International Journal of Innovative Computing, Information and Control
Volume **13**, Number **1**, February **2017**
ICIC International ©2017 ISSN 1349-4198
pp. 277–288

NON-FRAGILE ROBUST H_∞ CONTROL FOR NONLINEAR NETWORKED CONTROL SYSTEMS WITH TIME-VARYING DELAY AND UNKNOWN ACTUATOR FAILURES

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Received July 2016; revised November 2016

Abstract. *The problem of non-fragile robust H_∞ control for nonlinear networked control systems (NCSs) with time-varying delay and random actuator faults is addressed in this study. The system parameters are allowed to have time-varying uncertainties and the actuator faults are unknown but the upper and lower bounds of which are known. By taking the exogenous disturbance and network transmission delay into consideration, a delay nonlinear system model is constructed. Based on Lyapunov stability theory, linear matrix inequalities (LMIs) and free weighting matrix methods, the sufficient conditions for the existence of the non-fragile robust H_∞ controller gain are derived which can be obtained by solving the LMIs. Finally, a numerical example is provided to illustrate the effectiveness of the proposed methods.*

Keywords: Nonlinear networked control systems, Time-varying delay, Random actuator faults, Free weighting matrix methods

1. **Introduction.** The actuator failure problem has become a prevalent research focus in control engineering domain due primarily to its practical significance [1]. Over the last two decades, enormous research on the networked control systems (NCSs) has been carried out in search for new design methodologies, to deal with the actuator failures and maintain the acceptable system stability and performances [2-8]. The modeling of actuator faults is proposed with three types generally: 1) The actuator is either completely normal or completely fail and the actuator fault is known as constant values [2-5]; 2) The actuator fault is unknown but the lower and upper bounds are known [6,7]; 3) The actuator fault is stochastic variable with known expectation and variance [8-10]. Most of the literature focuses on the first and second types, but there has been seldom effort on the stochastic fault of actuator. Consider the different failure rates of the actuators and the actuator fault only has two situations [8,9]. Consider the different failure rate of the actuators and the actuator fault has a range [10]. Two sets of stochastic variable are proposed to describe stochastic fault of the actuator and sensor [11].

As is well known, time-delay phenomenon is very common in many real physical systems and the problem of time-delay is an important topic that has attracted considerable research interests [9,12-16]. Since the effects of time-delay are inevitable, it is important and necessary to take time-delay into account when considering the performances of systems [12]. Moreover, in practical systems, the study of control scheme with time-varying delay is more important than that with constant delays. Actually, the time-delay in many realistic control systems exists in a stochastic fashion [13-19]. For such case, if there is a time-varying perturbation on the nonzero delay, it is of great significance to consider the stability analysis and controller design of the systems with time-varying delay [17]. On the other hand, the design of control scheme in H_∞ setting has good advantages, and it is well known that the H_∞ performance is closely related to the capability of disturbance rejection [18]. The robust reliable H_∞ control with time-delay and time-varying norm-bounded parametric uncertainties is studied, but its actuators are considered as a disturbance signal to the system which is augmented with system disturbance input [20]. A new robust H_∞ filtering is investigated for a class of time-varying nonlinear systems with norm-bounded parameter uncertainties and probabilistic sensor gain faults [21]. In the above-mentioned literature, most of the literature only considers the case of continuous systems [12-23], and there is little research on discrete systems [21], but data transmission in the network is based on the existence of a discrete manner. To the best of our knowledge, the non-fragile robust H_∞ fault-tolerant control problem has received much less attention for networked time-delay nonlinear systems with randomly coming actuator failures and the exogenous disturbance, especially the articles about random actuator failure are few. These aspects constitute the main motivation of this paper.

Summarizing the aforementioned discussions, we aim to model the networked control systems as a discrete time nonlinear system with time-varying delay. Besides, randomly coming actuator failures, the exogenous disturbance, and norm-bounded parameter uncertainties are also considered. The sufficient condition of non-fragile robust H_∞ fault-tolerant control for networked control system is developed in terms of LMIs. The controller gain matrix can be figured out by a set of matrix inequalities. Finally, an example is used to illustrate the effectiveness and the feasibility of the proposed approach.

The rest of this paper is organized as follows. The system description and preliminaries are introduced in Section 2. In Section 3, the stability analysis and controller design of NCSs are addressed. A numerical example is given in Section 4. Finally, the concluding remarks are given in the last section.

2. **System Description and Preliminaries.**

In practical networked systems, the system parameters will produce perturbation due to external interference or the aging of the instruments themselves. Consider the discrete-time nonlinear model with the exogenous disturbance of the plant as:

$$\begin{cases} x(k+1) = \bar{A}_0 x(k) + \bar{A}_1(k-d(k)) + \bar{B}_0 u(k) + Rw(k) + f(k, x(k)) \\ z(k) = Cx(k) + Dw(k) \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the plant state, $u(k) \in R^m$ is the control input, $w(k) \in L_2[0, \infty)$ is the external disturbance, $z(k)$ is the control output, $f(k, x(k))$ is the nonlinear vector satisfying Lipschitz condition, and $\|f(k, x(k))\| \le \|F_1 x(k)\|$. $\bar{A}_0 = A_0 + \Delta A_0$, $\bar{A}_1 = A_1 + \Delta A_1$, $\bar{B}_0 = B_0 + \Delta B_0$, A_0, A_1, B_0, C, D, R and F_1 are constant matrices with appropriate dimensions. ΔA_0, ΔA_1 and ΔB_0 denote the uncertainty of the system and they are assumed norm-bounded. They could be time-varying, and described as

$$\begin{bmatrix} \Delta A_0 & \Delta A_1 & \Delta B_0 \end{bmatrix} = D_1 F(k) \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix} \tag{2}$$

where D_1, E_1, E_2 and E_3 are constant matrices which denote the uncertainty of structure. $F(k)$ is uncertain matrix function and satisfies $F(k)^T F(k) \le I$. $d(k)$ is a positive scalar and it is time-varying that satisfies

$$0 < d_1 \le d(k) \le d_2, \quad \forall k \tag{3}$$

Define $\tau = d_2 - d_1$, when $\tau = 0$, it indicates that $d(k)$ is time-invariant, so the time-delay is a scalar.

