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UNCERTAIN CHAOTIC GYROS SYNCHRONIZATION USING ADAPTIVE FUZZY PRESCRIBED PERFORMANCE CONTROL WITH UNKNOWN DEAD-ZONE INPUT

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Abstract. *This paper proposes an adaptive fuzzy prescribed performance control scheme for the synchronization of uncertain chaotic gyros with unknown dead-zone input. In order to eliminate the effects from external disturbance and dead-zone nonlinearity, we use error transformation to transform the original constrained system into an equivalent unconstrained one. Then, an adaptive fuzzy controller is designed for the equivalent unconstrained system, which can guarantee that the error remains an adjustable neighborhood of the origin with the prescribed performance bounds. In addition, the performance accuracy can be adjusted by an appropriate choice of the design parameters of the controller. Simulation results are provided to illustrate the effectiveness of the proposed method.*
Keywords: Gyro, Unknown dead-zone input, Predefined performance, Adaptive fuzzy control

1. Introduction.

The gyro system as an attractive nonlinear system, receiving attention by researchers in recent years. The main reason is that Gyros are applied in the navigational, aeronautical and space engineering domains. Recent research has confirmed that various forms of gyro systems maybe exhibit a diverse range of dynamic behavior, including both subharmonic and chaotic motions [1-3].

Recently, many researchers investigated the problem of the synchronization for two chaotic gyros, because this synchronization is used in areas of secure communications [4] and attitude control of long-duration spacecrafts [5].

Based on Lyapunov stability theory and Routh-Hurwitz criteria, Lei et al. [6] used the active control scheme to synchronize two nonlinear gyros. Hsu et al. [7] proposed a self-learning PID control system to dispel the approximation error between the ideal controller and PID controller and achieve the system stability in the Lyapunov sense. In practice, due to non-ideal characteristics of actuators used in physical implementations, the implementation of control input is usually faced with the problem of nonlinearity in control input. It has been shown that input nonlinearity, including saturation, backlash and dead-zone, can cause a serious degradation of the system performance if the controller is not well designed [8]. Therefore, it is clear that the effects of input nonlinearity must be taken into account when analyzing and implementing a synchronization control scheme. Moreover, the control gain of the controller may be unknown. For example, Yau [9] proposed a fuzzy sliding mode control (FSMC) scheme for the synchronization of two chaotic nonlinear gyros subject to uncertainties and external disturbances.

Dead-zone with unknown parameters in physical components may severely limit the performance of control, and its characteristics are quite commonly encountered in actuators in practical control systems. Roopaei et al. [10] proposed the adaptive fuzzy sliding mode control method to design a controller for the synchronization of chaotic gyros when uncertainties, disturbances and dead-zone nonlinearity input were presented. Based on the sliding mode control technique, Hung et al. [11] presented the control law such as two gyros chaotic systems with dead-zone nonlinearity achieving projective synchronization. However, the disadvantage of sliding mode controller is that the chatter will appear. In order to overcome the shortcoming, we propose an adaptive prescribed performance control scheme to achieve the synchronization of two uncertain gyros systems with unknown dead-zone input.

To the author's best knowledge, there is few literature to research the synchronization problem for uncertain gyros system with unknown control gain and unknown dead-zone input.

To handle unknown nonlinear functions in gyro system, fuzzy logic systems will be used in this paper. For adaptive fuzzy approaches, two of the main features are (i) they can be used to deal with those nonlinear systems without satisfying the matching conditions and (ii) they do not require the unknown nonlinear functions being linearly parameterized.

Compared with related works, there are three main contributions that are worth emphasizing:

(1) Compared with the results in [10,11], the uncertain gyros with unknown control gain is considered.

(2) The prescribed performance function (PPF) is incorporated into the control design.

(3) The controller will not show singular problem and chatter phenomenon.

