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ROBUST ADAPTIVE INVERSE DYNAMICS CONTROL FOR UNCERTAIN ROBOT MANIPULATOR

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Abstract. *On the basis of inverse dynamics controller as a nominal control portion, two types of novel robust adaptive inverse dynamics control schemes are proposed for the trajectory tracking control of robot manipulator with uncertain dynamics. They are composed of an adaptive fuzzy control algorithm and a nonlinear H_∞ tracking control model. The adaptive fuzzy control algorithm is employed to approximate the structured uncertainties, and the nonlinear robust H_∞ control model is designed to eliminate the effects of the unstructured uncertainties and approximation errors. The tuning parameters in the adaptive fuzzy control algorithm are derived through the Lyapunov method. Comparison studies of their control performances with the conventional inverse dynamics controller are carried out, and their validity is demonstrated by numerical simulations of a two-link rotary robot manipulator.*

Keywords: Inverse dynamics control, Robust H_∞ control, Adaptive fuzzy control, Robot manipulator with uncertain dynamics

1. **Introduction.** Motion control of robot manipulator is a difficult task, mainly because of highly nonlinear coupled and time-varying system with uncertainties such as load variations, friction and external disturbances [1]. These model uncertainties can be divided into the structured uncertainties stemming from the unknown kinematic parameters or nonlinear coupled dynamic model, and the unstructured uncertainties including changing payload, nonlinear friction and unknown external disturbance. Adaptive and robust control schemes are one of the most effective and popular means to handle these model uncertainties [2]. Although adaptive control scheme has the strong learning ability, its control performance may be affected by some unstructured uncertainties with the boundless energy [3]. On the other hand, robust control method is applicable to robot manipulator with the known upper bounds of the uncertainties, and it can also perform better in rejecting disturbances and compensating the unstructured uncertainties [4]. To overcome the shortcomings and to take advantage of the attractive features of the robust and adaptive control methods, some robust adaptive control strategies are proposed. Dou and Wang [5] proposed a robust adaptive synchronization motion controller for a two-link robot manipulator. Wu et al. [6] developed an adaptive sliding mode control scheme to guarantee the globally asymptotic convergence in the presence of unknown kinematic parameters and external disturbances. Yao et al. [7] presented a robust adaptive controller, but it neglects the effects of the external disturbances and nonlinear friction forces. Most robust adaptive control schemes have solely taken into account compensation of either nonlinear friction or uncertain kinematic parameters, or just provided an overall compensation control for structured and unstructured uncertainties. However, no attention has

been paid to separate compensation control for the structured and unstructured uncertainties. As a matter of fact, there exist great differences between the structured and unstructured uncertainties in many ways, for example, the structured uncertainties are characterized by the existence of an upper bound, but the unstructured uncertainties may not be bounded, even be of finite energy. Therefore, this paper will develop a novel robust adaptive control strategy to separately compensate these structured and unstructured uncertainties in terms of their character.

In addition, the adaptive control law in most robust adaptive control schemes is based on either the sliding mode control or the backsteping control methods. Under certain conditions, the sliding mode control is robust with respect to the perturbation and external disturbance [8], but it can also produce some drawbacks associated with large control chattering that may excite undesirable high-frequency dynamics [9]. Moreover, the backstepping control method suffers from the so-called problem of "explosion of complexity", which is caused by repeated differentiations of certain nonlinear functions, thus inevitably leads to a complicate algorithm with heavy computation burden [10]. One of the most effective model-based control approaches is the inverse dynamics controller, which can offer a large variety of advantages over model-free methods if the dynamic model is accurate enough [11]. Unfortunately, obtaining an accurate dynamic model is a challenging task, as simplifications are usually made and some uncertainties, such as friction or backlash, are not taken into account in the dynamic model [12]. It is well known that a fuzzy logic system is a particularly powerful tool for modeling uncertain nonlinear systems due to its universal approximation property [13-15]. Many adaptive fuzzy inverse dynamics controllers have been developed for a wide class of uncertain nonlinear systems. Mohan and Bhanot [16] conducted an investigation on three kinds of adaptive fuzzy inverse dynamics controllers; however, they may suffer from a tedious and cumbersome computation burden as it is a lookup table-based control scheme rather than a self-adaptive fuzzy control algorithm. Li et al. [17] developed an adaptive fuzzy output feedback control approach combining a fuzzy logic system with an adaptive fuzzy filter; however, the tuning number is so large that the learning time of the fuzzy logic system tends to become very long. In order to improve the adaptive control performances of the inverse dynamics controller, this paper will discuss a novel adaptive fuzzy control algorithm to approximate the structured uncertainties with the unknown upper bound. The main advantage of the adaptive fuzzy control algorithm is that no matter how many rules are utilized in the fuzzy logic system, only one tuning parameter will be adjusted on-line, which significantly reduces the computation burden. Another advantage is that the fuzzy logic system can approximate the structured uncertainties with the unknown upper bound.

2. Problem Formulation and Some Preliminaries.

2.1. Description of robot manipulator dynamic model.

The dynamic equation of an n degrees-of-freedom robot manipulator in joint space coordinates can be expressed as

$$D(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(q, \dot{q}) = \tau \tag{1}$$

where $q, \dot{q}, \ddot{q} \in R^n$ are the vectors of joint position, velocity and acceleration, respectively; $D(q) \in R^{n\times n}$ is a symmetric positive definite inertia matrix; $C(q, \dot{q})\dot{q} \in R^n$ expresses a vector of Coriolis and centrifugal forces; $G(q) \in R^n$ denotes a gravity vector; $F(q, \dot{q}) \in R^n$ includes friction terms and external disturbances; and $\tau \in R^n$ represents a vector of torque exerted on joints. For convenience, dynamic Equation (1) can be rewritten as follows.

