt = 0,
\end{aligned} \tag{47}$$

where

$$T_0(t) = \frac{1}{2}J_0\dot{\theta}^2(t) + \frac{1}{2}m\left[\left\{\ell\dot{\theta}(t) + \dot{\bar{u}}(t)\right\}^2 + \left\{\bar{u}(t)\dot{\theta}(t)\right\}^2\right] \tag{48}$$

$$\frac{d}{dt}\left(\frac{\partial T_0}{\partial \dot{\theta}}\right) = J_0\ddot{\theta}(t) + m\left\{\ell^2\ddot{\theta}(t) + \ell\ddot{\bar{u}}(t) + 2\bar{u}(t)\dot{\bar{u}}(t)\dot{\theta}(t) + \bar{u}^2(t)\ddot{\theta}(t)\right\} \tag{49}$$

$$\frac{d}{dt}\left(\frac{\partial T_0}{\partial \dot{\bar{u}}}\right) = m\left\{\ell\ddot{\theta}(t) + \ddot{\bar{u}}(t)\right\} \tag{50}$$

$$\hat{T}(t,x) = \frac{1}{2}\rho S\left[\left\{x\dot{\theta}(t) + \dot{u}(t,x)\right\}^2 + u^2(t,x)\dot{\theta}^2(t)\right] \tag{51}$$

$$\frac{d}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{\theta}}\right) = \rho S\left\{x^2\ddot{\theta}(t) + x\ddot{u}(t,x) + 2u(t,x)\dot{u}(t,x)\dot{\theta}(t) + u^2(t,x)\ddot{\theta}(t)\right\} \tag{52}$$

$$\frac{d}{dt}\left(\frac{\partial \hat{T}}{\partial \dot{u}}\right) = \rho S\left\{x\ddot{\theta}(t) + \ddot{u}(t,x)\right\} \tag{53}$$

$$\frac{\partial \hat{U}}{\partial u''} = EIu''(t,x), \;\; \frac{\partial}{\partial x}\left(\frac{\partial \hat{U}}{\partial u''}\right) = EIu'''(t,x), \;\; \frac{\partial^2}{\partial x^2}\left(\frac{\partial \hat{U}}{\partial u''}\right) = EIu''''(t,x) \tag{54}$$

$$\frac{\partial F_0}{\partial \dot{\theta}} = \mu\dot{\theta}(t), \;\; \frac{\partial \hat{F}}{\partial \dot{u}''} = c_D I\dot{u}''(t,x) \tag{55}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \hat{F}}{\partial \dot{u}''}\right) = c_D I\dot{u}'''(t,x), \;\; \frac{\partial^2}{\partial x^2}\left(\frac{\partial \hat{F}}{\partial \dot{u}''}\right) = c_D I\dot{u}''''(t,x). \tag{56}$$

Then, for considering $\bar{u}(t)$, we have

$$\bar{u}(t) = u(t,\ell) = \int_0^\ell u(t,x)\delta(x-\ell)dx \tag{57}$$

$$\begin{aligned}\delta\bar{u}(t) &= \bar{u}(t+\Delta t) - \bar{u}(t)\\ &= \int_0^\ell \{u(t+\Delta t,x) - u(t,x)\}\,\delta(x-\ell)dx = \int_0^\ell \delta u(t,x)\delta(x-\ell)dx.\end{aligned} \tag{58}$$

Substituting (48)-(56), (58) into (47), we obtain the following dynamics of the slave arm:

$$\begin{aligned}&J_0\ddot{\theta}(t) + \mu\dot{\theta}(t) - c_D I\{\dot{u}''(t,0) - \dot{u}''(t,\ell)\} - EI\{u''(t,0) - u''(t,\ell)\}\\ &+ m\bar{u}(t)\{2\dot{\bar{u}}(t)\dot{\theta}(t) + \bar{u}(t)\ddot{\theta}(t)\} + \int_0^\ell \rho S\{2u(t,x)u'(t,x)\dot{\theta}(t) + u^2(t,x)\ddot{\theta}(t)\}dx\\ &+ \int_0^\ell \rho Sxu(t,x)\dot{\theta}^2(t)dx + m\ell\bar{u}(t)\dot{\theta}^2(t) - \tau(t) = 0\end{aligned} \tag{59}$$

$$\begin{aligned}&\rho S\ddot{u}(t,x) + c_D I\dot{u}''''(t,x) + EIu''''(t,x) + \rho Sx\ddot{\theta}(t) + m\{\ell\ddot{\theta}(t) + \ddot{\bar{u}}(t)\}\delta(x-\ell)\\ &- \rho Su(t,x)\dot{\theta}^2(t) - m\bar{u}(t)\dot{\theta}^2(t)\delta(x-\ell) = 0\end{aligned} \tag{60}$$

$$\text{B.C.} : u(t,0) = u'(t,0) = u'(t,\ell) = u'''(t,\ell) = 0. \tag{61}$$