Inspired by [12-16], an improved prescribed performance function (PPF) is incorporated into the control design. An error transformed system is derived by applying the PPF on the original error system. Consequently, the error rates of the original error system can be guaranteed within the prescribed bound provided the transformed system is stable. For this purpose, an adaptive prescribed performance control (APPC) is designed for uncertain gyros in the presence of system uncertainties and external disturbance. A comparative example is given to emphasize the effectiveness of the proposed APPC scheme.

The organization of this paper is described as follows. In Section 2, system model is derived, and the assumptions are also given. In Section 3, a robust fuzzy adaptive feedback control approach is developed, and the stability of the closed-loop system is proved. The simulation results are presented to demonstrate the effectiveness of the proposed control scheme in Section 4. Conclusions are presented in Section 5.

2. System Descriptions and Problem Formulations.

The symmetric gyroscope mounted on a vibrating base is shown in Figure 1. The dynamics of a symmetrical gyro with linear-plus-cubic damping of the angle θ can be expressed as

$$\ddot{\theta} + \alpha^2 \frac{(1-\cos\theta)^2}{\sin^3\theta} - \beta\sin\theta + c_1\dot{\theta} + c_2\dot{\theta}^3 = f\sin\omega t\sin\theta, \tag{1}$$

where $f\sin\omega t$ represents a parametric excitation, $c_1\dot{\theta}$ and $c_2\dot{\theta}^3$ are linear and nonlinear damping terms, respectively, and $\alpha^2\left((1-\cos\theta)^2/\sin^3\theta\right) - \beta\sin\theta$ is a nonlinear resilience force. This gyro system exhibits complex irregular motion when $f = 35.7$, $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$ and states initial values of $\left[\theta(0), \dot{\theta}(0)\right]^T = [-1, 1]^T$, see Figure 2.