$$D(q)\ddot{q} + H(q, \dot{q}) + F(q, \dot{q}) = \tau \tag{2}$$

where $H(q,\dot{q}) = C(q,\dot{q})\dot{q}+G(q)$. And, the actual parameters $D(q)$ and $H(q,\dot{q})$ in Equation (1) are assumed to be separated as the nominal parts denoted by $\hat{D}(q)$ and $\hat{H}(q,\dot{q})$, and the uncertain parts defined by $\Delta D(q)$ and $\Delta H(q,\dot{q})$. Hence, these actual and nominal parameters satisfy the following relationships.

$$\begin{cases} \hat{D}(q) = D(q) - \Delta D(q) \\ \hat{H}(q) = H(q,\dot{q}) - \Delta H(q,\dot{q}) \end{cases} \tag{3}$$

The inverse dynamics control law can be written as follows.

$$\tau = \hat{D}\left(q\right)\left(\ddot{q}_d + K_v\dot{e} + K_p e\right) + \hat{H}\left(q,\dot{q}\right) \tag{4}$$

where K_v and K_p are derivative and proportional constant matrices, respectively; $q_d, \dot{q}_d, \ddot{q}_d \in R^n$ are the vector of joint position, velocity, and acceleration in the desired trajectories, respectively; and $e = q_d - q$ denotes as a vector of trajectory tracking error. Substituting Equation (4) into Equation (2) yields

$$\ddot{e} + K_v\dot{e} + K_p e = \psi\left(\chi_q\right) + \delta\left(\chi_q\right) \tag{5}$$

where $\psi(\chi_q)$ denotes the structured uncertainties, and its value is equal to $\psi(\chi_q) = \hat{D}(q)^{-1}(\Delta D(q)\ddot{q} + \Delta H(q,\dot{q}))$; $\delta(\chi_q) = \hat{D}(q)^{-1}F(q,\dot{q})$ expresses the unstructured uncertainties; and $\chi_q = \begin{bmatrix} q & \dot{q} & \ddot{q} \end{bmatrix}^T$.

From the closed loop tracking error dynamic Equation (5), it can be easily seen that the structured uncertainties $\psi(\chi_q)$ results from the unknown kinematic parameters or nonlinear coupled dynamic model, and the unstructured uncertainties $\delta(\chi_q)$ includes the nonlinear friction forces and external disturbance. A compensation control scheme composed of an adaptive fuzzy control algorithm and a robust H_∞ control model should be developed to ensure the control performances of the inverse dynamics controller. In this way, the compensation control law can be designed as

$$\tau = \tau_0 + \tau_c \tag{6}$$

where τ_0 denotes the control input torque of the inverse dynamics controller defined by Equation (4), and τ_c expresses the control input torque of the compensation control scheme. Replacing the term τ_0 in Equation (6) with Equation (4), and we can get the following overall control law: $\tau = \hat{D}(q)(\ddot{q}_d + K_v\dot{e} + K_p e) + \hat{H}(q,\dot{q}) + \tau_c$.

Then, substituting the overall control law τ into the inverse dynamics controller defined by Equation (4) yields the following closed loop tracking error dynamic equation.

$$\ddot{e} + K_v\dot{e} + K_p e = \psi(\chi_q) + \delta(\chi_q) - \hat{D}(q)^{-1}\tau_c \tag{7}$$

Given that state error vector is defined by $x = [e, \dot{e}]^T$, the state space form of the tracking error dynamic Equation (7) can be rewritten as follows.

$$\dot{x} = Ax + B(\psi(\chi_q) + \delta(\chi_q) - \hat{D}(q)^{-1}\tau_c) \tag{8}$$

where $A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ I \end{bmatrix}$.

From the above closed loop tracking error dynamic equation, a conclusion can be drawn that the main goal of this paper is to design a compensation control law τ_c combining an adaptive fuzzy control algorithm with a robust control model to eliminate the effects of the uncertainties on the control performances.

2.2. **Adaptive fuzzy logic system.** A fuzzy logic system consists of the following four parts: the knowledge base, the fuzzifier, the fuzzy inference engine and the defuzzifier. The knowledge base for a fuzzy logic system is composed of a collection of the following fuzzy IF-THEN rules:

$$R^l: \text{ If } x_1 \text{ is } F_1^l \text{ and } \dots \text{ and } x_n \text{ is } F_n^l, \text{ then } y \text{ is } G^l, \ l = 1, 2, \cdots, M$$

where $x = (x_1, \cdots, x_n)^T$ and y are the input and output vectors of the fuzzy logic system, respectively; F_i^l and G^l are two fuzzy sets associated with the fuzzy functions $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$; and M is the number of fuzzy IF-THEN rules.

Through the singleton function, center average defuzzification and product inference, the fuzzy logic system can be expressed as follows.

$$y(x) = \frac{\sum\limits_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)\bar{y}_l}{\sum\limits_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)} \tag{9}$$

where $\bar{y}_l = \max_{y \in R} \mu_{G^l}(y)$. And then, the fuzzy basis functions can be defined as

$$\xi_l(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum\limits_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)} \tag{10}$$

Let $\Theta = [\bar{y}_1, \bar{y}_2, \cdots, \bar{y}_M]^T$ and $\xi^T(x) = [\xi_1(x), \xi_2(x), \cdots, \xi_M(x)]$, then Equation (9) can be rewritten as

$$y(x) = \Theta^T \xi(x) \tag{11}$$

It has been proved that the fuzzy logic system defined by Equation (11) can approximate any continuous function $f(x)$ over a compact set $\Omega \subset R^n$ to an arbitrary accuracy [17].

$$f(x) = \Theta^{*T} \xi(x) + \varepsilon(x) \tag{12}$$

where $\varepsilon(x)$ is the fuzzy minimum approximation error, and Θ^* is an optimal weight matrix satisfying that

$$\Theta^* = \arg \min_{\Theta \in U} \left\{ \sup_{x \in \Omega} \left| f(x) - \Theta^T \xi(x) \right| \right\} \tag{13}$$

In order to approximate the structured uncertainties $\psi(\chi_q)$ with the unknown upper bound in the closed loop tracking error dynamic Equation (7), an adaptive fuzzy logic system and its adaptive control law are designed as follows in this paper.