For the problem of parameter perturbation, the design of the controller should be corresponding, so we use a non-fragile controller to make system have a good state. Consider the following state feedback controller:

$$u(k) = (K + \Delta K)x(k) = \hat{K}x(k) \tag{4}$$

where K is the controller gain matrix, ΔK denotes the uncertainty of the controller which is defined as $\Delta K = D_1 F(k) E_4$, and E_4 is a constant matrix with appropriate dimensions.

Actuator failures for some reasons cannot be measured, so it is important to take a different approach to handle random failures. If system (1) is under actuators failures whose matrix can be described as:

$$M = diag\{m_1, m_2, \ldots, m_n\} \tag{5}$$

where m_i are n unrelated random values, absolutely, '1' for normal and '0' for failure. When $m_i \neq 1$, it indicates that the actuator has partial failure. The expectation α_i and variance δ_i^2 of m_i are known values. Define $\bar{M} = E\{M\}$, and then

$$\bar{M} = diag\{\alpha_1, \alpha_2, \ldots, \alpha_n\} = \sum_{i=1}^{n} \alpha_i \Theta_i \tag{6}$$

where Θ_i is a diagonal matrix with the i elements being 1 and the other elements being 0. By taking the actuator failures into consideration, the control law (4) can be rewritten as

$$u(k) = M\hat{K}x(k) \tag{7}$$

Substituting (7) into (1) leads to the following closed-loop system:

$$\begin{cases} x(k+1) = A_k x(k) + \bar{A}_1 x(k - d(k)) + Rw(k) \\ z(k) = Cx(k) + Dw(k) \end{cases} \tag{8}$$

where $A_k = \bar{A}_0 - I + \bar{B}_0 M \hat{K} + F_1$.

3. **Main Results.** In this section, we will first present a sufficient condition of system (8) which is robustly stochastically stable, and then give a parameterized representation of the robust control laws in terms of the feasible solutions to certain LMIs. To begin with, we introduce the following lemmas which will be used in subsequent developments.

Lemma 3.1. *Let $Q = Q^T$, H and E be real matrices of appropriate dimensions with $F^T F \leq I$, and then the inequality $Q + HFE + E^T F^T H^T < 0$, if and only if there exists a positive scalar $\varepsilon > 0$ such that $Q + \varepsilon HH^T + \varepsilon^{-1} E^T E < 0$.*

Lemma 3.2. *Let X and Y be real matrices of appropriate dimensions, if there exists a positive scalar $\varepsilon > 0$ such that $X^T Y + Y^T X \leq \varepsilon X^T X + \varepsilon^{-1} Y^T Y$.*

Then, let us introduce the following definitions.

Definition 3.1. *When $w(k) = 0$, system (8) is said to be robustly stochastically stable if for every initial state x_0, $E\left[\sum_{k=0}^{\infty} \|x(k)\|^2\right] < \infty$.*

Definition 3.2. *For a given scalar $\gamma > 0$, the exogenous disturbance $w(k) \in L_2[0, \infty)$, if $\|z\|_2 < \gamma \|w\|_2$, where $\|z\|_2 = \left[\sum_{k=0}^{\infty} E\left[z^T(k) z(k)\right]\right]^{1/2}$, then system (8) is said to have a robust H_∞ performance γ.*

3.1. **Stability analysis.** The following theorem gives a sufficient condition for the stability of the networked control systems.

Theorem 3.1. *When $w(k) = 0$, the networked control system (8) is robustly stochastically stable if there exist symmetric positive-definite matrices $P = P^T > 0$, $Q = Q^T > 0$, $Z = Z^T > 0$, semi-positive-definite matrix $X = \begin{bmatrix} X_{11} & * \\ X_{21} & X_{22} \end{bmatrix} \geq 0$ and appropriate dimensions matrices N_1, N_2 such that the following matrix inequalities hold.*

$$\Phi = \begin{bmatrix} \varphi_{11} & * & * & * & * & * \\ \varphi_{21} & \varphi_{22} & * & * & * & * \\ (PR)^T & 0 & -\gamma^2 I & * & * & * \\ A_k & \bar{A}_1 & R & -P^{-1} & * & * \\ d_2 A_k & d_2 \bar{A}_1 & d_2 R & 0 & -d_2 Z^{-1} & * \\ C & 0 & D & 0 & 0 & -I \end{bmatrix} < 0 \tag{9}$$

$$\Psi = \begin{bmatrix} X_{11} & * & * \\ X_{21} & X_{22} & * \\ N_1^T & N_2^T & Z \end{bmatrix} \geq 0 \tag{10}$$

where

$$\varphi_{11} = PA_k + A_k^T P + N_1^T + N_1 + (\tau + 1)Q + d_2 X_{11},$$
$$\varphi_{21} = \bar{A}_1^T P - N_1^T + N_2 + d_2 X_{21},$$
$$\varphi_{22} = -N_2 - N_2^T - Q + d_2 X_{22},$$

and "$*$" is used as an ellipsis for terms induced by symmetry. Then the system has the H_∞ performance γ.