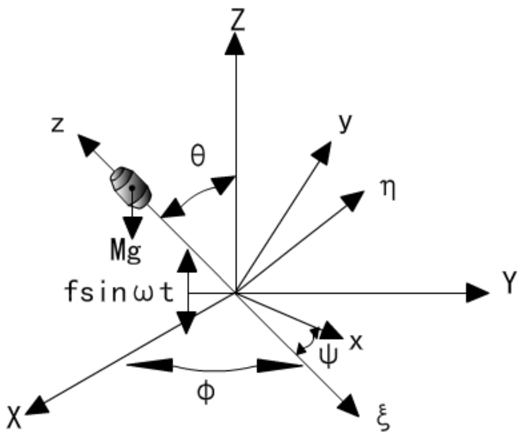

FIGURE 1. A schematic diagram of a symmetric gyroscope

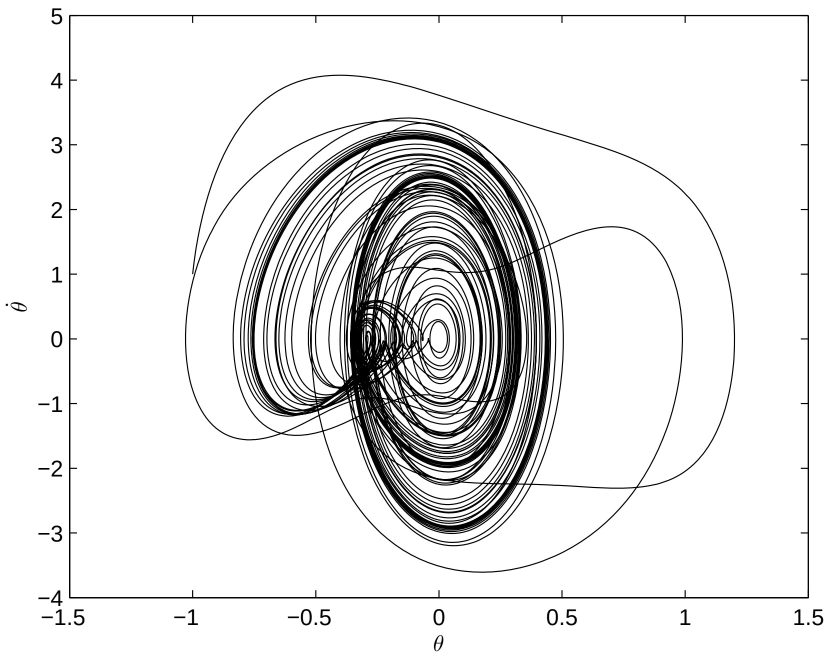

FIGURE 2. Phase plane trajectory of chaotic gyro

For simplicity, we introduce the following notations: $x = [x_1, x_2]^T = \left[\theta, \dot{\theta}\right]^T$, and then the dynamic model of (1) can be described by the following equations

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = h(x_1) - c_1 x_2 - c_2 x_2^2 + (\beta + f\sin(\omega t))\sin(x_1) \end{cases} \tag{2}$$

where $h(x_1) = -\alpha^2\left((1-\cos(x_1))^1/\sin^3(x_1)\right) - \beta\sin(x_1)$.

Let $f_1(t,x) = h(x_1) - c_1 x_2 - c_2 x_2^2 + (\beta + f\sin(\omega t))\sin(x_1)$, and consider two coupled, chaotic gyro systems of the form

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f_1(t,x), \end{cases} \tag{3}$$

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = f_1(t, y) + d(t, y) + g(t, y)\Gamma(u(t)), \end{cases} \tag{4}$$

where $d(t, y)$ is external disturbance, $g(t, y)$ is unknown control gain, and $\Gamma(u(t))$ is the dead-zone nonlinearity in the control input and described as follows:

$$\Gamma(u(t)) = \begin{cases} m(u(t) - b_1), & \text{for } u(t) \geq b_1, \\ 0, & \text{for } b_2 < u(t) < b_1, \\ m(u(t) - b_2), & \text{for } u(t) \leq b_2, \end{cases} \tag{5}$$

where m stands for the right and the left slope, and b_1 and b_2 represent the breakpoints of the input nonlinearity.

Assumption 2.1. *The dead-zone parameters: $m > 0$, b_1 and b_2 are all unknown bounded constants.*

Obviously, $\Gamma(u(t))$ can be rewritten as

$$\Gamma(u(t)) = mu(t) + \rho(u(t)),$$

where $\rho(u(t))$ can be calculated as

$$\rho(u(t)) = \begin{cases} -mb_1, & \text{for } u(t) \geq b_1 \\ -mu(t), & \text{for } b_2 < u(t) < b_1 \\ -mb_2, & \text{for } u(t) \leq b_2 \end{cases} \tag{6}$$

From Assumption 2.1, there exists an unknown positive constant d^* such as $|\rho(u(t))| \leq d^*$.

Let $e = [e_1 \ e_2]^T = [y_1 - x_1 \ y_2 - x_2]^T$, and the error dynamic equation can be written as:

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = f_2(t, y) - f_1(t, x) + g(t, y)mu(t) + g(t, y)\rho(u(t)), \end{cases} \tag{7}$$

where $f_2(t, y) = f_1(t, y) + d(t, y)$.

The objective of this paper is to construct a controller for system (7) such that the system states y_1 and x_1 can be synchronized and all the signals in the closed-loop system remain bounded.

To meet the objective, the following assumptions are made for system (7).

Assumption 2.2. *The state vectors x and y are measurable.*

Assumption 2.3. *$f_1(t, x)$, $f_2(t, y)$ and $g(t, y)$ are unknown but bounded.*

Assumption 2.4. *The control gain $g(t, y) \neq 0$ for all t and y.*

Remark 2.1. *It is known from the property of the dead-zone that the range of the dead-zone is relatively small. Therefore, Assumption 2.1 will always hold. Assumptions 2.2 and 2.3 are fairly common for the synchronization problem, see [9,10].*

Remark 2.2. *In [10], the unknown control gain $g(t, y)$ is assumed as $g(t, y) = 1$. And the method is sliding mode control scheme, so the chattering phenomena cannot be eliminated. In this paper, we employ prescribed performance control which can overcome these problems.*

Prescribed performance.

