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A CONSTRUCTIVE SOLUTION FOR ADAPTIVE STABILIZATION OF TCSC VIA IMMERSION AND INVARIANCE

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Abstract. *A new adaptive controller is designed based on immersion and invariance for TCSC. Compared with the available adaptive results in the literature, the proposed adaptive approach does not invoke certainty equivalence, nor requires a linear parameterization and allows for prescribed dynamics to be assigned to the parameter estimation error. By choosing an appropriate target dynamics and manifold, an adaptive state feedback controller has been synthesized constructively to ensure all signals of closed-loop system are globally bounded.*

Keywords: Immersion and invariance, TCSC, Adaptive control

1. **Introduction.** Maintaining power system stability is one of the main concerns in power systems. The design of an advanced control system to enhance the power system stability margin so as to achieve higher transfer limits is one of the major problems in power systems, which has attracted a great deal of research attention in recent years [1,2], and the references therein.

Improvements in the power electronics technology and in the new area of flexible AC transmission systems (FACTS) have considerable potential to enhance power system's transient stability [3]. Thyristor controlled series compensation (TCSC) is an important member of FACTS family. It is installed in long-distance transmission systems for rapid adjustment of the effective value of a capacitor in series with transmission line by making use of the short-time over-load capability of the capacitor [4].

Usually, there are uncertain parameters that cannot be measured accurately in power systems [5]. Recently, in [6] a robust adaptive modulation controller (RAMC) for TCSC in interconnected power systems to damp low frequency oscillation is proposed. In [7], the immersion and invariance (I&I) strategy has been used to stabilize the nonlinear swing equation model of SMIB using a CSC. The method of I&I for stabilization of nonlinear systems originated in Astolfi and Ortega [8] and was further developed in a series of publications that have been recently summarized in [9]. The I&I method in [8] was extended to adaptive I&I in [10]. The method relies upon the notions of system immersion and manifold invariance and, in principle, does not require the knowledge of a Lyapunov function. The resulting adaptive control schemes counter the effect of the uncertain parameters adopting a robustness perspective. This is in contrast with some of the existing adaptive designs that treat these terms as disturbances to be rejected [11-13].

This paper studies the control problem of TCSC system by I&I method. The damping coefficient uncertainty of TCSC is considered to enhance the transient stability. The adaptive I&I controller is designed to achieve stability of rotor angle, speed, and voltage. This paper is organized as follows. Section 2 gives an outline of immersion and invariance. In Section 3, the model of the power systems is described as well as the problem statement. In Section 4, we give main result to illustrate the performance of the proposed adaptive control scheme. The simulation plot is given in Section 5. We wrap up the paper with concluding remarks in Section 6.

2. **Adaptive Stabilization via Immersion and Invariance.** The main stabilization ideas in Astolfi and Ortega [8] can be used to design adaptive stabilizing controllers for classes of nonlinear systems with parametric uncertainties. To this end, consider the system

$$\dot{x} = f\left(x, u, \theta\right), \tag{1}$$

with state $x \in R^n$, control $u \in R^m$, unknown parameter $\theta \in R^q$, with an equilibrium point $x_* \in R^n$ to be stabilized, and define the augmented system

$$\dot{x} = f\left(x, u, \theta\right),\ \dot{\hat{\theta}} = w, \tag{2}$$

where $\hat{\theta} \in R^q$, and $w \in R^q$ is a new control signal. The adaptive stabilization problem can be posed, informally, as follows.

Find (if possible) a state feedback control law described by equations of the form

$$w = \varpi\left(x, \hat{\theta}\right),\ u = \upsilon\left(x, \hat{\theta}\right), \tag{3}$$

such that all trajectories of the closed-loop system (2)-(3) are bounded and

$$\lim_{t\to\infty} x\left(t\right) = x^*. \tag{4}$$

Note that, since $f\left(.\right)$ is only partially known, it is not required that $\hat{\theta}$ converges to any particular equilibrium, but merely that it remains bounded. The major result of [10] that constitutes the basis of the paper is the following theorem.

Theorem 2.1. *Consider the system (2) and a point $x_* \in R^n$. $p \leq n$, $\xi \in R^p$, $\zeta \in R^{n-p}$ and $z \in R^q$. Assume we can find mappings*

$$\alpha\left(\xi,\theta\right) \to R^p,\quad \pi\left(\xi,\theta\right) \to R^n,\quad c\left(\xi,\theta\right) \to R^m,\quad \beta\left(x\right) \to R^q,$$
$$\phi\left(x,\theta\right) \to R^{n-p},\quad u\left(x,\zeta,z+\theta\right) \to R^m,\quad \varpi\left(x,\zeta,z+\theta\right) \to R^q,$$

such that the following hold.

