ADAPTIVE INTERNAL MODEL CONTROLLER DESIGN USING DYNAMIC PARTIAL LEAST SQUARES DECOUPLING STRUCTURE

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ABSTRACT. A new adaptive internal model control (IMC) scheme based on a novel adaptive model is proposed to account for the process uncertainties in MIMO system. A dynamic partial least squares (PLS) decoupling structure which integrated tradition PLS model and auto-regressive exogenous (ARX) is introduced to model the dynamic process. In the presence of uncertainties, adaptive mechanisms including recursive PLS (RPLS) and recursive least squares (RLS) are used to update the model on-line. Accordingly, the adaptive IMC is designed based on the updated model. To demonstrate the modeling effect of the dynamic PLS decoupling structure and the control performance of the adaptive IMC scheme under uncertainties, a simulation example is presented.

Keywords: Partial least squares, Adaptive internal model control, Recursive partial least squares

1. Introduction. Due to the uncertainties stemming from disturbance signals, noise interference, unmodeled plant dynamics and plant-parameter variations, traditional control strategies usually fail to achieve good performance or even fail to stabilize the system. Under such circumstances, one may need to re-tune the controller parameters or even redesign the control strategies. Two kinds of control schemes are available to avoid redoing those work. One is robust control, the other is adaptive control. Both of them have been greatly developed in the past years. The essential difference between them lies in whether measurements are employed in the control scheme.

During robust control design, those uncertainties are unknown, but assuming bounded. Robust control guarantees that if the changes are within given bounds, the control law does not need to be changed. Usually, unsuitable bound would lead to pessimistic analysis of the system behavior and therefore produce potentially conservative designs.

In contrast with robust control, the adaptive control policy is concerned with control law changing. It does not need a priori information about the bounds of these uncertainties or time-varying parameters. An adaptive controller can use the measurements to adapt a controlled system with parameters which vary, or are initially uncertain. The foundation of adaptive control is parameter estimation. Traditional method of parameter estimation is recursive least squares (RLS) which is used to modify estimates in real time.

On the other hand, the internal model control (IMC) is a simple and popular control strategy in practice industrial process. A lot of attention has been attracted to expand IMC structure to adaptive control to handle uncertainties [1-6]. A complete theoretical design and analysis for adaptive IMC was first proposed by Datta and Ochoa. They
showed how one can adapt the internal model on-line and guarantee stability and asymptotic performance in the ideal case [4]. Rupp and Guzzella presented an adaptive internal model controller for stable non-minimum phase SISO plant and applied to the air/fuel ratio control system of a spark-ignited engine [2]. A new discrete-time adaptive IMC which is robust for changes in modeling was proposed by Silva and Datta. In their paper, two structures of the adaptive IMC system, H2 optimal control and pole-placement, are presented and simulated using the uncertain model [3]. However, those contributions only focus on SISO system. And there are no theoretical guarantees for extending SISO adaptive IMC to MIMO system. In this paper, a novel adaptive IMC scheme is proposed to handle uncertainties in MIMO system by introducing a partial least squares decoupling structure.

Partial least squares (PLS) is a well-known multivariable chemometric tool and has been successfully applied to process monitoring, fault detection, process modeling and recently multivariable control system design. PLS is a robust alternative to the standard least squares in the analysis of highly correlated data. Kaspar and Ray first proposed the idea of moving the conventional control scheme into dynamic PLS framework [7,8]. An MIMO system can be decomposed into several SISO subsystems in this dynamic PLS framework and loop pairing is automatically implemented at the same time. Based on their work, Lakshminarayanan et al. extended the PLS to a dynamic one by constructing an auto-regressive exogenous (ARX) inner model [9]. Hu et al. proposed a multi-loop IMC scheme in conjunction with feed-forward strategy based on the dynamic linear PLS framework [10] and proposed multi-loop nonlinear IMC design based on nonlinear dynamic PLS framework using ARX neural network model [11]. There are also some discusses on the adaptive control using PLS structure. Chen and Cheng designed multi-loop adaptive PID controllers based on a PLS decoupling structure [12,13]. Based on those work, a new adaptive IMC scheme is proposed in this paper. The model of the plant is constructed using the PLS decoupling structure presented in [12,13]. The model updating includes two parts, RLS and recursive partial least squares (RPLS) [14,15]. The inverse-model which is used in controller is updated accordingly. Since the PLS decoupling structure has decomposed the original MIMO system into several SISO subsystems, this adaptive IMC scheme is actually implemented in several SISO loops. Hence, those conclusions drawn from SISO adaptive IMC can be inherited, including stability, asymptotic performance and parameter convergence [1,4,16].