Definition 2.1. *A smooth function $\mu(t) : R^+ \to R^+$ is called a prescribed performance function (PPF) if $\mu(t)$ is decreasing and $\lim_{t\to\infty} \mu(t) = \mu_\infty$.*

In this paper, we select $\mu(t)$ as

$$\mu(t) = (\mu_0 - \mu_\infty)e^{-\kappa t} + \mu_\infty, \tag{8}$$

where $\mu_0 > \mu_\infty$ and $\kappa > 0$ are design parameters.

It is sufficient to achieve the control objective if condition (9) holds

$$-\delta_{\min}\mu(t) < e_1(t) < \delta_{\max}\mu(t), \quad \forall t \geq 0, \tag{9}$$

where $\delta_{\min}$ and $\delta_{\max}$ are design constants.

Remark 2.3. *The transient and steady-state performances can be designed a priori by tuning the parameters $\delta_{\min}$, $\delta_{\max}$, μ_0, κ, μ_∞.*

To represent (9) by an equality form, we employ an error transformation as

$$e_1(t) = \mu(t)s(z), \tag{10}$$

where z is the transformed error, and $s(\cdot)$ is smooth, strictly increasing function, and satisfies the following condition

$$\begin{cases} -\delta_{\min} < s(z) < \delta_{\max}, \quad \forall z \in L_\infty, \\ \lim_{z\to-\infty} s(z) = -\delta_{\min}, \\ \lim_{z\to+\infty} s(z) = \delta_{\max}. \end{cases} \tag{11}$$

Note that $s(z)$ are strictly increasing functions, and we have

$$z = s^{-1}\left(\frac{e_1(t)}{\mu(t)}\right). \tag{12}$$

Note that for any initial condition $e(0)$, if parameters μ_0, $\delta_{\min}$ and $\delta_{\max}$ are selected such that $-\delta_{\min}\mu(0) < e_1(0) < \delta_{\max}\mu(0)$ and z can be controlled to be bounded, then $-\delta_{\min} < s(z) < \delta_{\max}$ holds. Thus, the condition $-\delta_{\min}\mu(t) < e_1(t) < \delta_{\max}\mu(t)$ can be guaranteed. Now, the synchronization problem of system (7) is now transformed to stabilize the transformed system (12).

Differentiating (12) with respect to time t yields

$$\dot{z} = \frac{\partial s^{-1}}{\partial\left(\frac{e_1(t)}{\mu(t)}\right)} \frac{1}{\mu(t)} \left[e_2 - \frac{e_1(t)\dot{\mu}(t)}{\mu(t)}\right]. \tag{13}$$

Let $0 < r = \frac{\partial s^{-1}}{\partial\left(\frac{e_1(t)}{\mu(t)}\right)} \frac{1}{\mu(t)} < r_m$, and r_m is a positive constant. Then (13) can be rewritten as

$$\dot{z} = r\left[e_2 - \frac{e_1\dot{\mu}}{\mu}\right]. \tag{14}$$

Moreover, we obtain

$$\ddot{z} = rF_1 + rf_2(t, y) - rf_1(t, x) + rg(t, y)mu + rg(t, y)\rho(u), \tag{15}$$

where $F_1 = \frac{\dot{r}}{r}(e_2 - e_1\dot{\mu}/\mu) - (e_2\dot{\mu}/\mu + e_1\ddot{\mu}\mu/\mu^2 - e_1\dot{\mu}^2/\mu^2)$ is known nonlinear function, $f_1(t, x)$, $f_2(t, y)$, $g(t, y)$ are unknown nonlinear functions and m is an unknown positive constant.