Proof: Supposing

$$y(l) = x(l + 1) - x(l), \tag{11}$$

then we can get

$$x(k + 1) = x(k) + y(k), \tag{12}$$

and

$$x(k) - x(k - d_k) - \sum_{l=k-d_k}^{k-1} y(l) = 0. \tag{13}$$

Choose the following Lyapunov-Krasovskii function as

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{14}$$

where

$$V_1(k) = x^T(k)Px(k),$$
$$V_2(k) = \sum_{\theta=-d_2+1}^{0} \sum_{l=k-1+\theta}^{k-1} y^T(l)Zy(l),$$
$$V_3(k) = \sum_{\theta=-d_2+1}^{-d_1+1} \sum_{l=k-1+\theta}^{k-1} x^T(l)Qx(l).$$

Define $\Delta V(k) = V(k + 1) - V(k)$, and then we can get

$$\Delta V_1(k) = 2x^T(k)Py(k),$$
$$\Delta V_2(k) = y^T(k)(P + d_2 Z)y(k) - \sum_{l=k-d_2}^{k-1} y^T(j)Zy(j),$$

$$\Delta V_3(k) \leq (\tau+1)x^T(k)Qx(k) - x^T(k-d_k)Qx(k-d_k).$$

Using the free weighting matrix method for any matrices N_i $(i = 1, 2)$ with appropriate dimensions, we can obtain that

$$2\left[x^T(k)N_1 + x^T(k-d_k)N_2\right] \times \left[x(k) - x(k-d_k) - \sum_{l=k-d_k}^{k-1} y(l)\right] = 0 \tag{15}$$

The following formula is true for semi-positive-definite matrix $X = \begin{bmatrix} X_{11} & * \\ X_{21} & X_{22} \end{bmatrix} \geq 0$.

$$\sum_{l=k-d_2}^{k-1} \zeta_1^T(k)X\zeta_1(k) - \sum_{l=k-d_k}^{k-1} \zeta_1^T(k)X\zeta_1(k) = d_2\zeta_1^T(k)X\zeta_1(k) - \sum_{l=k-d_k}^{k-1} \zeta_1^T(k)X\zeta_1(k) \geq 0 \tag{16}$$

where $\zeta_1(k) = \begin{bmatrix} x^T(k) & x^T(k-d_k) \end{bmatrix}^T$.

Calculating $V(k)$ and adding the left side of (15) and (16), $\Delta V(k)$ can be written as

$$\Delta V(k) \leq \zeta_2^T(k)\left\{\Xi + \Gamma_1^T(P + d_2Z)\Gamma_1\right\}\zeta_2(k) - \sum_{l=k-d_k}^{k-1} \zeta_3^T(k,l)\Psi\zeta_3(k,l) + \gamma^2 w^T(k)w(k) \tag{17}$$

where $\zeta_2(k) = \begin{bmatrix} x^T(k) & x^T(k-d_k) & w^T(k) \end{bmatrix}^T$, $\zeta_3(k,l) = \begin{bmatrix} x^T(k) & x^T(k-d_k) & y^T(l) \end{bmatrix}^T$, $\Xi = \begin{bmatrix} \varphi_{11} & * & * \\ \varphi_{21} & \varphi_{22} & * \\ (PB_1)^T & 0 & -\gamma^2 I \end{bmatrix}$, $\Gamma_1 = \begin{bmatrix} A_k & \bar{A}_1 & R \end{bmatrix}$.

We can prove Theorem 3.1 from two aspects. On the one hand, system (8) is robustly stochastically stable; on the other hand, system (8) has the H_∞ performance γ. Firstly, when $w(k) = 0$ if the matrix inequality (9) holds, the following matrix inequality is true:

$$\begin{bmatrix} \varphi_{11} & * & * & * \\ \varphi_{21} & \varphi_{22} & * & * \\ A_k & \bar{A} & -P^{-1} & * \\ d_2A_k & d_2\bar{A} & 0 & -d_2Z^{-1} \end{bmatrix} < 0 \tag{18}$$

If $\Psi \geq 0$, using Lemma 3.1 and Formula (17), we can obtain that $\Delta V(k) < 0$. So system (8) is asymptotically stable from the Lyapunov-Krasovskii stability theorem.

When $w(k) \neq 0$, the following inequality is true from Formula (17).

$$\begin{aligned} &\Delta V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) \\ &\leq \zeta_2^T(k)\left\{\Xi + \Gamma_1^T(P + d_2R)\Gamma_1 + \Gamma_2^T\Gamma_2\right\}\zeta_2(k) - \sum_{l=k-d_k}^{k-1} \zeta_3^T(k,l)\Psi\zeta_3(k,l) \end{aligned}$$

where $\Gamma_2 = \begin{bmatrix} C & 0 & D \end{bmatrix}$.

If the matrix inequalities (9) and (10) hold and use Lemma 3.1, we can get

$$\Delta V(k) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) < 0 \tag{19}$$

Summing from 0 to ∞ for the above formula of both sides, we can obtain that $\sum_{k=0}^{\infty}\left[z^T(k)z(k) - \gamma^2 w^T(k)w(k)\right] < V(0) - V(\infty)$, and when $V(0) = 0$, we have

$$\sum_{k=0}^{\infty}\left[z^T(k)z(k) - \gamma^2 w^T(k)w(k)\right] < 0 \tag{20}$$

that is, $\|z\|_2 < \gamma\,\|w\|_2$.