2.1. Partial least squares. PLS is a well-known dimensionality reduction technique. The traditional PLS model consists of an inner model in which the algebraic relationship between latent variables is obtained and an outer model which extracts the principal components [17]. Consider two blocks of scaled dataset $X = (x_{ij})_{n \times r}$ and $Y = (y_{ij})_{n \times v}$, where $n$, $r$ and $v$ denote the number of observation, independent variables and dependent variables respectively. The outer model can be expressed as follows:

$$X - E^* = \sum_{a=1}^{A} t_a p_a^T = TP^T$$  \hspace{1cm} (1)

$$Y - F^* = \sum_{a=1}^{A} u_a q_a^T = UQ^T$$  \hspace{1cm} (2)

where $A$ denotes the number of latent variables, which is determined based on the percentage of variance or by a statistical method such as cross validation. $T$ and $U$ denote
the score matrices; \( P \) and \( Q \) are the loading matrices of \( T \) and \( U \) respectively. \( t_a \) and \( u_a \) are the \( a \)th orthogonal vectors of \( T \) and \( U \), \( p_a \), \( q_a \) are the \( a \)th loading vectors of \( P \) and \( Q \). \( E^* \) and \( F^* \) are residual matrices of \( X \) and \( Y \), respectively.

In the inner model, an algebraic relationship between the input and output latent variable is obtained by least squares (LS) method,

\[
u_a = b_a t_a \Rightarrow b_a = u_a^T t_a / t_a^T t_a
\]

Finally, the PLS model can be expressed as,

\[
Y - F^* = \sum_{a=1}^{A} b_a t_a q_a^T = TBQ^T
\]

where \( B = \text{diag}(b_1, b_2, \ldots, b_A) \) is a diagonal matrix containing the inner model regression coefficients. Actually, PLS decouples the multivariate regression problem into a series of univariate regression problems. The PLS modeling is generally implemented by means of numerical approach, called NIPALS (non-linear iterative partial least squares) algorithm which is proposed by Höskuldsson [18]. However, with the algebraic structure, the traditional PLS model can only deal with the static relationship. Therefore, it is necessary to incorporate dynamics into the PLS model.

2.2. Dynamic partial least squares. In recent years, dynamic PLS modeling that developed to obtain a better representation of dynamic process is presented in literature [7-9,12,13,19-21]. Among them, modifying traditional PLS by incorporating the previous data into each observation vector [19,21] is an intuitive method. However, these approaches require a substantial increase in the dimension of the \( X \) matrix. Kaspar and Ray proposed another method by including filters before applying the standard PLS algorithm [7,8]. However, the dynamic filter is designed either based on a-prior knowledge or by minimizing the sum of squares of the output residuals. Lakshminarayanan et al. proposed a dynamic extension of the PLS algorithm that incorporating the dynamic regression relationship, like ARX, into the PLS inner model [9]. This method modified the standard PLS algorithm. Chen et al. proposed a dynamic PLS method by combining the standard PLS model and ARX model [12,13]. The standard PLS model can decouple the MIMO system into several SISO subsystems and pair the loops automatically. While the dynamic characteristics of the system can be inferred from the analysis of an ARX model fitted to the observations of the system [13]. The model structure is shown in Figure 1. The input \( x(t) \) is scaled and centered by matrix \( W x \) to be the input vector of standard PLS. Using PLS model, one can obtain the output vector of PLS, \( y^{PLS}(t) \). \( G_p \) represents the process plant. The output \( y(t) \) is scaled and centered by matrix \( W y \), then passes the delay operators, \( q^{-1}, q^{-2}, \ldots, q^{-N_a} \), to be the input vector of ARX part. The prediction output \( \hat{y}(t + 1) \) is expressed as a weighted sum of the past outputs plus the outputs from the PLS model.

This dynamic PLS structure can be formulated as two parts,

\[
y^{PLS}(t) = PLS(x(t))
\]

\[
\hat{y}(t + 1) = \sum_{j=1}^{N_a} A_j y(t + 1 - j) + y^{PLS}(t - d)
\]

where \( N_a \) is the number of lagged, \( A_j \) is the ARX model parameters and \( d \) is the pure delay.