Remark 2.4. *In general, $s(z)$ is chosen as $s(z) = \frac{\delta_{\max}e^z - \delta_{\min}e^{-z}}{e^z + e^{-z}}$. So, we can calculate that $r = \frac{1/(\lambda+\delta_{\min}) - 1/(\lambda-\delta_{\max})}{2\mu}$ such that $0 < r < \frac{\delta_{\max}+\delta_{\min}}{\mu_\infty\delta_{\max}\delta_{\min}}$, where $\lambda = \frac{e_1}{\mu}$.*

3. **Main Results.** Define the filtered error as

$$\nu = \iota z + \dot{z},$$

where $\iota > 0$ is a positive constant. The transformed error z is bounded as long as ν is bounded.

Let $g_1(t, y) = g(t, y)m$, if $f_1(t, x)$, $f_2(t, y)$, $g(t, y)$, m and d^* are known, we can consider the following control law

$$u = -\frac{\iota\left(e_2 - \frac{e_1\dot{\mu}}{\mu}\right) + F_1 + f_2(t, y) - f_1(t, x) + k_0\nu + u_s \mathrm{sign}(\nu)}{g_1(t, y)}, \tag{16}$$

where $u_s = |g(t, y)|d^*$ and k_0 is a designed positive constant. Consider the Lyapunov function $V_0 = \frac{1}{2}\nu^2$, and substituting (16) into $\dot{\nu}$, we have $\dot{V}_0 = \nu\dot{\nu} = -rk_0\nu^2 - r(|\nu|u_s - \nu g(t, y)\rho(u)) \le -rk_0\nu^2 \le 0$. So, ν is bounded. And then the objective can achieve.

Due to the fact that $f_1(t, x)$, $f_2(t, y)$, $g(t, y)$, m and d^* are unknown, we need to use fuzzy logic system to approximate the nonlinear unknown functions. We can use the following fuzzy systems to approximate $f_1(t, x)$, $f_2(t, y)$, $g_1(t, y)$ and u_s:

$$\begin{aligned} \hat{f}_1\left(x, \hat{\theta}_{f_1}\right) &= \hat{\theta}_{f_1}^T \psi_{f_1}(x), \quad \hat{f}_2\left(y, \hat{\theta}_{f_2}\right) = \hat{\theta}_{f_2}^T \psi_{f_2}(y), \\ \hat{g}_1\left(y, \hat{\theta}_{g_1}\right) &= \hat{\theta}_{g_1}^T \psi_{g_1}(y), \quad \hat{u}_s\left(y, \hat{\theta}_{u_s}\right) = \hat{\theta}_{u_s}^T \psi_{u_s}(y), \end{aligned} \tag{17}$$

where $\psi_{f_1}(x)$, $\psi_{f_2}(y)$, $\psi_{g_1}(y)$ and $\psi_{u_s}(y)$ are fuzzy function vectors, and $\hat{\theta}_{f_1}^T$, $\hat{\theta}_{f_2}^T$, $\hat{\theta}_{g_1}^T$ and $\hat{\theta}_{u_s}^T$ are the parameter vectors of each fuzzy system design later.

Now, we can modify the control law (16) as follows:

$$u = \frac{\left[\iota\left(e_2 - \frac{e_1\dot{\mu}}{\mu}\right)\nu + F_1\nu + \hat{f}_2\left(y, \hat{\theta}_{f_2}\right)\nu - \hat{f}_1\left(x, \hat{\theta}_{f_1}\right)\nu + k_0\nu^2 + |\nu|\hat{u}_s\left(y, \hat{\theta}_{u_s}\right) + u_r\right]\nu}{-\nu^2\hat{g}_1\left(y, \hat{\theta}_{g_1}\right) + \xi\nu^2 + u_r^2}, \tag{18}$$

where $\xi = \epsilon + \left|g_1\left(y, \hat{\theta}_{g_1}\right)\right|$, ϵ is a small positive constant, and u_r is an auxiliary controller, which will be designed later.

