A MODIFIED REAL-CODED GENETIC ALGORITHM CONSIDERING WITH FITNESS-BASED VARIABILITY

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ABSTRACT. A genetic algorithm (GA) is a search algorithm based on the mechanism of natural genetics. In various GAs, a real-coded GA (RCGA) employing individuals represented by real valued-genes has been proposed to solve the optimization problem in the continuous searching space. However, the conventional RCGA yields ineffective searches due to insufficient genetic diversity in the selection process. In this paper, we propose a modified RCGA with variability operator maintaining the genetic diversity of the population. In the proposed method, a variability term is newly added to the individuals selected by the ordinary selection. The degree of the variability is decided considering the fitness value of the individual. The searching performance of the proposed method is better than the conventional methods. The effectiveness and the validity of the proposed method are verified by applying it to optimization problems of continuous benchmark functions and signal sources localization.

Keywords: Genetic algorithm, Real-coded genetic algorithm, Genetic diversity, Mutation, Variability operator

1. Introduction. A genetic algorithm (GA) is a search algorithm based on the mechanism of natural selection and natural genetics [1]. In general, the GA solves optimization problems by using a set of individuals, which are represented by bit-strings. A real-coded GA (RCGA) employing individuals represented by real valued-genes has been proposed to solve function optimization problems [2, 3].

In the traditional GAs, the search is achieved by iteratively carrying out the selection, crossover, mutation and so on. These methods generally employ copy-based selection strategies, e.g., roulette wheel selection (RWS), tournament selection and ranking selection [2]. The conventional selection methods are simply realized by only the copy-based procedure.

However, the conventional RCGAs cause a loss of the genetic diversity [4, 5] which means the number of base points in the searching space, because the lack of the genetic diversity corresponds to loss of the base points. As a result, a drop in the genetic diversity leads to an ineffective search.

To ensure adequate genetic diversity, there are two kinds of approaches, a selection-based method and a mutation-based method. The selection-based method performs the
search by generating better offspring with keeping the genetic diversity. As the representative selection-based method, a self-organizing map (SOM [6])-based selection operator was proposed [7, 8]. On the other hand, the mutation-based method realizes the effective search by making variety offspring or changing mutation probability [9, 10, 11]. Especially, an adaptive directed mutation (ADM) operator [11] is the state-of-the-art algorithm in the mutation-based methods.

The SOM-based selection and the ADM operators can maintain the genetic diversity, because these operators generate new varied individuals without copying. However, a computational cost of the SOM-based selection is bigger than that of the copy-based conventional selection operator, because the SOM-based selection requires some calculation processes.

On the other hand, the ADM operator can keep the genetic diversity without time consuming computational processes. The individual after applying the ADM is decided by considering the variations of the individual and its fitness value. However, the convergence property of the ADM is not good, because the ADM operator based on the elitism is not applied to the individual with the highest fitness value.

In order to achieve the effective search with small computational costs, the simple algorithm is desirable. Furthermore, the algorithm needs to find the good solution in early generation and the fast convergence. The motivation of this study is to establish the new operator which encourages the fast search and convergence with small computational costs in the framework of RCGA.

In this paper, we propose a variability operator in RCGA. In the proposed operator, to maintain the genetic diversity, the varied individuals are generated by adding the variability term to each individual selected by the conventional copy-based selection. The variability is given by random values. In this study, we consider two kinds of random values generated from normal and uniform distributions. A degree of the variability is based on fitness values of the individuals. When the fitness value of the individual is high, a small variability is added. On the other hand, when the fitness value of the individual is low, a large variability is added. In the proposed operator, the variation is controlled by a searching range of each individual in accordance with its fitness value.

The feature of the proposed mutation is the simpleness of the algorithm. Furthermore, the proposed mutation realizes the faster search with the better convergence property compared with the conventional methods.

The effectiveness of the proposed method is verified by applying it to optimization problems of continuous benchmark functions and signal source localization.

2. Conventional Selection and Mutation Operators in RCGA. In the ordinary GA, a chromosome of the individual is represented by a bit-string. On the other hand, in the RCGA [2], a chromosome is represented by a string of real values. In the RCGA, the i-th individual is represented as an m-dimensional vector \( \mathbf{x}_i = (x_{i1}, \ldots, x_{im}) \), where each element \( x_{ij} \) is a real value.