It follows from Definition 3.1 that the result of the theorem is true. Therefore, the proof of this theorem is complete.

3.2. **Controller design.** Next, we will show that the above sufficient condition for the existence of robust controller is equivalent to the feasibility of LMIs.

Theorem 3.2. *The networked control system (8) is robustly stochastically stable and has an H_∞ performance γ if there exist symmetric positive-definite matrices $P = P^T > 0$, $\bar{Q} > 0$, $Z > 0$, semi-positive-definite matrix $\bar{X} = \begin{bmatrix} \bar{X}_{11} & * \\ \bar{X}_{21} & \bar{X}_{22} \end{bmatrix} \geq 0$, positive scalar $\varepsilon_i > 0$ $(i = 1, 2, 3, 4)$ and appropriate dimensions matrices $\bar{N}_1$, $\bar{N}_2$, such that the following matrix inequalities hold. A state feedback controller can be constructed via $K = \bar{K}P$.*

$$\Phi_1 = \begin{bmatrix} \Omega_{11} & * \\ \Omega_{21} & \Omega_{22} \end{bmatrix} < 0, \tag{21}$$

$$\Psi_1 = \begin{bmatrix} \bar{X}_{11} & * & * \\ \bar{X}_{21} & \bar{X}_{22} & * \\ \bar{N}_1^T & \bar{N}_2^T & 2P^{-1} - Z^{-1} \end{bmatrix} \geq 0. \tag{22}$$

where

$$\Omega_{11} = \begin{bmatrix} v_{11} & * & * & * & * & * \\ v_{21} & v_{22} & * & * & * & * \\ R^T & 0 & -\gamma^2 I & * & * & * \\ \begin{matrix}\bar{A}_k P^{-1} \\ +\varepsilon_4 H_1\end{matrix} & A_1 P^{-1} & R & \begin{matrix}-P^{-1} + \varepsilon_3 D_1 D_1^T \\ +\varepsilon_4 H_1\end{matrix} & * & * \\ \begin{matrix}d_2 \bar{A}_k P^{-1} \\ +\varepsilon_4 d_2 H_1\end{matrix} & d_2 A_1 P^{-1} & d_2 R & \begin{matrix}\varepsilon_3 d_2 D_1 D_1^T \\ +\varepsilon_4 d_2 H_1\end{matrix} & \begin{matrix}-d_2 Z^{-1} \\ +\varepsilon_3 d_2^2 D_1 D_1^T \\ +\varepsilon_4 d_2^2 H_1\end{matrix} & * \\ CP^{-1} & 0 & D & 0 & 0 & -I \end{bmatrix},$$

$$\Omega_{21} = \begin{bmatrix} E_1 P^{-1} + E_3 M \bar{K} + \varepsilon_4 H_2 & E_2 P^{-1} & 0 & \varepsilon_4 H_2 & \varepsilon_4 d_2 H_2 & 0 \\ E_2 P^{-1} & 0 & 0 & 0 & 0 & 0 \\ E_1 P^{-1} + E_3 M \bar{K} + \varepsilon_4 H_2 & 0 & 0 & \varepsilon_4 H_2 & \varepsilon_4 d_2 H_2 & 0 \\ E_4 P^{-1} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{22} = \begin{bmatrix} -\varepsilon_3 I + \varepsilon_4 H_3 & * & * & * \\ 0 & -\varepsilon_2 I & * & * \\ \varepsilon_4 H_3 & 0 & -\varepsilon_1 I + \varepsilon_4 H_3 & * \\ 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix},$$

$$\begin{aligned} v_{11} = {} & \bar{A}_k P^{-1} + P^{-1} \bar{A}_k^T + P^{-1} N_1 P^{-1} + P^{-1} N_1^T P^{-1} + (\tau + 1) P^{-1} Q P^{-1} \\ & + d_2 P^{-1} X_{11} P^{-1} + (\varepsilon_1 + \varepsilon_2) D_1 D_1^T + \varepsilon_4 \beta_i H_1, \end{aligned}$$

$\beta_i = \alpha_i + \delta_i^2$, $\quad v_{21} = P^{-1} A_1^T - P^{-1} N_1^T P^{-1} + P^{-1} N_2 P^{-1} + d_2 P^{-1} X_{21} P^{-1}$,
$v_{22} = -P^{-1} N_2 P^{-1} - P^{-1} N_2^T P^{-1} - P^{-1} Q P^{-1} + d_2 P^{-1} X_{22} P^{-1}$, $\quad \bar{N}_1 = P^{-1} N_1 P^{-1}$,
$\bar{N}_2 = P^{-1} N_2 P^{-1}$, $\quad \bar{X}_{11} = P^{-1} X_{11} P^{-1}$, $\quad \bar{X}_{21} = P^{-1} X_{21} P^{-1}$, $\quad \bar{X}_{22} = P^{-1} X_{22} P^{-1}$,

$$\bar{Q} = P^{-1}QP^{-1}, \quad \bar{K} = KP^{-1}, \quad \bar{A}_k = A_0 - I + B_0MK + F_1,$$

$$H_1 = \sum_{i=1}^{m} (\beta_i B_0 \theta_i D_1 (B_0 \theta_i D_1)^T), \quad H_2 = \sum_{i=1}^{m} (\beta_i E_3 \theta_i D_1 (B_0 \theta_i D_1)^T),$$

$$H_3 = \sum_{i=1}^{m} (\beta_i E_3 \theta_i D_1 (E_3 \theta_i D_1)^T).$$

Proof: These uncertainties such as ΔA_0, ΔA_1 and ΔB_0 in Formula (9), can be written as (23) using Lemma 3.2 and Schur Lemma.