The procedure of the dynamic PLS modeling is presented as follows.
(1) Set the initial guess value of \( Na \) and \( d \), as well as corresponding searching band, then initialize \( A_j \).

(2) Using the given ARX model and the output \( y(t) \), one can obtain the \( y^{PLS}(t) \) based on Equation (6).

\[
y^{PLS}(t) = y(t + 1 + d) - \sum_{j=1}^{Na} A_j y(t + 1 + d - j)
\]

Combine \( y^{PLS}(t) \) with scaled \( x(t) \) to construct the PLS model and obtain the PLS output \( \hat{y}_{PLS}(t) \).

(3) Using \( \hat{y}^{PLS}(t) \) and \( \hat{y}(t) \), one can calculate model prediction output \( \hat{y}(t + 1) \) using Equation (6). And the ARX model parameters \( A_j \) can be updated using \( \hat{y}^{PLS}(t) \) and \( y(t) \) again.

(4) Repeat steps (2) and (3) until the model converges.

(5) If the model prediction output \( \hat{y}(t + 1) \) is not approaching the real output \( y(t + 1) \), modify \( Na \) and \( d \) then go to step (2).

Since adaptive IMC will be designed based on this framework, recursive updating the model is needed. In the ARX part, model parameter can be updated online via recursive least squares (RLS). While the PLS part is updated using the recursive partial least squares (RPLS).

2.3. Recursive partial least squares. RPLS is firstly proposed by Helland et al. [14]. And some issues which were not clearly considered in Helland’s work was supplemented by Qin [15]. There is a minor modification of standard PLS in their method. The score matrix \( T \) is normalized by transferring the lengths of \( T \) to the loading matrices \( P \) and \( Q \) via matrix \( L \). The new representations are shown below.

\[
L = \{l_{ij}\}; \quad l_{ii} = \sqrt{t_i^T t_i}; \quad l_{ij} = 0, \quad i \neq j
\]

\[
S = TL^{-1}; \quad S^T S = I
\]

\[
R = PL
\]

\[
H = QBL
\]

\[
X = TP^T + E^* = TL^{-1}LP^T + E^* = SR^T + E^*
\]
\[ Y = TBQ^T + F^* = TL^{-1}LBQ^T + F^* = SH^T + F^* \] (12)

In order to accord with Equations (1)-(3), here \( T, P, Q, \) and \( U \) are still used to describe PLS model. However, \( T \) is normalized as \( T^TT = I \). This modification makes it easy to derive the RPLS regression algorithm.

Like in least squares, minimizing the square residuals, \( ||Y - XC||^2 \), it can obtain,
\[
(X^TX)C = X^TY \tag{13}
\]

The PLS regression coefficient matrix can be denoted as
\[
C_{PLS} = (X^TX)^+X^TY \tag{14}
\]
where \((\cdot)^+\) denotes the ‘PLS generalized inverse’. When a new dataset \((X_1, Y_1)\) is available, the whole dataset used to modeling is augmented as
\[
X_{new} = \begin{bmatrix} X \\ X_1 \end{bmatrix}, \quad Y_{new} = \begin{bmatrix} Y \\ Y_1 \end{bmatrix}
\]

Now the PLS regression coefficient matrix is
\[
C_{new}^{PLS} = \left( \begin{bmatrix} X \\ X_1 \end{bmatrix}^T \begin{bmatrix} X \\ X_1 \end{bmatrix} \right)^+ \begin{bmatrix} X \\ X_1 \end{bmatrix}^T \begin{bmatrix} Y \\ Y_1 \end{bmatrix} \tag{15}
\]

Since \( T \) is mutually orthonormal,
\[
X^TX = PTT^TP^T = PP^T, \quad X^TY = PTT^TBQ^T = PBQ^T \tag{16}
\]

So, Equation (15) can be rewritten as
\[
C_{new}^{PLS} = \left( \begin{bmatrix} P^T \\ X_1 \end{bmatrix}^T \begin{bmatrix} P^T \\ X_1 \end{bmatrix} \right)^+ \begin{bmatrix} P^T \\ X_1 \end{bmatrix}^T \begin{bmatrix} BQ^T \\ Y_1 \end{bmatrix} \tag{17}
\]

Above is the basic concept of RPLS. RPLS updates the model using the old model and the new data. The procedures of RPLS modeling is summarized as follows.