The popular RCGA as well as the ordinary bit-string GA employs selection, crossover and mutation operators. The crossover and mutation methods of the RCGA are different from those of the bit-string GA. In the RCGA, the one-point crossover, BLX-\( \alpha \) crossover and unimodal normal distribution crossover (UNDX) [12] methods are used generally.

As the selection operator, the roulette wheel, tournament and ranking selections are employed in both the RCGA and the bit-string GA. These selections can be regarded as the copy-based methods. For example, in the roulette wheel selection (RWS), the survival
A modified GA considering with fitness-based variability

Figure 1. A loss of genetic diversity and preferable location after genetic operation. Each axis and dashed circles represent the element of chromosomes and contours of fitness value, respectively. Area in the circle with smaller radius means that fitness value is higher. (a) Location of individuals before selection. (b) Location of individuals after copy-based selection. (c) Preferable location of individuals after genetic operations.

The probability \( p_{x_i(t)} \) of the \( i \)-th individual to the next generation is calculated by:

\[
p_{x_i(t)} = \frac{f(x_i(t))}{\sum_{j=1}^{N} f(x_j(t))},
\]

where \( t \) and \( N \) represent a present generation and a population size, respectively. \( f(x_i(t)) \) stands for the fitness value of the individual \( x_i(t) \). In the copy-based selection, the individual with low fitness value is hardly copied to the next generation. Contrastively, the individual with high fitness value is easily copied to the next generation. However, the copy-based selection operator causes a loss of the genetic diversity and an ineffective search.

Figures 1(a) and 1(b) show locations of individuals before and after applying the selection operator, respectively. It seems that the genetic diversity of the population decreases in Figure 1(b). It appears that a population size has got smaller because this phenomenon results from the fact that some individuals take on identical string, although the population size is exactly the same from beginning to end of iteration. The individuals are the searching points in the present generation, and they are also base points of crossover in the next generation. Thus, the decrease of genetic diversity leads to ineffectiveness of the search. To encourage an effective search, a genetic operation generating various individuals shown in Figure 1(c) is needed.

To keep the genetic diversity in the selection process in the RCGA, a self-organizing map (SOM)-based selection was proposed [7, 8]. In the ordinary SOM, reference vectors are used for vector quantization, data clustering, 2-dimensional approximation of distribution of high-dimensional data and so on [13]. In the SOM-based selection method, the reference vectors of SOM after learning are employed as a set of new individuals of next generation. Furthermore, the modified SOM, which employs new coefficients with respect to the fitness values, is utilized to approximate the distribution of the individuals with only high fitness value. Thus, the individuals of next generation are newly generated based on the learning of the modified SOM. The SOM-based selection method can realize a search with maintaining the genetic diversity. However, the search becomes ineffective in a high-dimensional space, because the approximation performance in the SOM-based selection method decreases with increasing the dimension of searching space. In addition,
Table 1. Mutation strategies in ADM

<table>
<thead>
<tr>
<th>Condition (1): $\Delta f(t-1) \cdot \Delta f(t) &gt; 0$</th>
<th>Mutation strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x_{id}(t-1) \cdot \Delta x_{id}(t) &gt; 0$</td>
<td>Directional small scale</td>
</tr>
<tr>
<td>$\Delta x_{id}(t-1) \cdot \Delta x_{id}(t) &lt; 0$</td>
<td>Random small scale</td>
</tr>
<tr>
<td>$\Delta x_{id}(t-1) \cdot \Delta x_{id}(t) = 0$</td>
<td>Random medium scale</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Condition (2): $\Delta f(t-1) \cdot \Delta f(t) &lt; 0$</th>
<th>Mutation strategy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(t) \geq \overline{f}(t)$ and $\Delta x_{id}(t-1) \cdot \Delta x_{id}(t) &lt; 0$</td>
<td>Directional small scale</td>
</tr>
<tr>
<td>$f(t) &lt; \overline{f}(t)$ and $\Delta x_{id}(t-1) \cdot \Delta x_{id}(t) &lt; 0$</td>
<td>Random small scale</td>
</tr>
<tr>
<td>$\Delta x_{id}(t-1) \cdot \Delta x_{id}(t) = 0$</td>
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</thead>
<tbody>
<tr>
<td>$\Delta f(t-1) = 0$ and $\Delta f(t) \neq 0$</td>
<td>Directional small scale</td>
</tr>
<tr>
<td>$\Delta f(t-1) \neq 0$ and $\Delta f(t) = 0$</td>
<td>Random small scale</td>
</tr>
<tr>
<td>$\Delta f(t-1) = 0$ and $\Delta f(t) = 0$</td>
<td>Random large scale</td>
</tr>
</tbody>
</table>

A computational cost of the SOM-based selection is higher than those of the copy-based selections.