(H1) (Target system) *The system*

$$\dot{\xi} = \alpha\left(\xi,\theta\right), \tag{5}$$

with state $\xi \in R^p$, has a globally asymptotically stable equilibrium at $\xi^* \in R^p$ and $x^* = \pi\left(\xi^*,\theta\right)$.

(H2) (Immersion condition) *For all $\xi \in R^p$*

$$f\left(\pi\left(\xi,\theta\right), c\left(\xi,\theta\right), \theta\right) = \frac{\partial \pi}{\partial \xi}\alpha\left(\xi,\theta\right). \tag{6}$$

(H3) (Implicit manifold) *The set identity*

$$\left\{x \in R^n \,|\, \phi\left(x,\theta\right) = 0\right\} = \left\{x \in R^n \,|\, x = \pi\left(\xi,\theta\right),\ \xi \in R^p\right\} \tag{7}$$

holds.

(H4) (Manifold attractivity and trajectory boundedness) *All trajectories of the system*

$$\dot{\zeta} = \frac{\partial\phi\left(x,\hat{\theta}\right)}{\partial x} f\left(x, u\left(x,\zeta,z+\theta\right),\theta\right) + \frac{\partial\phi\left(x,\hat{\theta}\right)}{\partial\hat{\theta}}\varpi\left(x,\zeta,z+\theta\right), \tag{8}$$

$$\dot{z} = \bar{\omega}\left(x,\zeta,z+\theta\right) + \frac{\partial\beta}{\partial x} f\left(x,u\left(x,\zeta,z+\theta\right),\theta\right), \tag{9}$$

$$\dot{x} = f\left(x,u\left(x,\zeta,z+\theta\right),\theta\right), \tag{10}$$

are bounded and satisfy

$$\lim_{t\to\infty}\zeta\left(t\right) = 0, \tag{11}$$

$$\lim_{t\to\infty}\left[\phi\left(x\left(t\right),z\left(t\right)+\theta\right) - \phi\left(x\left(t\right),\theta\right)\right] = 0. \tag{12}$$

Then all trajectories of the closed-loop system

$$\begin{aligned} \dot{x} &= f\left(x,u\left(x,\phi\left(\hat{\theta}+\beta\left(x\right)\right),\hat{\theta}+\beta\left(x\right)\right),\theta\right) \\ \dot{\hat{\theta}} &= \varpi\left(x,\phi\left(x,\hat{\theta}+\beta\left(x\right)\right),\hat{\theta}+\beta\left(x\right)\right) \end{aligned} \tag{13}$$

are bounded and satisfy (4).

Finally

$$\lim_{t\to\infty}\left[x\left(t\right) - \pi\left(\xi\left(t\right),\theta\right)\right] = 0$$

and, if $\phi\left(x\left(0\right),\hat{\theta}\left(0\right)\right) = 0$, $\hat{\theta}\left(0\right) - \theta + \beta\left(x\left(0\right)\right) = 0$ *and* $x\left(0\right) = \pi\left(\xi\left(0\right),\theta\right)$, $x\left(t\right) = \pi\left(\xi\left(t\right),\theta\right)$, *for all* $t \geq 0$.

Definition 2.1. *The system (2) is said to be adaptively I&I stabilizable with target dynamics (5) if (H1)-(H4) of Theorem 2.1 are satisfied.*

3. **Model and Problem Statement.** Consider the following single-machine infinite-bus system with TCSC shown in Figure 1. Let the generator be the constant voltage source after the transient reactance. One-order inertial link being equivalent to the TCSC dynamic process, the model of the system controlled by TCSC is expressed as follows [1]:

$$\begin{cases} \dot{\delta} = \omega - \omega_0 \\ \dot{\omega} = \dfrac{\omega_0}{H}\left(P_m - E_q V_s y_{t\,\mathrm{csc}}\sin\delta - \dfrac{D}{\omega_0}\left(\omega-\omega_0\right)\right) \\ \dot{y}_{t\,\mathrm{csc}} = -\dfrac{1}{T_{t\,\mathrm{csc}}}\left(y_{t\,\mathrm{csc}} - y_{t\,\mathrm{csc}\,0} - u\right) \end{cases}, \tag{14}$$

where δ is the angle and ω is the relative speed of generator rotor; P_m is the mechanical power on the generator shaft; H is the inertial constant. $T_{t\,\mathrm{csc}}$ is the time constant of TCSC; D and E_q are damping coefficients and inner generator voltage, respectively; V_s is the infinite bus voltage. $y_{t\,\mathrm{csc}} = \frac{1}{X_{d\sum}+X_{t\,\mathrm{csc}}}$ is the admittance of the whole system. $X_{t\,\mathrm{csc}}$ is the equivalent reactance of TCSC. $X_{d\sum}$ is the external reactance. u is the reactance modulated input of TCSC. Other units without special instructions are standard per unit.