$$\Gamma_2 = \begin{bmatrix} \begin{array}{c}\varepsilon_1 PD_1(PD_1)^T \\ +\varepsilon_2 PD_1(PD_1)^T\end{array} & * & * & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_3 D_1 D_1^T & * & * & * & * & * \\ 0 & 0 & 0 & \varepsilon_3 d_2 D_1 D_1^T & \varepsilon_3 d_2^2 D_1 D_1^T & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * \\ \left(E_1 + E_3 M \hat{K}\right) & E_2 & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * & * \\ 0 & E_2 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 I & * \\ \left(E_1 + E_3 M \hat{K}\right) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 I \end{bmatrix} \tag{23}$$

Note that the form of ΔK is the same as ΔA, so using Lemma 3.2 and Schur lemma, we can get

$$\Pi_0 = \begin{bmatrix} \varepsilon_4 PC_1P & * & * & * & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * & * & * & * \\ \varepsilon_4 C_1 P & 0 & 0 & \varepsilon_4 C_1 & * & * & * & * & * & * \\ \varepsilon_4 d_2 C_1 P & 0 & 0 & \varepsilon_4 d_2 C_1 & \varepsilon_4 d_2^2 C_1 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\ \varepsilon_4 C_2 P & 0 & 0 & \varepsilon_4 C_2 & \varepsilon_4 d_2 C_2 & 0 & \varepsilon_4 C_3 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\ \varepsilon_4 C_2 P & 0 & 0 & \varepsilon_4 C_2 & \varepsilon_4 d_2 C_2 & 0 & \varepsilon_4 C_3 & 0 & \varepsilon_4 C_3 & * \\ E_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} \tag{24}$$

where $C_1 = B_0MD_1(B_0MD_1)^T$, $C_2 = E_3MD_1(B_0MD_1)^T$, $C_3 = E_3MD_1(E_3MD_1)^T$. Considering the fault's specific form such as (6), we can get

$$\begin{aligned} E(C_1) &= E\left\{B_0MD_1(B_0MD_1)^T\right\} = E\left\{B_0(M - \bar{M} + \bar{M})D_1(B_0(M - \bar{M} + \bar{M})D_1)^T\right\} \\ &= \sum_{i=1}^{m} \left(\delta_i^2 B_0 \Theta_i D_1 \left(B_0 \Theta_i D_1\right)^T + \alpha_i B_0 \Theta_i D_1 \left(B_0 \Theta_i D_1\right)^T\right) \\ &= \sum_{i=1}^{m} \left(\beta_i B_0 \Theta_i D_1 \left(B_0 \Theta_i D_1\right)^T\right), \end{aligned}$$

$$E(C_2) = \sum_{i=1}^{m} \left(\beta_i E_3 \Theta_i D_1 \left(B_0 \Theta_i D_1 \right)^T \right),$$
$$E(C_3) = \sum_{i=1}^{m} \left(\beta_i E_3 \Theta_i D_1 \left(E_3 \Theta_i D_1 \right)^T \right).$$

Then, using Lemma 3.2, we can obtain that

$$\Pi_1 = \begin{bmatrix} \varepsilon_4 P H_1 P & * & * & * & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * & * & * & * \\ \varepsilon_4 H_1 P & 0 & 0 & \varepsilon_4 H_1 & * & * & * & * & * & * \\ \varepsilon_4 d_2 H_1 P & 0 & 0 & \varepsilon_4 d_2 H_1 & \varepsilon_4 d_2^2 H_1 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\ \varepsilon_4 H_2 P & 0 & 0 & \varepsilon_4 H_2 & \varepsilon_4 d_2 H_2 & 0 & \varepsilon_4 H_3 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\ \varepsilon_4 H_2 P & 0 & 0 & \varepsilon_4 H_2 & \varepsilon_4 d_2 H_2 & 0 & \varepsilon_4 H_3 & 0 & \varepsilon_4 H_3 & * \\ E_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} \quad (25)$$

From (9), (24) and (25), we can get

$$\Phi_2 = \begin{bmatrix} \Upsilon_{11} & * \\ \Upsilon_{21} & \Upsilon_{22} \end{bmatrix}, \quad (26)$$

where

$$\Upsilon_{11} = \begin{bmatrix} w_{11} & * & * & * & * & * \\ w_{21} & w_{22} & * & * & * & * \\ R^T P & 0 & -\gamma^2 I & * & * & * \\ \bar{A}_k + \varepsilon_4 H_1 P & A_1 & R & \begin{matrix} -P^{-1} + \varepsilon_3 D_1 D_1^T \\ +\varepsilon_4 H_1 \end{matrix} & * & * \\ d_2 \left(\bar{A}_k + \varepsilon_4 d_2 H_1 P \right) & d_2 A_1 & d_2 R & \begin{matrix} \varepsilon_3 d_2 D_1 D_1^T \\ +\varepsilon_4 d_2 H_1 \end{matrix} & \begin{matrix} -d_2 Z^{-1} \\ +\varepsilon_3 d_2^2 D_1 D_1^T \\ +\varepsilon_4 d_2^2 H_1 \end{matrix} & * \\ C & 0 & D & 0 & 0 & -I \end{bmatrix},$$