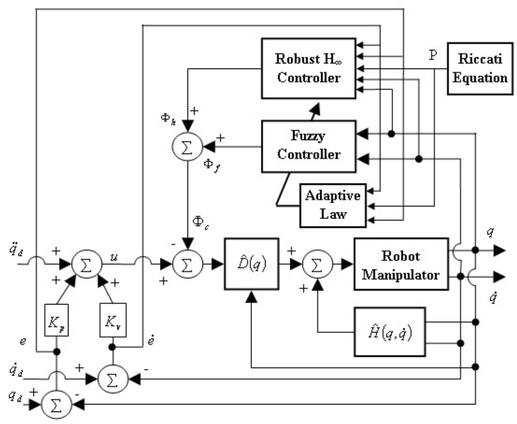
$$\Phi_f\left(x, \hat{\Theta}\right) = \rho\left(x, \hat{\Theta}\right) \tanh\left(\frac{\rho\left(x, \hat{\Theta}\right) B^T P x}{\varepsilon}\right) \tag{14}$$

$$\dot{\hat{\Theta}} = -\lambda \hat{\Theta} + L\xi(x) \left\| B^T P x \right\| \tag{15}$$

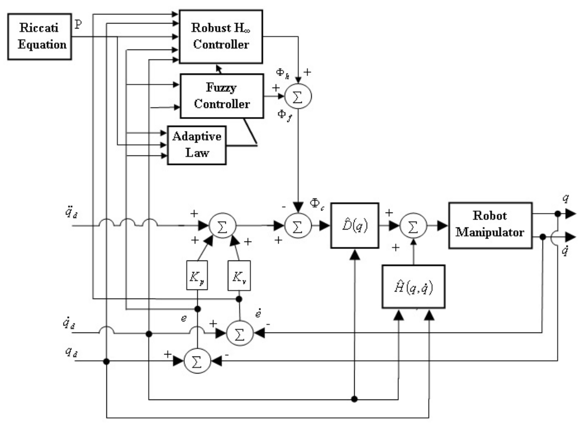
where $\rho\left(x, \hat{\Theta}\right) = \hat{\Theta}^T \xi(x)$ is the output vector of the adaptive fuzzy controller defined by Equation (11); $x = \begin{bmatrix} q & \dot{q} \end{bmatrix}^T$ is the input vector of the adaptive fuzzy controller; $\lambda \in (0, \infty)$, $L = diag\{l_1, l_2, \cdots, l_p\}$, $l_i \in (0, \infty)$, p is the dimension of the vector Θ, ε is an arbitrary small positive constant; $\lambda, l_i, \varepsilon$ are the parameters determined by the designer; and P is a symmetric positive definite matrix satisfying a Riccati-like equation.

3. **Design of Robust Adaptive Inverse Dynamics Controller.** In this section, two types of robust adaptive inverse dynamics controllers combining an adaptive fuzzy control algorithm with a nonlinear robust H_∞ tracking control model will be developed to deal with the control problem of robot manipulator with the structured and unstructured uncertainties.

3.1. **Robust adaptive inverse dynamics control schemes.** Figure 1(a) demonstrates the robust adaptive control scheme based on feedback compensator (FBC-based), which takes the actual output commands as the input vectors of the fuzzy logic system, and utilizes the trajectory tracking errors as the tuning parameters of adaptive fuzzy controller. Another robust adaptive control scheme based on feedforward compensator (FFC-based) is illustrated in Figure 1(b). The two types of robust adaptive control schemes have a common adaptive learning concept, that is, the trajectory tracking errors are employed as the tuning parameters. Moreover, the output control torque τ_c in the two control schemes is used to cancel out the uncertainties. However, a closer investigation reveals many differences in the two control schemes, such as the type of training signals and the process of taming uncertainties. The main difference is that the input vectors in the FBC-based robust adaptive control scheme are calculated as a function of the actual positions $q(t)$ and velocities $\dot{q}(t)$, while in the FFC-based robust adaptive control scheme, the input vectors are expressed as a function of the desired positions $q_d(t)$ and velocities $\dot{q}_d(t)$.



(a)



(b)

FIGURE 1. Configurations of robust adaptive inverse dynamics control schemes: (a) FBC-based; (b) FFC-based

The corresponding control laws in the two control schemes are depicted as follows.

$$\tau_{ff} = \hat{D}(q_d)(\ddot{q}_d + K_v\dot{e} + K_p e + \tau_c) + \hat{H}(q_d, \dot{q}_d) \tag{16}$$

$$\tau_{fb} = \hat{D}(q)\left(\ddot{q} + K_v\dot{e} + K_p e + \tau_c\right) + \hat{H}(q, \dot{q}) \tag{17}$$

where τ_{ff} and τ_{fb} are the output control torques of the FBC-based and FFC-based robust adaptive control schemes, respectively.

For convenience, the control law in the two control schemes are rewritten as follows.

$$\tau = \hat{D}(\ddot{q} + K_v\dot{e} + K_p e + \tau_c) + \hat{H} \tag{18}$$

Hence, their closed loop tracking error dynamic equation is denoted as follows.

$$\ddot{e} + K_v\dot{e} + K_p e = \hat{D}^{-1}(\Delta D\,(q)\,\ddot{q} + \Delta H(q, \dot{q}) + F(q, \dot{q})) - \tau_c \tag{19}$$

where $\Delta D(q) = D(q) - \hat{D}(q)$, and $\Delta H(q, \dot{q}) = H(q, \dot{q}) - \hat{H}(q, \dot{q})$.