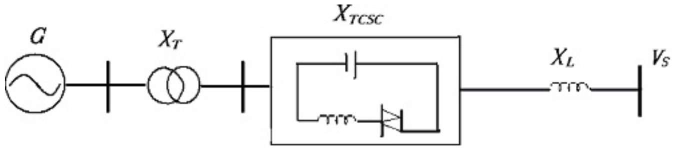

FIGURE 1. A single-machine infinite-bus system with TCSC

For system (14), letting $x_1 = \delta - \delta_0$, $x_2 = \omega - \omega_0$, $x_3 = y_{t\,\mathrm{csc}} - y_{t\,\mathrm{csc}\,0}$, where δ_0, ω_0, $y_{t\,\mathrm{csc}\,0}$ are the initial value of corresponding variables, system (1) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{D}{H}x_2 + \dfrac{\omega_0}{H}\left(P_m - E_q V_s \left(x_3 + y_{t\,\mathrm{csc}\,0}\right)\sin\left(x_1 + \delta_0\right)\right) \\ \dot{x}_3 = \dfrac{1}{T_{t\,\mathrm{csc}}}\left(-x_3 + u\right) \end{cases}, \tag{15}$$

Usually, the damping coefficient D cannot be measured accurately in practical engineering applications [1]. Hence $\theta = -D/H$ is taken as an unknown and/or uncertain constant parameter that has to be estimated on-line in real time. Let $k_1 = \omega_0/H, k_2 = \omega_0 E_q V_s/H$, and they are known constants. Then system (15) is readily transformed as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \theta x_2 + k_1 P_m - k_2\left(x_3 + y_{t\,\mathrm{csc}\,0}\right)\sin\left(x_1 + \delta_0\right) \\ \dot{x}_3 = \dfrac{1}{T_{t\,\mathrm{csc}}}\left(-x_3 + u\right) \end{cases}. \tag{16}$$

4. I&I-Based Adaptive Controller Design for TCSC.

4.1. **Control objective.** As mentioned earlier, x_* denotes the operating stable equilibrium. We assume that x_* is known to us and state the control objective as 'to design a control law u in order to make the system (16) asymptotically stable at x_*, and to improve the transient response of the closed-loop system'.

4.2. **Controller design.** We proceed to verify the H1-H4 of Theorem 2.1.

(H1) (Target system) The key idea is to immerse a low-dimensional system into a high-dimensional one. Thus, we define the target dynamics as

$$\Sigma_T : \begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -l_1 \sin\left(\xi_1\right) - l_2 \xi_2 \end{cases} \tag{17}$$

where ξ_1, $\xi_2 \in R$, $l_1 > 0$, $l_2 > 0$ which are to be chosen. The target system (17) has an asymptotically stable equilibrium at $(0, 0)$.

(H2) (Immersion condition) Given the control objectives and our choice of target dynamics, a natural selection of the mapping π is

$$\pi\left(\xi, \theta\right) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3\left(\xi, \theta\right) \end{bmatrix}, \tag{18}$$

where $\pi_3\left(\xi, \theta\right)$ is a function to be defined. With this choice of $\pi\left(\xi, \theta\right)$ and the target dynamics above, Equation (6) becomes

$$\begin{bmatrix} \xi_2 \\ \theta\xi_2 + k_1 P_m - k_2\left(\pi_3 + y_{t\,\mathrm{csc}\,0}\right)\sin\left(\xi_1 + \delta_0\right) \\ \frac{1}{T_{t\,\mathrm{csc}}}\left(-\pi_3 + c\left(\xi, \theta\right)\right) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial \pi_3(\xi,\theta)}{\partial \xi_1} & \frac{\partial \pi_3(\xi,\theta)}{\partial \xi_2} \end{bmatrix} \begin{bmatrix} \xi_2 \\ -l_1 \sin\left(\xi_1\right) - l_2 \xi_2 \end{bmatrix} \tag{19}$$