$$\Upsilon_{21} = \begin{bmatrix} E_1 + E_3 M_0 K + \varepsilon_4 H_2 P & E_2 & 0 & \varepsilon_4 H_2 & \varepsilon_4 d_2 H_2 & 0 \\ E_2 & 0 & 0 & 0 & 0 & 0 \\ E_1 + E_3 M_0 K + \varepsilon_4 H_2 P & 0 & 0 & \varepsilon_4 H_2 & \varepsilon_4 d_2 H_2 & 0 \\ E_4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Upsilon_{22} = \Omega_{22},$$

$$w_{11} = P\bar{A}_k + \bar{A}_k^T P + N_1 + N_1^T + (\tau + 1)Q + d_2 X_{11} + (\varepsilon_1 + \varepsilon_2) P D_1 D_1^T P + \varepsilon_4 P H_1 P,$$
$$w_{21} = A_1^T P - N_1^T + N_2 + d_2 X_{21}, \quad w_{22} = -N_2 - N_2^T - Q + d_2 X_{22}.$$

Pre- and post-multiplying (26) by the matrix $diag\{P^{-1}, P^{-1}, I, I, I, I, I, I, I, I\}$, and pre- and post-multiplying (10) by the matrix $diag\{P^{-1}, P^{-1}, P^{-1}\}$, then we can get (21) and (27)

$$\Psi_2 = \begin{bmatrix} \bar{X}_{11} & * & * \\ \bar{X}_{21} & \bar{X}_{22} & * \\ \bar{N}_1^T & \bar{N}_2^T & P^{-1}ZP^{-1} \end{bmatrix} \geq 0. \tag{27}$$

Obviously, since there has nonlinear term $P^{-1}ZP^{-1}$ in (30), it is not strict linear matrix inequalities.

Note that Z and P are positive matrix, and then we can obtain $(Z^{-1} - P^{-1})V(Z^{-1} - P^{-1}) \geq 0$, which implies

$$P^{-1}ZP^{-1} \geq 2P^{-1} - Z^{-1}. \tag{28}$$

From (27) and (28), we can get (22). If (21) and (22) are true and from Theorem 3.1, system (8) is robustly stochastically stable and has an H_∞ performance γ. The proof is complete.

3.3. **The optimization of performance.** Let $e = \gamma^2$, and we can obtain the minimum disturbance rejection rate $\gamma_{\min} = \sqrt{e}$ of system (8), which satisfies non-fragile fault-tolerant H_∞ control if the following optimization problem holds.

$$\begin{aligned} &\min e \\ &\text{s.t. } (21),\ (22),\ P > 0,\ \bar{Q} > 0,\ Z > 0,\ \bar{X} \geq 0,\ \varepsilon_i > 0\ (i = 1, 2, 3, 4) \end{aligned} \tag{29}$$

4. **Numerical Example.** In this section, a numerical example is provided to illustrate the effectiveness of the proposed design method. Consider the discrete-time networked control system (8), in which the system parameters are given as follows:

$$A_0 = \begin{bmatrix} 0 & 0.1 \\ -0.14 & 0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 0.01 & 0.02 \\ 0.02 & 0.02 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.2 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.2 \end{bmatrix},$$

$$E_4 = \begin{bmatrix} 0.01 & 0.02 \\ 0.01 & 0.02 \end{bmatrix}, \quad R = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad D = 0.6, \quad d_1 = 1, \quad d_2 = 4.$$

The nonlinearities F_1 and $F(k)$ are chosen as follows:

$$F_1 = \begin{bmatrix} 0.1\sin(0.01k) \\ 0.1\cos(0.01k) \end{bmatrix} \Big/ k, \quad F(k) = \begin{bmatrix} \sin(0.01k) & 0 \\ 0 & \cos(0.01k) \end{bmatrix}.$$

The exogenous disturbance $w(k) = \begin{cases} 0.3 & 60 \leq k \leq 65 \\ 0 & \text{otherwise} \end{cases}$.

The actuators fault matrices are given in Tables 1 and 2. By solving the linear matrix inequalities (21) and (22) in Theorem 3.2, the state feedback controller K and H_∞ performance γ can be obtained. The results obtained can be optimized and we can get K^*, γ^* which are listed in Tables 1 and 2.

TABLE 1. K^* and γ^* for $\bar{M} = diag\{0.9, 1.1\}$ with different variance

Expectation	$\bar{M} = diag\{0.9, 1.1\}$	$\bar{M} = diag\{0.9, 1.1\}$	$\bar{M} = diag\{0.9, 1.1\}$
Variance	$\delta_i^2 = 0$	$\delta_i^2 = 0.09$	$\delta_i^2 = 0.36$
K^*	$\begin{bmatrix} 0.304 & -0.138 \\ 0.117 & -0.481 \end{bmatrix}$	$\begin{bmatrix} 0.304 & -0.138 \\ 0.117 & -0.481 \end{bmatrix}$	$\begin{bmatrix} 0.304 & -0.138 \\ 0.117 & -0.481 \end{bmatrix}$
γ^*	1.6205	1.6220	1.6263

TABLE 2. $K^*$and γ^* for $\bar{M} = diag\{1, 1\}$ with different variance

Expectation	$\bar{M} = diag\{1, 1\}$	$\bar{M} = diag\{1, 1\}$	$\bar{M} = diag\{1, 1\}$
Variance	$\delta_i^2 = 0$	$\delta_i^2 = 0.09$	$\delta_i^2 = 0.36$
K^*	$\begin{bmatrix} 0.274 & -0.125 \\ 0.129 & -0.529 \end{bmatrix}$	$\begin{bmatrix} 0.274 & -0.124 \\ 0.129 & -0.529 \end{bmatrix}$	$\begin{bmatrix} 0.274 & -0.124 \\ 0.129 & -0.589 \end{bmatrix}$
γ^*	1.6214	1.6228	1.6272

From Tables 1 and 2, we can see that γ^* in Table 1 is smaller than that in Table 2 when the variance is the same. In other words, the capability of disturbance rejection with the actuators having partial failures is better than with the actuators being normal in the effect of H_∞ fault-tolerant controller. Therefore, it can be seen that the proposed method provides the desired H_∞ control.