As shown in Figure 1, the compensation controller is composed of the adaptive fuzzy controller, which is utilized to approximate the structured uncertainties $\psi(\chi_q)$, and the roust H_∞ controller, which is designed to eliminate the effects of the unstructured uncertainties $\delta(\chi_q)$. Hence, the compensation control torque τ_c is the sum of the control torque τ_f of the adaptive fuzzy controller and the control torque τ_h of the robust H_∞ controller, and it can be described as follows.

$$\tau_c = \tau_h + \tau_f \tag{20}$$

where $\tau_h = \hat{D}(q)\Phi_h$ and $\tau_f = \hat{D}(q)\Phi_f$, in which Φ_h and Φ_f are the output vectors of the robust H_∞ controller and the adaptive fuzzy controller, respectively. In the end, the corresponding state space tracking error dynamic equation can be derived as follows.

$$\dot{x} = Ax + B(\psi(\chi_q) + \delta(\chi_q) - (\Phi_h + \Phi_f)) \tag{21}$$

where $\Phi_h = \hat{D}(q)^{-1}\tau_h$, $\Phi_f = \hat{D}(q)^{-1}\tau_f$, $\psi(\chi_q) = \hat{D}(q)^{-1}(\Delta D(q)\ddot{q} + \Delta H(q, \dot{q}))$ and $\delta(\chi_q) = \hat{D}(q)^{-1}F(q, \dot{q})$.

3.2. **Derivation of robust adaptive inverse dynamics controller.** In this section, tuning parameters of the adaptive fuzzy controller are derived, and the stability of the closed loop control system is proved through the Lyapunov stability theorem. The boundaries of the terms $\hat{D}(q)^{-1}$ and the unknown structured uncertainties $\psi(\chi_q)$ are assumed as follows.

$$\begin{cases} b_{\min} \le \left\| \hat{D}(q)^{-1} \right\| \le b_{\max} \\ \|\psi(\chi_q)\| \le \Theta^T \xi(x) + \varepsilon \end{cases} \tag{22}$$

where $\xi(x)$ is a vector of fuzzy base function; Θ is the tuning parameters of the fuzzy logic system; and ε denotes an arbitrary small positive constant.

Lemma 3.1. *The following inequality holds for any $\varepsilon > 0$ and for any $\eta \in R$*

$$0 \le |\eta| - \eta \tanh\left(\frac{\eta}{\varepsilon}\right) \le \kappa\varepsilon \tag{23}$$

where κ is a constant that satisfies $\kappa = e^{-(\kappa+1)}$, i.e., $\kappa = 0.2785$.

The proof was given by Polycarpou and Ioannou [18].

Lemma 3.2. *The following inequality holds for any two matrices $X \in R^{n\times m}$ and $Y \in R^{n\times m}$*

$$2X^TY \le \varepsilon X^TX + \varepsilon^{-1}Y^TY \tag{24}$$

where ε denotes an arbitrary small positive constant.

Theorem 3.1. *If there exists a continuous function $V(\bullet) : R^n \to R^+$ for a nonlinear system described by Equation (8) with the following properties: (1) there are scalars $q \ge 1$, $\omega_1 > 0$ and $\omega_2 > 0$ such that $\omega_1\|x\|^q \le V(x) \le \omega_2\|x\|^q$ for all $x(t) \in R^n$; (2) there are scalars $\bar{V}$ and $\underline{V}$, such that whenever $0 < \underline{V} \le V(x) \le \bar{V} < \infty$; (3) V is continuously differentiable and $\dot{V} = \frac{\partial V}{\partial t} f(x, t) \le -q\alpha[V(x) - \underline{V}]$ for all $t \in R^+$, then, the nonlinear control system is uniformly exponentially convergent towards a residual set $S = \Phi(r)$ with rate α.*

The proof was given by Corless and Leitman [19].

Theorem 3.2. *If the control laws Φ_f and Φ_h of the adaptive fuzzy controller and robust H_∞ controller is designed as follows.*

$$\Phi_f = \rho\left(x, \hat{\Theta}\right) \tanh\left(\frac{\rho\left(x, \hat{\Theta}\right) B^T P x}{\varepsilon}\right) \tag{25}$$

$$\dot{\hat{\Theta}} = -\lambda\hat{\Theta} + L\xi(x)\left\|B^T P x\right\| \tag{26}$$

$$\Phi_h = R^{-1} B^T P x \tag{27}$$

where $\rho\left(x, \hat{\Theta}\right) = \hat{\Theta}^T \xi(x)$ is the output vector of the adaptive fuzzy controller defined by Equation (11); $x = \left[\begin{array}{cc} q & \dot{q} \end{array}\right]^T$ is the input vector of the adaptive fuzzy controller; $\lambda \in (0, \infty)$, $L = diag\{l_1, l_2, \cdots, l_p\}$, $l_i \in (0, \infty)$, p is the dimension of the vector Θ, ε is an arbitrary small positive constant; $\lambda, l_i, \varepsilon$ are the parameters determined by the designer; and P is a symmetric positive definite matrix satisfying the following Riccati-like equation.

$$A^T P + PA + Q + PB\left(\frac{1}{\varepsilon} I - R^{-1}\right) B^T P = 0 \tag{28}$$

where $Q > 0$ is chosen by the designer; and ε is an arbitrary small positive constant.

Then, the proposed robust adaptive control law can guarantee the closed loop system to convergent towards a residual set $\Phi(r)$ with rate $\mu/2$, where $\mu = \frac{1}{2}\min\left\{\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}, \lambda\right\}$, $r = \sqrt{\frac{\bar{\varepsilon}}{\mu}}$, $\bar{\varepsilon} = \gamma\left[\frac{\lambda\|\Theta\|}{2l_{\min}} + \varepsilon\right] + \frac{1}{2}\varepsilon\|F\|^2$, $l_{\min} = \min\{l_1, l_2, \cdots, l_p\}$.

Proof: Define a Lyapunov function candidate as follows:

$$V = \frac{1}{2}x^T P x + \frac{1}{2}\gamma\tilde{\Theta}^T L^{-1}\tilde{\Theta} \tag{29}$$

where $\tilde{\Theta}^T = \hat{\Theta}^T - \Theta^T$.