On the other hand, in order to ensure adequate genetic diversity in the RCGA, various mutation methods have been proposed [9, 10, 11]. In those methods, an adaptive directed mutation (ADM) operator [11] is the state-of-the-art mutation operator. The ADM operator considers the variation of fitness value and individual in order to make many kinds of individuals. The $\ell$-th element $x_{i\ell}(t+1)$ of the $i$-th individual after applying the ADM operator is shown as follows:

$$x_{i\ell}(t+1) = x_{i\ell}(t) + g(\Delta f(x_i(t)), \Delta f(x_i(t-1)), \Delta x_{i\ell}(t), \Delta x_{i\ell}(t-1), x_{i\ell}^{UB}, x_{i\ell}^{LB}) \cdot p_m(t),$$

where

$$\Delta f(t) = f(x_i(t)) - f(x_i(t-1)),$$

$$\Delta f(t-1) = f(x_i(t-1)) - f(x_i(t-2)),$$

$$\Delta x_{i\ell}(t) = x_{i\ell}(t) - x_{i\ell}(t-1),$$

and

$$\Delta x_{i\ell}(t-1) = x_{i\ell}(t-1) - x_{i\ell}(t-2).$$

In Equation (2), $x_{i\ell}^{UB}$ and $x_{i\ell}^{LB}$ are the upper and lower bounds of the $x_{i\ell}$, respectively. $p_m(t)$ is an adaptive probability of the mutation, and it is calculated as follows:

$$p_m(t) = \begin{cases} 
  0.5 \cdot \frac{f_{\max}(t) - f(x_i(t))}{f_{\max}(t) - \overline{f}(t)}, & \text{if } f(x_i(t)) \geq \overline{f}(t), \\
  0.5, & \text{otherwise},
\end{cases}$$

where $f_{\max}(t)$ and $\overline{f}(t)$ are the highest and average fitness values in the population of the $\ell$-th generation, respectively. $g(\cdot)$ is the function in order to choose the mutation strategy from four strategies, “directional small-scale mutation”, “random small-scale mutation”, “random medium-scale mutation” and “random large-scale mutation”. In the function $g(\cdot)$, the mutation strategy is selected based on nine patterns of evolutionary trends in the three consecutive generations ($t-2, t-1, t$) shown in Table 1.

In Table 1, “Directional small-scale mutation” is performed by:

$$x_{i\ell}(t+1) = x_{i\ell}(t) + \text{sign}(\Delta f(t)) \cdot \Delta x_{i\ell}(t) \cdot p_m(t),$$
where \( \text{sign}(z) \) is 1, if \( z > 0 \); \( \text{sign}(z) \) is \(-1\), if \( z < 0 \); and \( \text{sign}(z) \) is 0, if \( z = 0 \). “Random small-scale mutation” is defined as:
\[
x_{id}(t + 1) = x_{id}(t) + |\Delta x_{id}(t)| \cdot r_s \cdot p_m(t),
\]
where \( |\Delta x_{id}(t)| \) is an absolute value of the individual. \( r_s \) is uniform distributed random number in \([-1, 1]\]. “Random medium-scale mutation” is defined as:
\[
x_{id}(t + 1) = x_{id}(t) + x_{id}(t) \cdot r_s \cdot p_m(t).
\]
“Random large-scale mutation” is defined as:
\[
x_{id}(t+1) = \begin{cases} x_{id}(t) + (x_i^{UB}(t) - x_{id}(t)) \cdot r_s \cdot p_m(t), & \text{if } r < 0.5, \\ x_{id}(t) + (x_{id}(t) - x_i^{LB}(t)) \cdot r_s \cdot p_m(t), & \text{if } r \geq 0.5, \end{cases}
\]
where \( r \) is a uniform distributed random number in \([0, 1]\).

The ADM operator can keep the genetic diversity without time consuming computational processes. However, the convergence property of the ADM may not be good, because the ADM does not operate to the elite individual, i.e., \( f_{\text{max}}(t) - f(x_i(t)) = 0 \) in Equation (7). Furthermore, the processes of the ADM operator are too complicated, because the ADM uses the information on the individuals and their fitness values of three consecutive generations.