1. Scale the dataset \((X, Y)\) to zero mean and unit variance.
2. Use the scaled dataset to derive a PLS model \((T, P, B, Q)\). It is to be noted that enough principal components are needed to make the residual matrices \(E^*\) satisfy \( ||E^*|| \leq \varepsilon \) \((\varepsilon > 0 \) is the error tolerance). 
3. When a new dataset \((X_1, Y_1)\) is available, scale it in the same way as step (1). Formulate
\[
X = \begin{bmatrix} P^T \\ X_1 \end{bmatrix}, \quad Y = \begin{bmatrix} BQ^T \\ Y_1 \end{bmatrix}
\]
and go to step (2).

Remark 2.1. RPLS updates the model without increasing the size of data matrices.

Remark 2.2. When the new datasets are available as time goes on, the mean and variance of the whole dataset may change. Step (3) may not scale the new dataset to zero mean and unit variance. In this case, a modified RPLS is needed. It is beyond the scope of this paper. One can see [15] for more detail. In the simulation of our paper, the mean and variance are assumed not to change, since it is not very long running time considered in our example. So this assumption is reasonable. Because that not many datasets are accumulated, the changes of mean and variance are very limited.
RPLS is used to update only the PLS part in the dynamic PLS decoupling structure, the ARX part is also needed to be updated when the new dataset available. Traditional RLS can be applied to update ARX model. Due to RLS is well-known and widely used, no description will be presented here again. It is notable that, the model is updated only when the mismatch between model and plant exceeds a threshold.

3. **Adaptive Internal Model Control Scheme.** The conventional SISO IMC consists of a forward model of the plant, its inverse-model and a low-pass filter is presented in Figure 2. The controller is designed using the inverse-model. And the low-pass filter is used to make the system robust.

![Figure 2. The basic structure of IMC scheme](image)

To make the conventional IMC adaptive, both the adaptive forward model and inverse-model are required. The forward model is PLS model connected in series with ARX model in this paper. The model parameters can be updated online using RPLS and RLS. The inverse-model is the inverse of the forward model’s minimum phase part, so it can be updated accordingly. Since the forward model is a part of IMC and the controller is the inverse-model, the whole IMC is updated as well. The adaptive IMC scheme is illustrated in Figure 3. The PLS-ARX is the forward model. RPLS+RLS are the adaptive mechanisms and used to update the forward model and controller simultaneously.

![Figure 3. Adaptive IMC based on dynamic PLS decoupling structure](image)

The design of IMC mainly includes two parts: the low-pass filter and the inverse-model. The low-pass filter can be selected as

$$F(q^{-1}) = \left( \frac{1 - \lambda}{1 - \lambda q^{-1}} \right)^N$$

(18)

where the positive integer $N$ is chosen to make the IMC controller semi-proper or proper. The low-pass filter parameter $\lambda$ is chosen to make a balance between the robustness and
sensitivity. As for the inverse-model, the controller output is calculated as follows.

\[ e(t) = y_{\text{set}}(t) - y(t) + y_m(t) \]  \hspace{1cm} (19)
\[ x(t) = ginv(F(q^{-1})e(t)) = ginv(ef(t)) \]  \hspace{1cm} (20)

`ginv()` is the inverse of the PLS-ARX model. So it also includes two parts. According to Equation (5), Equation (6) and Figure 3, the inverse-model can be formulated as,

\[ e^{PLS}(t) = ef(t) - \sum_{j=1}^{Na} A_jy(t - j) \]  \hspace{1cm} (21)
\[ x(t) = invPLS(e^{PLS}(t)) \]  \hspace{1cm} (22)

Equation (21) is used to calculate the expected PLS output vector, \( e^{PLS}(t) \). Equation (22) is the inverse process of standard PLS, i.e., using the output vector to calculate the input vector. By now, the controller input, \( x(t) \) is obtained.