Assume that the initial states of the system are $x(0) = \begin{bmatrix} 1 & -0.5 \end{bmatrix}^T$, and the state trajectories of the networked control systems stabilized by the above controllers in Table 1 are shown in Figures 1-3.

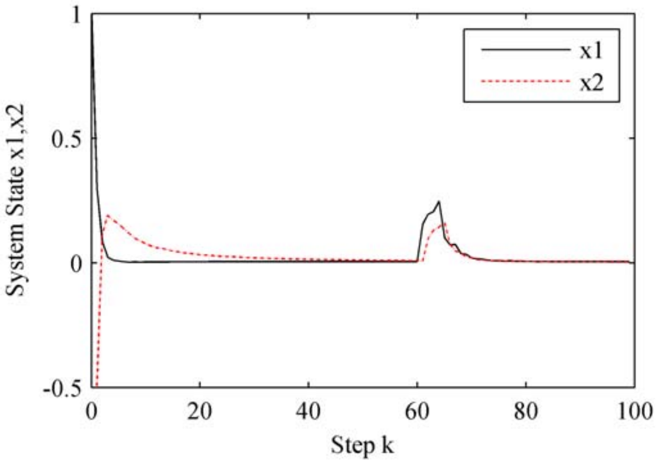

FIGURE 1. The system state when $\bar{M} = diag\{0.9, 1.1\}$ and $\delta_i^2 = 0$

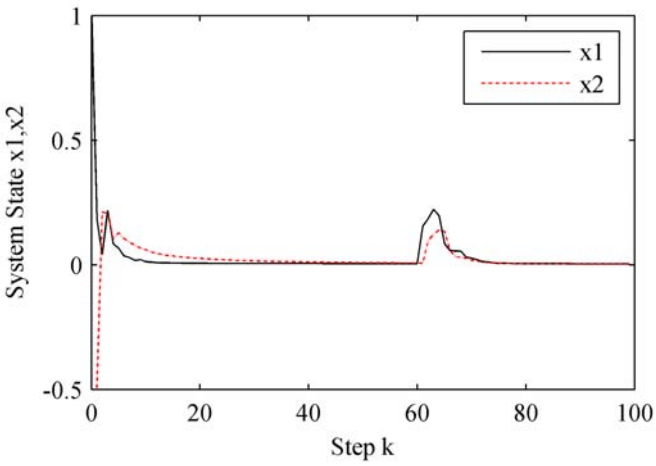

FIGURE 2. The system state when $\bar{M} = diag\{0.9, 1.1\}$ and $\delta_i^2 = 0.09$

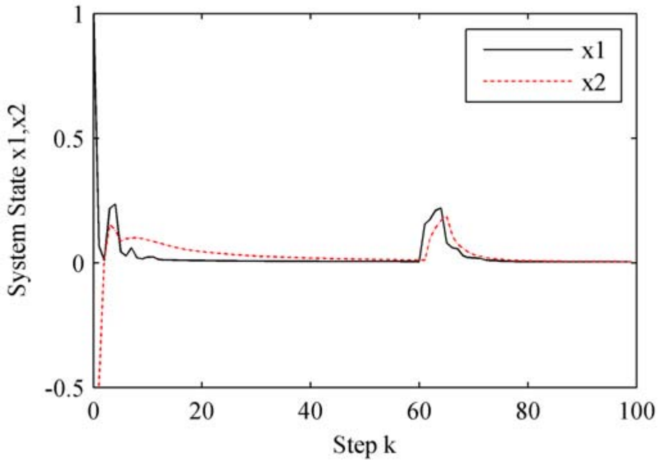

FIGURE 3. The system state when $\bar{M} = diag\{0.9, 1.1\}$ and $\delta_i^2 = 0.36$

It can be seen that the system with random actuator faults is stable in the end even having exogenous disturbance under the effects of the designed controller, which demonstrates the usefulness of the method presented in this article. Besides, when the actuator faults' expectation is the same, the bigger the variance is, the faster the system becomes stable from Figures 1-3. Therefore, the proposed method provides the desired fault-tolerant control.

5. **Conclusions.** In this paper, the effects of time-delay, the exogenous disturbance, norm-bounded parameter uncertainties, stochastic nonlinearities and random actuator faults are considered. By constructing an appropriate Lyapunov-Krasovskii function, the sufficient conditions for the existence of the non-fragile robust fault-tolerant H_∞ controller are obtained in terms of LMIs. The numerical example shows the feasibility of the proposed method. Further, the output feedback guaranteed cost control and model predictive control strategies for NCSs will be our future topics of research.

Acknowledgments. This work was supported by the National Natural Science Foundation of China (61403168) and the Research Innovation Program for College Graduates of Jiangsu Province (KYLX15_1194).