3. Proposed Variability Operator. In this paper, we propose a new variability operator which achieves an effective search with maintaining the genetic diversity by simple procedures. In the proposed operator, a variability vector is introduced in order to ensure adequate genetic diversity. Specifically, the new individuals for the next generation are generated by adding a variability vector to the individual selected by the copy-based conventional selection method. The variability vector is generated by random values. In the proposed operator, the dimension of the variability vector \( \mathbf{v}_i = (v_{i1}, \ldots, v_{il}, \ldots, v_{im}) \) for the \( i \)-th individual is the same size as that of the individuals. Each element \( v_{id} \) is given by the random values.

In this paper, we consider two kinds of random values generated by the following distributions: \( N(0, \sigma^2) \) or \( U[-\delta, \delta] \). \( N(0, \sigma^2) \) and \( U[-\delta, \delta] \) mean the normal and the uniform distributions, respectively. The parameters “0” and \( \sigma^2 \) of \( N(0, \sigma^2) \) stand for a mean value and a variance of the normal distribution. The conceptual sketch of the proposed method based on the normal distribution is shown in Figure 2(a). In Figure 2(b), the parameter “\( \delta \)” means a range to generate the random values based on the uniform

![Figure 2](image-url)

**Figure 2.** A conceptual sketch of the proposed operator. (a) Variability vector generated from normal distribution. (b) Variability vector generated from uniform distribution.
distribution. As shown in Figure 2, the proposed operator is regarded as a kind of local mutation with adaptive range.

In the RCGA with the proposed operator, all individuals are first initialized by random values. Then, the new individuals are generated by the crossover and mutation operators. Furthermore, an individual is selected by the copy-based selection operator. Here, the selected individual is represented as \( \mathbf{x}_i^*(t) \). In the proposed operator, the element \( x_{it}^{*\prime}(t) \) of new individual \( \mathbf{x}_i(t + 1) \) for the next generation is calculated by the selected individual \( \mathbf{x}_i^*(t) \) and the variability vector \( \mathbf{v}_i(t) \):

\[
x_{it}^{*\prime}(t + 1) = x_{it}^*(t) + v_{it}(t),
\]

where each element \( v_{it}(t) \) is generated by random values based on \( N(0, \sigma_i^2) \) or \( U[-\delta_i, \delta_i] \). \( \sigma_i^2 \) and \( \delta_i \) are defined by:

\[
\sigma_i^2 = \alpha \cdot \frac{1}{1 + f(\mathbf{x}_i^*(t))},
\]

\[
\delta_i = \alpha \cdot \frac{1}{1 + f(\mathbf{x}_i^*(t))},
\]

\( \sigma_i^2 \) and \( \delta_i \) are varied in accordance with the fitness value of \( \mathbf{x}_i^*(t) \). In Equations (13) and (14), \( \alpha \) is determined 0.1 empirically. These operations are repeated until the given conditions are satisfied.

In the proposed operator, the individual with low fitness value moves away by adding the large variability vector. On the other hand, the individual with high fitness value moves nearby adding the small variability vector. The proposed operator adaptively encourages the local and the global searches of each individual by considering its fitness value. Therefore, the proposed operator can maintain the genetic diversity with low computational cost by employing simple procedures, and realizes the effective search.

4. Experiments.

4.1. Continuous benchmark functions optimization. To evaluate the effectiveness and convergence property of the proposed operator, it is first applied to optimization problems of continuous benchmark functions (\( F_1(\mathbf{x}) \): Sphere, \( F_2(\mathbf{x}) \): Rosenbrock, \( F_3(\mathbf{x}) \): Rastrigin, \( F_4(\mathbf{x}) \): Griewank), which are defined as:

\[
F_1(\mathbf{x}) = \sum_{\ell=1}^{M} x_{\ell}^2, \quad (-64 < x_\ell < 64),
\]

\[
F_2(\mathbf{x}) = \sum_{\ell=1}^{M} \left\{ 100(x_{\ell+1} - x_\ell^2)^2 + (x_\ell - 1)^2 \right\}, \quad (-2.048 < x_\ell < 2.048),
\]

\[
F_3(\mathbf{x}) = \sum_{\ell=1}^{M} \left\{ x_\ell^2 - 10 \cos(2\pi x_\ell) + 10 \right\}, \quad (-5.12 < x_\ell < 5.12),
\]

\[
F_4(\mathbf{x}) = \frac{1}{4000} \sum_{\ell=1}^{M} x_\ell^2 - \prod_{\ell=1}^{M} \cos \left( \frac{x_\ell}{\sqrt{\ell}} \right) + 1, \quad (-512 < x_\ell < 512).
\]

The dimension \( M \) of each function is set to 2 and 5. In this experiment, to convert these functions from the minimization problems into the maximization problems, we use the following conversion function:

\[
f_k(\mathbf{x}_i) = \frac{1}{1 + F_k(\mathbf{x}_i)},
\]
where \( f_k(x_i) \) is the fitness value of \( k \)-th optimization problem.