The derivative of the Lyapunov function V with respect to time along the tracking error dynamic Equation (21) is given by

$$\begin{aligned} \dot{V} = &\frac{1}{2}x^T(A^T P + PA)x - x^T PB\Phi_f + x^T PB\psi(\chi_q) \\ &- x^T PB\Phi_h + x^T PB\delta(\chi_q) + \gamma\tilde{\Theta}^T L^{-1}\dot{\hat{\Theta}} \end{aligned} \tag{30}$$

where $\psi(\chi_q) = \hat{D}(q)^{-1}(\Delta D(q)\ddot{q} + \Delta H(q, \dot{q}))$ and $\delta(\chi_q) = \hat{D}(q)^{-1}F(q, \dot{q})$.

From the boundaries of the structured uncertainties in Equation (22), we can derive the third term in Equation (30) as $x^T PB\psi(\chi_q) \le \|\psi(\chi_q)\|\left\|B^T P x\right\| \le \Theta^T\xi(x)\left\|B^T P x\right\|$. According to the adaptive fuzzy control law Φ_f in Equation (25), the second term in Equation (30) can be derived as follows: $-x^T PB\Phi_f = -x^T PB\hat{\Theta}^T\xi(x)\tanh(\frac{\hat{\Theta}^T\xi(x)B^T Px}{\varepsilon})$. Then, using Lemma 3.1 with $\eta = \hat{\Theta}^T\xi(x)B^T Px$, we can get the following inequality of the two terms of $-x^T PB\Phi_f + x^T PB\psi(\chi_q)$ in Equation (30).

$$\begin{aligned} &- x^T PB\Phi_f + x^T PB\psi(\chi_q) \\ &\le \hat{\Theta}^T\xi(x)\left\|B^T Px\right\| - x^T PB\hat{\Theta}^T\xi(x)\tanh\left(\frac{\hat{\Theta}^T\xi(x)B^T Px}{\varepsilon}\right) - \tilde{\Theta}^T\xi(x)\left\|B^T Px\right\| \\ &\le \gamma\varepsilon - \gamma\tilde{\Theta}^T\xi(x)\left\|B^T Px\right\| \end{aligned} \tag{31}$$

where γ is an arbitrary small positive constant.

Since $\frac{1}{2}(\tilde{\Theta}+\Theta)^T L^{-1}(\tilde{\Theta}+\Theta)\geq 0$, such that $\tilde{\Theta}^T L^{-1}\tilde{\Theta}\tilde{\Theta}^T L^{-1}\Theta \geq \frac{1}{2}(\tilde{\Theta}^T L^{-1}\tilde{\Theta}\Theta^T L^{-1}\Theta)$. And, the term $\dot{\tilde{\Theta}} = -\lambda\left(\tilde{\Theta}+\Theta\right)+L\xi\left(x\right)\left\|B^T Px\right\|$ can also be inferred from the adaptive fuzzy control law in Equation (26). Hence, the final term of $\gamma\tilde{\Theta}^T L^{-1}\dot{\tilde{\Theta}}$ in Equation (30) can be rewritten as

$$\begin{aligned}\gamma\tilde{\Theta}^T L^{-1}\dot{\tilde{\Theta}} &= -\gamma\lambda\tilde{\Theta}^T L^{-1}\left(\tilde{\Theta}+\Theta\right)+\gamma\tilde{\Theta}^T\xi\left(x\right)\left\|B^T Px\right\| \\ &\leq -\frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta+\gamma\tilde{\Theta}^T\xi\left(x\right)\left\|B^T Px\right\|\end{aligned} \tag{32}$$

Substituting Equations (31) and (32) into Equation (30) yields the following inequality.

$$\dot{V} \leq \frac{1}{2}x^T\left(A^T P+PA\right)x+\gamma\varepsilon-\frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta-x^T PB\Phi_h+x^T PB\delta \tag{33}$$

Taking into account Lemma 3.2, the term of $x^T PB\delta\left(\chi_q\right)$ in Equation (30) can be rewritten as follows.

$$x^T PB\delta\left(\chi_q\right)\leq\frac{1}{2}\left(\varepsilon\delta\left(\chi_q\right)^T\delta\left(\chi_q\right)+\varepsilon^{-1}x^T PBB^T Px\right) \tag{34}$$

Substituting Equation (34), the robust control law $\Phi_h = R^{-1}B^T Px$ in Equation (27), and the Riccati-like Equation (28) into Equation (33), the derivative of the Lyapunov function $\dot{V}$ can be bounded as

$$\begin{aligned}\dot{V} &\leq \frac{1}{2}x^T\left(A^T P+PA\right)x+\gamma\varepsilon-\frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta \\ &\quad -x^T PB\Phi_h+\frac{1}{2}\left(\varepsilon\delta\left(\chi_q\right)^T\delta\left(\chi_q\right)+\varepsilon^{-1}x^T PBB^T Px\right) \\ &\leq \frac{1}{2}x^T\left(A^T P+PA+\varepsilon^{-1}PBB^T P-PBR^{-1}B^T P\right)x+\gamma\varepsilon \\ &\quad -\frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta+\frac{1}{2}\varepsilon\delta\left(\chi_q\right)^T\delta\left(\chi_q\right) \\ &\leq -\frac{1}{2}x^T Qx+\gamma\varepsilon-\frac{1}{2}\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta+\frac{1}{2}\varepsilon\delta\left(\chi_q\right)^T\delta\left(\chi_q\right) \\ &\leq \frac{1}{2}\left(-x^T Qx-\gamma\lambda\tilde{\Theta}^T L^{-1}\tilde{\Theta}\right)\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta+\gamma\varepsilon+\frac{1}{2}\varepsilon\left\|\hat{D}\left(q\right)^{-1}\right\|^2\left\|F\left(q,\dot{q}\right)\right\|^2\end{aligned} \tag{35}$$

Given that $\bar{Q}=\frac{1}{2}\begin{bmatrix} Q & 0 \\ 0 & \gamma\lambda L^{-1}\end{bmatrix}$, the above expression can be given as follows

$$\dot{\bar{V}} \leq -z^T\bar{Q}z+\bar{\varepsilon} \tag{36}$$

where $z=\left[x,\tilde{\Theta}\right]^T$ and $\bar{\varepsilon}=\frac{1}{2}\gamma\lambda\Theta^T L^{-1}\Theta+\gamma\varepsilon+\frac{1}{2}\varepsilon\left\|\hat{D}\left(q\right)^{-1}\right\|^2\left\|F\left(q,\dot{q}\right)\right\|^2$.