Next we choose $\pi_3\left(\xi\right)$ and $c\left(\xi, \theta\right)$ to satisfy the above equation as follows: the first row of (11) is already satisfied. From the second row we have

$$\pi_3 = \frac{\theta\xi_2 + k_1 P_m + l_1 \sin\left(\xi_1\right) + l_2 \xi_2}{k_2 \sin\left(\xi_1 + \delta_0\right)} - y_{t\,\mathrm{csc}\,0}. \tag{20}$$

From the third row we have

$$c(\xi,\theta) = T_{t\,\mathrm{csc}}\left[\frac{\partial \pi_3(\xi,\theta)}{\partial \xi_1}\xi_2 + \frac{\partial \pi_3(\xi,\theta)}{\partial \xi_2}\left(-l_1 \sin(\xi_1) - l_2\xi_2\right)\right] + \pi_3(\xi,\theta). \tag{21}$$

(H3) (Implicit manifold) The manifold is $\mathcal{M}$ implicitly described by $\mathcal{M} = \{x \in R^3 | \phi(x,\theta) = 0\}$, with

$$\phi(x,\theta) = x_3 - \pi_3\left([x_1, x_2]^{\mathrm{T}}, \theta\right) = x_3 - \frac{\theta x_2 + k_1 P_m + l_1 \sin(x_1) + l_2 x_2}{k_2 \sin(x_1 + \delta_0)} + y_{t\,\mathrm{csc}\,0}. \tag{22}$$

(H4) (Manifold attractivity and trajectory boundedness) The off-the-manifold coordinates are $\zeta = \phi\left(x, \hat{\theta}\right)$, $z = \hat{\theta} - \theta + \beta(x_1, x_2)$ and straightforward calculations show that

$$\begin{aligned}
\dot{\zeta} &= \frac{(-x_3 + u)}{T_{t\,\mathrm{csc}}} - \frac{w x_2 + \left(\frac{\partial\beta}{\partial x_1}x_2 + l_1\sin(x_1)\right)x_2 + \left(\frac{\partial\beta}{\partial x_2}x_2 + l_2 + \hat{\theta} + \beta\right)\dot{x}_2}{k_2 \sin(x_1 + \delta_0)} \\
&\quad + \frac{\left[\left(\hat{\theta} + \beta\right)x_2 + k_1 P_m + l_1 \sin(x_1) + l_2 x_2\right]\cos(x_1 + \delta_0)\, x_2}{k_2^2 \sin^2(x_1 + \delta_0)} \\
\dot{z} &= w + \frac{\partial\beta}{\partial x_1}x_2 + \frac{\partial\beta}{\partial x_2}\left(\theta x_2 + k_1 P_m - k_2\left(x_3 + y_{t\,\mathrm{csc}\,0}\right)\sin(x_1 + \delta_0)\right) \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -k_2 \zeta \sin(x_1 + \delta_0) - l_1 \sin(x_1) - l_2 x_2 - z x_2.
\end{aligned} \tag{23}$$

By choosing control laws u and w as follows, and $\beta = \frac{\gamma}{2}x_2^2$

$$\begin{aligned}
u &= x_3 + T_{t\,\mathrm{csc}}\left\{\frac{\dot{\hat{\theta}} x_2 + l_1 \sin(x_1)\, x_2}{k_2 \sin(x_1 + \delta_0)}\right. \\
&\quad + \frac{\left(\frac{3\gamma}{2}x_2^2 + l_2 + \hat{\theta}\right)\left(\left(\hat{\theta} + \frac{\gamma}{2}x_2^2\right)x_2 + k_1 P_m - k_2\left(x_3 + y_{t\,\mathrm{csc}\,0}\right)\sin(x_1 + \delta_0)\right)}{k_2 \sin(x_1 + \delta_0)} \\
&\quad \left. - \frac{\left[\left(\hat{\theta} + \beta\right)x_2 + k_1 P_m + l_1 \sin(x_1) + l_2 x_2\right]\cos(x_1 + \delta_0)\, x_2}{k_2^2 \sin^2(x_1 + \delta_0)} - \sigma\varsigma\right\} \\
\dot{\hat{\theta}} &= -\gamma x_2^2\left(\hat{\theta} + \frac{\gamma}{2}x_2^2\right) - \gamma x_2\left[k_1 P_m - k_2\left(x_3 + y_{t\,\mathrm{csc}\,0}\right)\sin(x_1 + \delta_0)\right] \\
\sigma &= \lambda + \varepsilon\left(\frac{3\gamma}{2}x_2^2 + l_2 + \hat{\theta}\right)^2.
\end{aligned} \tag{24}$$

The system (23) can be rewritten as

$$\begin{aligned}
\dot{\zeta} &= -\sigma\zeta - \frac{\left(\frac{3\gamma}{2}x_2^2 + l_2 + \hat{\theta}\right)x_2 z}{k_2 \sin(x_1 + \delta_0)} \\
\dot{z} &= -\gamma x_2^2 z \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -k_2 \zeta \sin(x_1 + \delta_0) - l_1 \sin(x_1) - l_2 x_2 - z x_2.
\end{aligned} \tag{25}$$

4.3. **Stability result.** Finally, we establish boundedness of the trajectories of the closed-loop system (16) with the control law (24) and the off-the-manifold coordinate z.