The searching performance of the proposed operator is compared with the conventional copy-based selection and the ADM operator. As the copy-based selection, the RWS is employed. Table 2 shows details of the proposed and the conventional methods in this experiment. In the experiment, to evaluate the fundamental performance of the proposed operator, we use simple crossover method. Thus, the crossover operator in all cases employs one-point crossover. As the mutation operator, a uniform mutation is used. Then, their crossover and mutation probabilities are 0.3 and 0.05, respectively. The population size is 100 for each method.

These parameters are often used in various practical situations. In addition, for the practical use of the proposed method, the parameter \( \alpha \) has to be provided depending on the size of the searching space appropriately. In this experiment, \( \alpha \) is set to 0.1 empirically.

<table>
<thead>
<tr>
<th>Methods</th>
<th>Selection</th>
<th>Crossover</th>
<th>Mutation</th>
<th>Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conv. (RWS)</td>
<td>RWS</td>
<td>One point</td>
<td>Uniform</td>
<td>-</td>
</tr>
<tr>
<td>Conv. (ADM)</td>
<td>RWS</td>
<td>One point</td>
<td>ADM</td>
<td>-</td>
</tr>
<tr>
<td>Prop. (Norm.)</td>
<td>RWS</td>
<td>One point</td>
<td>Uniform</td>
<td>Normal distribution</td>
</tr>
<tr>
<td>Prop. (Unif.)</td>
<td>RWS</td>
<td>One point</td>
<td>Uniform</td>
<td>Uniform distribution</td>
</tr>
</tbody>
</table>

Table 3. Mean best fitness values (100 trials) in case with \( \alpha = 0.1 \)

<table>
<thead>
<tr>
<th>( f_k )</th>
<th>( D )</th>
<th># of Gen.</th>
<th>Conv. (RWS)</th>
<th>Conv. (ADM)</th>
<th>Prop. (Norm.)</th>
<th>Prop. (Unif.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 )</td>
<td>2</td>
<td>100</td>
<td>0.946</td>
<td>0.912</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.996</td>
<td>0.912</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.999</td>
<td>0.912</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>100</td>
<td>0.656</td>
<td>0.985</td>
<td>0.999</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.983</td>
<td>0.986</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.996</td>
<td>0.986</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>( f_2 )</td>
<td>2</td>
<td>100</td>
<td>0.924</td>
<td>0.986</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.951</td>
<td>0.986</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
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<td>1000</td>
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<tr>
<td></td>
<td>5</td>
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<tr>
<td></td>
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<td>0.473</td>
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<td></td>
<td></td>
<td>1000</td>
<td>0.486</td>
<td>0.844</td>
<td>0.842</td>
<td>0.974</td>
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<tr>
<td>( f_3 )</td>
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<td>0.939</td>
<td>0.960</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.996</td>
<td>0.960</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
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<td>0.960</td>
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<tr>
<td></td>
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<td>0.622</td>
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<td>0.438</td>
<td>0.967</td>
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<td></td>
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<td>0.465</td>
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<td>( f_4 )</td>
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<td>0.949</td>
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<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.980</td>
<td>0.992</td>
<td>0.993</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.990</td>
<td>0.993</td>
<td>0.997</td>
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<tr>
<td></td>
<td>5</td>
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<td>0.805</td>
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<td></td>
<td></td>
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<td>0.928</td>
<td>0.920</td>
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<td>0.960</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1000</td>
<td>0.963</td>
<td>0.924</td>
<td>0.977</td>
<td>0.974</td>
</tr>
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</table>
The searching performance of each method is evaluated by the mean best fitness (MBF) value after 100, 500 and 1,000 generations. To suppress the stochastic effects by the initialization, the number of trials is 100.

Table 3 shows the MBF values of each method. As shown in Table 3, the ADM operator works better than the conventional RWS in the 2-dimensional searching space. However, it is observed that the searching performance of the ADM operator becomes worse, as the searching space is large, e.g., $F_2(x)$ and $F_3(x)$. Furthermore, it is also seen that the convergence property of the ADM is not good.