4. Example. To demonstrate the modeling effect of the dynamic PLS decoupling structure and the control performance of the adaptive IMC scheme under uncertainties, a 2×2 process [22] is considered as follows.

\[ G(s) = \begin{bmatrix}
\frac{3(s + 3)(s + 5)}{(s + 1)(s + 2)(s + 4)} & \frac{6(s + 1)}{(s + 2)(s + 4)} \\
\frac{2}{(s + 3)(s + 5)} & \frac{1}{s + 1}
\end{bmatrix} \]  \hspace{1cm} (23)

In the nominal case, the process is excited by two pseudorandom signals to produce modeling datasets. Using those datasets, the model parameters of dynamic PLS decoupling structure are got as follows.

\[ P = \begin{bmatrix}
0.455715 & 0.864725 \\
0.890126 & -0.50225
\end{bmatrix} \quad Q = \begin{bmatrix}
0.519524 & 0.883665 \\
0.854456 & -0.46812
\end{bmatrix} \]

\[ B = \begin{bmatrix}
1.02692 & 0.780334
\end{bmatrix} \]

The whole model is

\[ y_1(t) = [y_1(t-1) \quad y_1(t-2) \quad y_1(t-3)] \ast [0.9642 \quad -0.2507 \quad 0.0084]^T + y_1^{PLS}(t) \]
\[ y_2(t) = [y_2(t-1) \quad y_2(t-2) \quad y_2(t-3)] \ast [0.8360 \quad -0.1592 \quad -0.0037]^T + y_2^{PLS}(t) \]

If there is a large change in the process, the control strategy without an adaptive law may not make the system stable. Two types of changes in the model structure will be used to simulate the uncertainties in the process.

4.1. Steady state gain changes. Assuming the steady state gain in the diagonal of Equation (23) is enlarged to double. Activity variation of catalyst in the reaction process belongs to this kind of steady state gain change. The model identified in nominal case cannot describe the process or predict the output exactly. With adaptive law, RPLS and RLS, the model can be modified on-line. As seen in Figure 4, the model prediction output is close to the actual output with small error. Figure 5 shows the changes of parameters in ARX part of the model. The ARX model parameters are convergent. It means that a new model is established to adapt the process change. Hence, the RLS and RPLS used in dynamic PLS decoupling structure can adjust the model effectively in the case of process gain changes.

Accordingly, adaptive IMC scheme is used to track the setpoint in the case of steady state gain changes. The control response is presented in Figure 6. As can be seen, the adaptive IMC tracks the setpoint well with the adaptive mechanism, RPLS and PLS.
As comparison, IMC without adaptive mechanism is also carried out in this case. As seen in Figure 7, IMC can track the setpoint with a long time. And a very intense oscillation response is presented which is undesired in the process. Intense oscillation means frequent control action which may shorten the life of actuators. Therefore, adaptive IMC outperforms IMC in the control performance.

**Figure 4.** The prediction output in the case of process gain change

**Figure 5.** The change of ARX parameter in the case of process gain change
4.2. Process model pole changes. Another common change in the process behavior is the pole changes in the transfer function. Parameters of equipment changing with temperature which are very common in the continuous process belong to this kind of uncertainty. Here, assuming that the transfer function in Equation (23) is varied as

\[
G(s) = \begin{bmatrix}
\frac{3(s + 3)(s + 5)}{(s + 4)(s + 2)(s + 4)} & \frac{6(s + 1)}{(s + 2)(s + 4)} \\
\frac{1}{(s + 3)(s + 5)} & \frac{1}{s + 3}
\end{bmatrix}
\]
The pole changes make the original model no longer matching the process. With the model adaptation, the output prediction and actual output are compared in Figure 8. Good prediction performance is presented with small error. The parameters of ARX part are also shown in Figure 9 to validate the effectiveness of RLS in the adaptive approach. Thus, this adaptive PLS decoupling structure can update the model effectively in the case of pole changes.
Consistently, adaptive IMC scheme is used to track the setpoint under the condition of process pole changes. Comparison with IMC is also carried out. Figure 10 shows the control performance obtained by them. Although both IMC and adaptive IMC can track the setpoint, adaptive IMC has a good control performance than IMC in term of response time.

5. **Conclusions.** In this paper, a new adaptive IMC scheme is proposed with a novel model adaptive method to deal with the process uncertainties in MIMO system. A PLS structure which combines the traditional PLS and ARX is used to describe the dynamic process and decouple the MIMO system into several SISO subsystems. The adaptive mechanism includes two parts, RPLS is used to update the PLS and RLS is used to update the ARX. The proposed adaptive IMC scheme can guarantee good control performance in the presence of uncertainties. The proposed modeling method and control scheme are demonstrated to be effectiveness in the simulation example.

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