From (17) and (18), we can obtain that states x_1, x_2 are bounded and converge to equilibrium x_{1*}, x_{2*}. We know that the off-the-manifold coordinate ζ is bounded and $\lim_{t\to\infty} \zeta(t) = 0$ from (25). Next we have $x_3 = \zeta + \pi_3$, from (20) we have π_3 which is bounded, and hence we can conclude boundedness of x_3.

The above discussion on the control synthesis can be summarized in the following proposition which is one of the main results of this paper.

Proposition 4.1. *The system (16) with the control law (24) is asymptotically stable at* x_*.

Proof: From the derivations above it is clear that the proposition can be easily proved, but omitted here for brevity.

5. **A Practical Example.** In this section, we use a practical example which is taken from [1] to illustrate the effectiveness and merit of our result. Simulation comparison of the proposed design method and adaptive backstepping in [6] has been carried out by using Matlab software and the parameters of TCSC system are taken from [1]:

$$
\begin{aligned}
&D = 1 \text{ p.u.},\ H = 7 \text{ s},\ v_s = 0.995 \text{ p.u.},\\
&E_q = 1.067 \text{ p.u.},\ P_m = 0.9 \text{ p.u.},\\
&T_{t\,\mathrm{csc}} = 0.05 \text{ s},\ \omega_0 = 1 \text{ p.u.},\\
&\delta_0 = 0.7854 \text{ rad},\ y_{t\,\mathrm{csc}\,0} = 1.1991 \text{ p.u.},\\
&\lambda = 2,\ \varepsilon = 1,\ l_1 = 4,\ l_2 = 0.8, \gamma = 5.
\end{aligned}
$$

The initial states are set $[x_1(0), x_2(0), x_3(0)] = [0.2, 0, 0]$, $\theta(0) = 0$.

As seen from Figure 2, we achieve faster convergence speed and smaller magnitude with the proposed method, so we can say that the proposed adaptive I&I controller has better performance than adaptive backstepping controller.

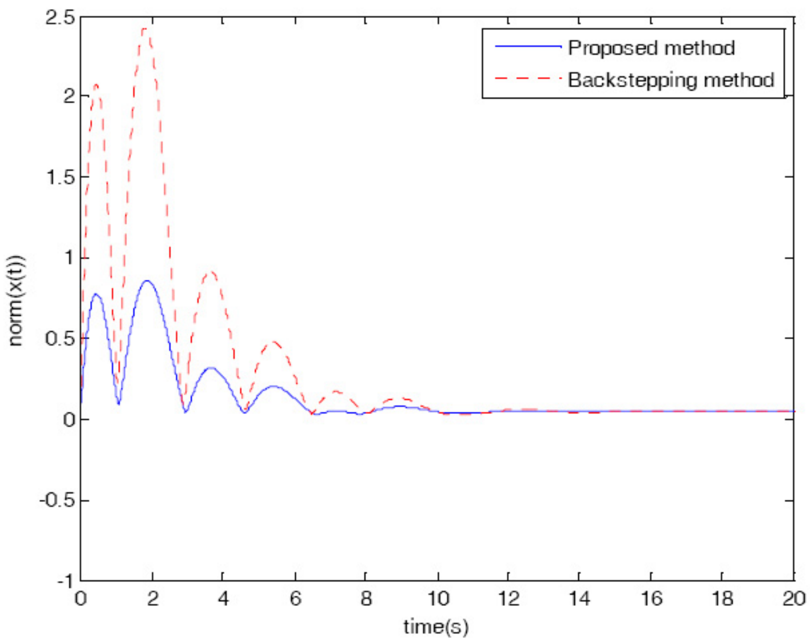

FIGURE 2. Comparative result for different controllers

6. **Conclusions.** In this paper, the nonlinear adaptive controller is designed for TCSC using adaptive immersion and invariance. The proposed method does not need Lyapunov function, and it deals with uncertain parameter in a robustness perspective. This is different from some of exiting methods relying on certain matching condition which treat these terms as disturbances to be rejected.

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