On the other hand, it is observed that the proposed method realizes the better search than the conventional RWS and the ADM. Thus, it can be said that the proposed method can realize the effective search even if the dimension of the searching space becomes high. Furthermore, it is seen that the convergence property of the proposed method is better than that of the ADM. From these results, the proposed method can achieve more effective search than the conventional methods. In addition, the uniform distribution seems good on average for generating the variability vectors in the proposed method for various benchmark functions.

4.2. Signal source localization. In recent years, many researches that brain activities (signal sources) are estimated from potentials (signals) recorded with an electroencephalograph have been reported [14]. The localization of spikes (transient signals) caused by the abnormal neural firing is an important problem. Therefore, to perform brain surgeries such as intractable epilepsy safely, the localization of signal sources with high accuracy is required.

In order to evaluate the performance of the proposed method in the practical use, we used a single-layered sphere whose electric-conduction rate is 0.33 [S/m] as a head model shown in Figure 3. Potentials generated by current sources were recorded with 20 electrodes arranged on the surface of the sphere. The potentials were computed in accordance with the method in [15]. The problem to be solved was to search the position of the current source $p = (x, y, z)^T$ from the recorded potential $v = (v_1, \cdots, v_{20})^T$. To simplify the problem, the moment (direction) of the current source was set to $z = (0.63, 0.0, 0.63)^T$. In this problem, solution spaces should be 3 dimensional spaces for single current source.

![Figure 3. A simple head model and positions of inductive electrodes. Each electrode is represented by a dot on the sphere.](image-url)
In the localization simulation, the solution (the position of the current source) was defined at first. We call them correct solutions \( p' = (2, 0, 1.5, 8.6)^T \). The potentials of electrodes for the correct solution were calculated and were called observed potentials \( v^o = (v^o_1, \cdots, v^o_{20})^T \). The individuals of the RCGAs are candidates for the solution, that is, the position of the current source. Therefore, the individual for single sources is \( \hat{p} = (\hat{x}, \hat{y}, \hat{z})^T \). The potential \( v^e = (v^e_1, \cdots, v^e_{20})^T \) for each individuals is calculated and the evaluation value of the individuals \( E \) is given by:

\[
E = \exp \left( \frac{-e}{\sigma} \right),
\]

\[
e = \sqrt{\frac{|v^o - v^e|^2}{\|v^e\|^2}},
\]

where, \( \sigma \) is a parameter and is set to 0.1 in this simulation. This evaluation (fitness) function gives a high value when the potential calculated for the individuals is similar to the observed potential. It means that the individuals \( \hat{p} \) is similar to the correct solution \( p' \). The maximum value of \( E \) is 1.0.

In common with the continuous benchmark functions optimization, the searching performance of the proposed operator is also compared with the conventional RWS and the ADM operator. The crossover operator in this experiment employs BLX-\( \alpha \) crossover [2]. As the mutation operator, a uniform mutation is used. Then, their crossover and mutation probabilities are 0.3 and 0.05, respectively. The population size is 100 for each method. In the proposed method, the variability operator by employing uniform distribution is used. The parameter \( \alpha \) of the proposed method is also set to 0.1 empirically.

The searching performance of each method is evaluated by the mean best fitness (MBF) value until 1,000 generations. To suppress the stochastic effects by the initialization, the number of trials is 100.

Figure 4 shows the experimental results. As shown in Figure 4, the ADM operator performs slightly better search than the conventional RWS. However, it is observed that the RCGA with the ADM operator cannot find the optimum solution until 1,000-th generation.

On the other hand, it is observed that the proposed method realizes better search than the conventional RWS and the ADM. The RCGA with the proposed method can find the optimum solution in about 800-th generation.

With these results, the advantages of the proposed method are to be able to find the optimum solution faster than the conventional methods. Furthermore, the convergence property of the proposed method is better than the conventional methods.
5. **Conclusions.** In this paper, we proposed a modified RCGA with the variability operator. The proposed method maintains genetic diversity by adding variability term to the individuals. Experiments confirmed that the proposed method ensures effective search. In the high-dimensional searching space, the proposed method can find the optimum solution faster than the conventional methods. Furthermore, the convergence property of the proposed method is good. The effectiveness of the proposed method was also verified by applying it to the signal source localization problem.

As future work, the establishment of the automatic decision method of the parameter $\alpha$ still remains.

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