Substituting the parameters given in Theorem 3.1 into the above expression, we get

$$\dot{\bar{V}} \leq -2\mu\bar{V}+\bar{\varepsilon} \tag{37}$$

Then, using Theorem 3.1, one can see that the tracking error converges towards a residual set $\Phi(r)$ with the convergence rate $\mu/2$.

4. **Numerical Simulation and Discussions.** In this section, the proposed control schemes are applied to a two-link planar rotary robot manipulator (shown in Figure 2)

gripping an unknown load. Through the Euler-Lagrangian approach, its dynamic equation is derived as follows.

$$\begin{bmatrix} D_{11}(q_2) & D_{12}(q_2) \\ D_{21}(q_2) & D_{22}(q_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -C_{12}(q_2)\dot{q}_2 & -C_{12}(q_2)(\dot{q}_1+\dot{q}_2) \\ C_{12}(q_2)\dot{q}_1 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} G_1(q_1,q_2)g \\ G_2(q_1,q_2)g \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad (38)$$

where $D_{11}(q_2) = (m_1+m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2\cos q_2$, $D_{12}(q_2) = D_{21}(q_2) = m_2r_2^2 + m_2r_1r_2\cos q_2$, $D_{22}(q_2) = m_2r_2^2$, $C_{12}(q_2) = m_2r_1r_2\sin q_2$, $G_1(q_1,q_2) = (m_1+m_2)r_1\cos q_2 + m_2r_2\cos(q_1+q_2)$, $G_2(q_1,q_2) = m_2r_2\cos(q_1+q_2)$, in which q_i $(i=1,2)$ denote angular position (rad), m_i $(i=1,2)$ are the links masses (kg), and r_i $(i=1,2)$ represent the links length (m).

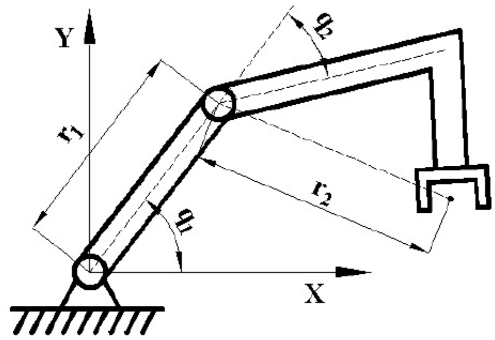


FIGURE 2. A two-link planar rotary robot manipulator

The kinematics parameters of the two-link planar rotary robot manipulator are defined as follows: $r_1 = 0.115$(m), $m_1 = 2.74$(kg), $r_2 = 0.130$(m), $m_2 = 2.01$(kg), and their corresponding nominal values are assumed as: $\hat{r}_1 = 0.08$(m), $\hat{m}_1 = 1.9$(kg), $\hat{r}_2 = 0.1$(m), $\hat{m}_2 = 1.2$(kg). Furthermore, an uncertain payload $\delta(t) = [10\sin(t), 10\cos(t)]^T$ is assumed to be attached to the second link, and a viscous friction force $F(q,\dot{q}) = 80sgn(\dot{q}) + 10\dot{q}$ is added to each joint. In the following four simulation cases, all initial joint positions are assumed as $q_1(0) = q_2(0) = 4$(rad), and initial joint velocities are zeros.

Case 1: conventional inverse dynamics controller. Firstly, the conventional inverse dynamics controller is applied to robot manipulator with uncertain dynamics. In other words, the control scheme is designed according to nominal kinematic parameters instead of actual kinematic parameters. In order to prevent the robot manipulator from exhibiting overshoot when a desired trajectory terminates at the surface of a work piece, the control parameters are usually selected for the critical damping; hence, the control parameters are picked as $K_v = diag\{20, 20\}$ and $K_p = diag\{100, 100\}$. A set of simulation results of the two-link planar rotary robot manipulator are illustrated in Figure 3.

As shown in Figure 3, a large tracking errors occur at the initial stages, and after the transient response, the actual trajectories severely deviate from the desired trajectories. Especially, a large gap between the actual and desired trajectories in the second joint may destroy the stability of the closed loop system. Therefore, in order to eliminate the effects of the uncertainties, some additional compensation controllers should be appropriately added to the inverse dynamics controller.

Case 2: an improved inverse dynamics controller. In this case, an improved inverse dynamics controller including an adaptive fuzzy compensator proposed by Chen et al. [20] is used to control robot manipulator with uncertain dynamics. The uncertain payload and viscous friction forces in this case are the same that in the first case. The