REFERENCES

[1] Q. Shen, B. Jiang and V. Cocquempot, Adaptive fault-tolerant back stepping control against actuator gain faults and its applications to an aircraft longitudinal motion dynamics, *International Journal of Robust and Nonlinear Control*, vol.23, no.15, pp.1753-1779, 2013.
[2] H. J. Fang, H. Ye and M. Y. Zhong, Fault diagnosis of networked control systems, *Annual Reviews in Control*, vol.31, no.1, pp.55-68, 2007.
[3] Z. Huo, Z. Yuan and X. Chang, A simplified state switching control strategy for survivable variable-speed induction generators, *Electric Machines and Drives Conference*, vol.34, no.2, pp.182-187, 2011.
[4] L. B. Wu, G. H. Yang and D. Ye, Robust adaptive fault-tolerant control for linear systems with actuator failures and mismatched parameter uncertainties, *IET Control Theory and Applications*, vol.8, no.6, pp.441-449, 2014.
[5] W. Wu and H. Fang, Integrated fault-tolerant control methodology for non-linear dynamical networked control systems subject to time-varying delay, *IET Control Theory and Applications*, vol.8, no.8, pp.535-545, 2014.
[6] D. Zhang, H. Su, S. Pan et al., LMI approach to reliable guaranteed cost control with multiple criteria constraints: The actuator faults case, *International Journal of Robust and Nonlinear Control*, vol.19, no.8, pp.884-899, 2009.

[7] F. Yang, H. Zhang, B. Jiang et al., Adaptive reconfigurable control of systems with time-varying delay against unknown actuator faults, *International Journal of Adaptive Control and Signal Processing*, vol.28, no.11, pp.1206-1226, 2014.

[8] H. Wang, B. Zhou, C. C. Lim et al., H_∞ fault-tolerant control of networked control systems with actuator failures, *IET Control Theory and Applications*, vol.8, no.12, pp.1127-1136, 2014.

[9] P. Pan, S. Jiang and F. Pan, Event-triggered robust control for networked control systems with actuator failures and time-varying transmission delays, *International Journal of Intelligent Computing and Cybernetics*, vol.8, no.2, pp.172-186, 2015.

[10] G. Wei, Z. Wang and H. Shu, Robust filtering with stochastic nonlinearities and multiple missing measurements, *Automatica*, vol.45, no.3, pp.836-841, 2009.

[11] E. Tian, C. Peng and Z. Gu, Fault tolerant control for discrete networked control systems with random faults, *International Journal of Control Automation and Systems*, vol.10, no.2, pp.444-448, 2012.

[12] Q. Zhu and J. Cao, Exponential stability of stochastic neural networks with both markovian jump parameters and mixed time delays, *IEEE Trans. Systems Man and Cybernetics – Part B, Cybernetics: A Publication of the IEEE Systems, Man, and Cybernetics Society*, vol.41, no.2, pp.341-353, 2011.

[13] H. Li, W. K. Wong and Y. Tang, Global synchronization stability for stochastic complex dynamical networks with probabilistic interval time-varying delays, *Journal of Physics A Mathematical and Theoretical*, vol.152, no.2, pp.496-516, 2012.

[14] A. Ray, Output feedback control under randomly varying distributed delays, *Journal of Guidance Control and Dynamics*, vol.17, no.4, pp.71-83, 1994.

[15] Y. Dong, T. Engang, Z. Yijun et al., Delay-distribution-dependent stability and stabilization of T-S fuzzy systems with probabilistic interval delay, *IEEE Trans. Systems Man and Cybernetics – Part B: Cybernetics*, vol.39, no.2, pp.503-516, 2009.

[16] Y. Wei, W. X. Zheng and S. Xu, Robust output feedback control of uncertain time-delay systems with actuator saturation and disturbances, *Journal of the Franklin Institute*, vol.352, no.5, pp.2229-2248, 2015.

[17] P. Vadivel, R. Sakthivel, K. Mathiyalagan et al., Robust stabilisation of non-linear uncertain Takagi-Sugeno fuzzy systems by H_∞ control, *IET Control Theory and Applications*, vol.6, no.16, pp.2556-2566, 2012.

[18] F. Lei and C. M. Yue, Passive control for singular time-delay system with actuator saturation, *Applied Mathematics and Computation*, vol.289, pp.181-193, 2016.

[19] Q. M. Guo, H. L. Xing, L. Q. Lin et al., Finite-time event-triggered H_∞ control for switched systems with time-varying delay, *Neurocomputing*, vol.207, pp.828-842, 2016.

[20] M. S. Alwan, X. Liu and W. C. Xie, On design of robust reliable H_∞ control and input-to-state stabilization of uncertain stochastic systems with state delay, *Communications in Nonlinear Science and Numerical Simulation*, vol.18, no.4, pp.1047-1056, 2013.

[21] X. He, Z. Wang and D. Zhou, Robust filtering for time-delay systems with probabilistic sensor faults, *IEEE Signal Processing Letters*, vol.16, no.5, pp.442-445, 2009.

[22] G. Ming, S. Li and R. L. Yu, Robust H_∞ control for T-S fuzz systems subject to missing measurements with uncertain missing probabilities, *Neurocomputing*, vol.193, pp.235-241, 2016.

[23] H. L. Shao and M. Z. Shou, H_∞ control for a class of discrete-time singular systems via dynamic feedback controller, *Applied Mathematics Letters*, vol.58, pp.110-118, 2016.