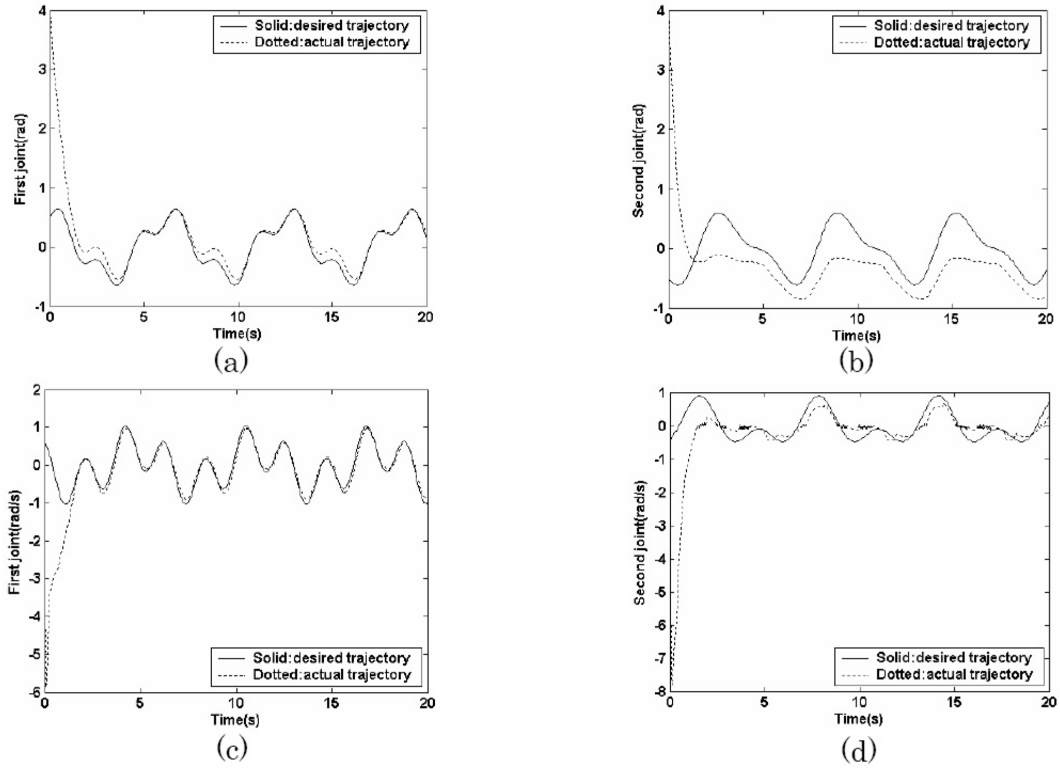


FIGURE 3. Control performances in the first case: (a) position of the first joint; (b) position of the second joint; (c) velocity of the first joint; (d) velocity of the second joint

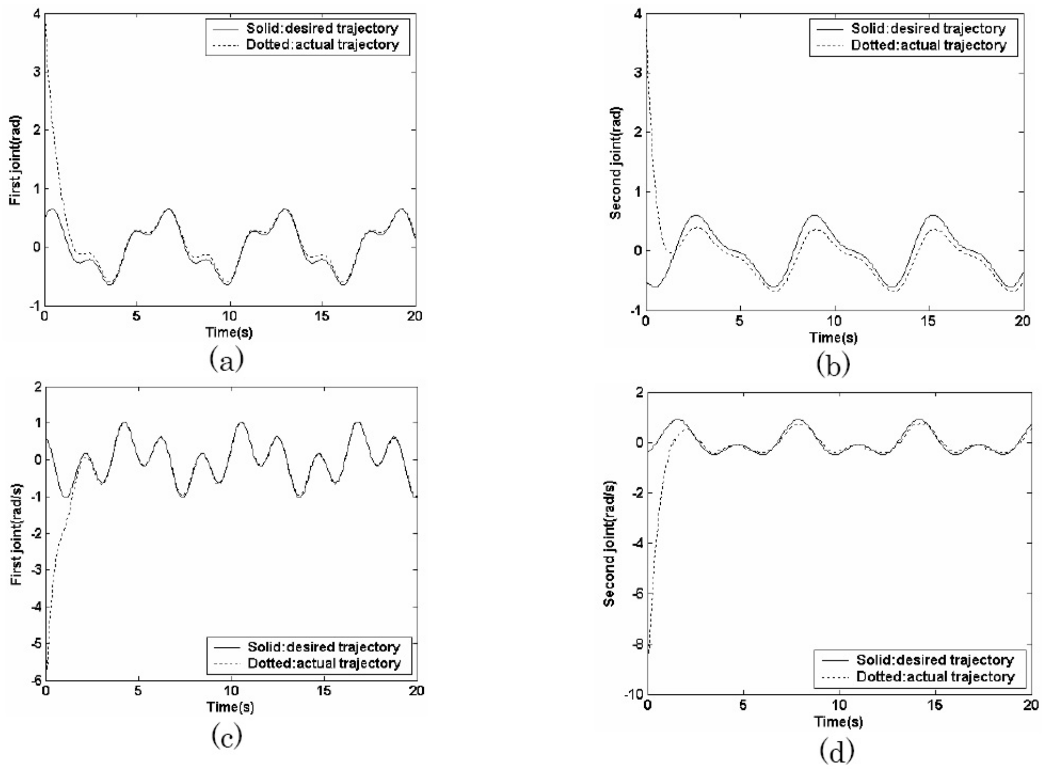


FIGURE 4. Control performances in the second case: (a) position of the first joint; (b) position of the second joint; (c) velocity of the first joint; (d) velocity of the second joint

input vector of the adaptive fuzzy compensator is defined as $X = \{x_i \,|i = 1, 2, \cdots, 4\} = \{q_1, \dot{q}_1, q_2, \dot{q}_2\}$, the universe of discourse of each fuzzy input vector is divided into five fuzzy labels, i.e., *NB, NS, ZO, PS, PB*, and their corresponding membership functions are defined as $\mu_{A_i^l}(x_i) = \exp\left[-\frac{(x_i-C_i)^2}{2\sigma_i^2}\right]$, where C_i are -1, -0.5, 0, 0.5, and 1, σ_i is equal to 0.2124. Figure 4 demonstrates the control performances of two joints when the inverse dynamics controller plus adaptive fuzzy compensator is applied to the two-link planar rotary robot manipulator with uncertain dynamics.

As can be seen from Figures 3 and 4, the oscillations of the tracking errors in this case are remarkably smaller than the first case, which verifies that the adaptive fuzzy compensator can effectively compensate some uncertainties. However, it can be obviously seen from Figure 4 that there is a big error between the desired and actual trajectories. Thus, only adaptive fuzzy compensator cannot completely eliminate the effects of the structured and unstructured uncertainties, and a novel compensation controller should be developed to separately eliminate the structured and unstructured uncertainties on the control performances.

Case 3: robust adaptive inverse dynamics controller. In this case, the proposed FFC-based robust adaptive inverse dynamics controller combining an adaptive fuzzy control algorithm with a nonlinear robust H_∞ controller is utilized to control the two-link planar rotary robot manipulator with uncertain dynamics. Here, the adaptive fuzzy control algorithm is designed to approximate the structured uncertainties, and the robust H_∞ control model is employed to eliminate the effect of the unstructured uncertainties. The exclusive difference between the second case and this case is that a nonlinear robust H_∞ control model in this case is employed to eliminate the unstructured uncertainties.

Figure 5 illustrates the control performances of two joints when the proposed controller is applied to the robot manipulator with uncertain dynamics. It can be seen that the proposed FFC-based robust adaptive inverse dynamics controller performs much better than the inverse dynamics controller plus adaptive fuzzy compensator in the previous case.

Case 4: comparison studies between FBC-based and FFC-based controllers. In this case, comparison studies between FBC-based and FFC-based controllers are carried out. In order to show the effectiveness of the proposed controller, the tracking errors of the position and velocity are calculated by the following tracking error equations over one training cycle of a trajectory.

$$E_p = \frac{1}{N}\sum_{i=1}^{N} \|q_{di} - q_i\|^2 \quad (\mathrm{rad})^2, \qquad E_v = \frac{1}{N}\sum_{i=1}^{N} \|\dot{q}_{di} - \dot{q}_i\|^2 \quad (\mathrm{rad/\,sec})^2 \tag{39}$$

where E_p is the tracking error of the position; E_v is the tracking error of the velocity; N is the number of the position vectors; and q_{di}, q_i are the desired and actual trajectories, respectively.

From these calculation results summarized in Table 1, one can see that the proposed two controllers demonstrate extremely good control performances compared with the conventional inverse dynamics controller. Moreover, the FFC-based robust adaptive inverse dynamics controller performs slightly better in the circular trajectory than the FBC-based robust adaptive inverse dynamics controller. It can also be observed that the tracking errors of the position and velocity in the first joint under the proposed FFC-based robust adaptive controller are reduced by 66.39% and 78.06% compared with the FBC-based robust adaptive inverse dynamics controller. Hence, the proposed FFC-based robust adaptive inverse dynamics controller exhibits better tracking performances than the FBC-based controller.

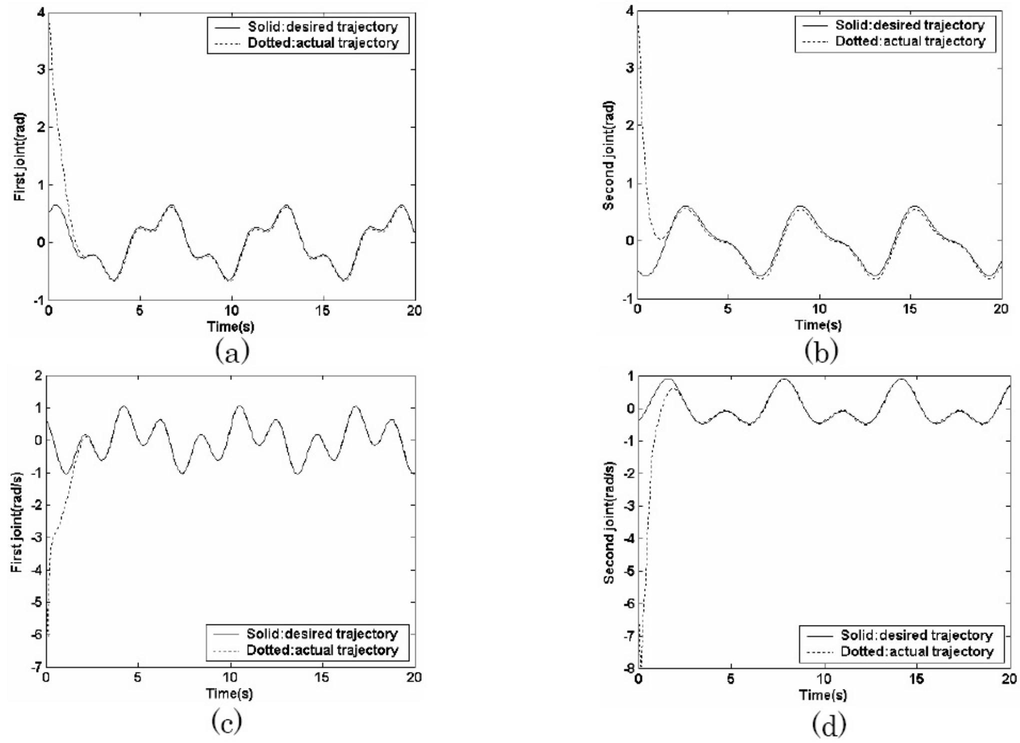


FIGURE 5. Control performances in the third case: (a) position of the first joint; (b) position of the second joint; (c) velocity of the first joint; (d) velocity of the second joint

TABLE 1. Comparison results of tracking error

Control schemes	Errors			
	First joint		Second joint	
	$E_p(\text{rad})^2$	$E_v(\text{rad/sec})^2$	$E_p(\text{rad})^2$	$E_v(\text{rad/sec})^2$
FFC-based robust adaptive	0.00099324	0.00034113	0.0029166	0.0008082
FBC-based robust adaptive	0.0029552	0.0015551	0.025762	0.007903
Inverse dynamics controller	0.024389	0.009591	0.30657	0.11172

5. **Conclusion.** This paper addresses trajectory tracking control problems of robot manipulator with the structured and unstructured uncertainties. On the basis of inverse dynamics controller as a nominal control portion, two types of novel robust adaptive inverse dynamics control schemes combining an adaptive fuzzy control algorithm with a nonlinear H_∞ tracking controller are designed to handle these inevitable uncertainties. Comparison studies of the control performances of the proposed controllers with the conventional inverse dynamics controllers in the presence of model uncertainties are carried out, and comparison results demonstrate that the robust adaptive inverse dynamics control schemes are an effective approach to improve control performances in terms of uncertainties. The stability of the two control schemes are also proved through the Lyapunov method. Computer simulation of a two-link rotary robot manipulator is carried out. Simulation results demonstrate the proposed FFC-based robust adaptive control scheme is the most effective and superiority.

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