Remark 3.1. *For control law (18), $-\nu^2\hat{g}_1\left(y, \hat{\theta}_{g_1}\right) + \xi\nu^2 + u_r^2$ is not equal to zero. So, this design can avoid the singular problem.*

Optimal parameters $\hat{\theta}_{f_1}^*$, $\hat{\theta}_{f_2}^*$, $\hat{\theta}_{g_1}^*$ and $\hat{\theta}_{u_s}^*$ can be defined such that

$$\begin{aligned} \hat{\theta}_{f_1}^* &= \arg\min_{\hat{\theta}_{f_1}} \left[\sup\left|f_1(t, x) - \hat{f}_1\left(x, \hat{\theta}_{f_1}\right)\right|\right], \\ \hat{\theta}_{f_2}^* &= \arg\min_{\hat{\theta}_{f_2}} \left[\sup\left|f_2(t, y) - \hat{f}_2\left(y, \hat{\theta}_{f_2}\right)\right|\right], \\ \hat{\theta}_{g_1}^* &= \arg\min_{\hat{\theta}_{g_1}} \left[\sup\left|g_1(t, y) - \hat{g}_1\left(y, \hat{\theta}_{g_1}\right)\right|\right], \\ \hat{\theta}_{u_s}^* &= \arg\min_{\hat{\theta}_{u_s}} \left[\sup\left|u_s(t, y) - \hat{u}_s\left(y, \hat{\theta}_{u_s}\right)\right|\right]. \end{aligned} \tag{19}$$

Define the parameter estimation errors $\tilde{\theta}_{f_1} = \hat{\theta}^*_{f_1} - \hat{\theta}_{f_1}$, $\tilde{\theta}_{f_2} = \hat{\theta}^*_{f_2} - \hat{\theta}_{f_2}$, $\tilde{\theta}_{g_1} = \hat{\theta}^*_{g_1} - \hat{\theta}_{g_1}$, $\tilde{\theta}_{u_s} = \hat{\theta}^*_{u_s} - \hat{\theta}_{u_s}$ and the fuzzy approximation errors as follows:

$$\begin{aligned} &\varepsilon_{f_1} = f_1(t,x) - \hat{f}_1\left(x, \hat{\theta}^*_{f_1}\right), \quad \varepsilon_{f_2} = f_2(t,y) - \hat{f}_2\left(y, \hat{\theta}^*_{f_2}\right), \\ &\varepsilon_{g_1} = g_1(t,y) - \hat{g}_1\left(y, \hat{\theta}^*_{g_1}\right), \quad \varepsilon_{u_s} = u_s(t,y) - \hat{u}_s\left(y, \hat{\theta}^*_{u_s}\right). \end{aligned} \tag{20}$$

Assumption 3.1. *The fuzzy approximation errors ε_{f_1}, ε_{f_2}, ε_{g_1} and ε_{u_s} are bounded.*

Let $u_0 = \frac{\iota\left(e_2 - \frac{e_1\dot{\mu}}{\mu}\right)\nu + F_1\nu + \hat{f}_2\left(y,\hat{\theta}_{f_2}\right)\nu - \hat{f}_1\left(x,\hat{\theta}_{f_1}\right)\nu + k_0\nu^2 + |\nu|\hat{u}_s\left(y,\hat{\theta}_{u_s}\right) + u_r}{-\nu^2\hat{g}_1\left(y,\hat{\theta}_{g_1}\right) + \xi\nu^2 + u_r^2}$, and we have $u = u_0\nu$. Substituting (18) and (20) into $\nu\dot{\nu}$, we can obtain

$$\begin{aligned} \nu\dot{\nu} &= r\nu\Bigg(\iota\left(e_2 - \frac{e_1\dot{\mu}}{\mu}\right) + F_1 + f_2(t,y) - f_1(t,x) + g_1(t,y)u + g(t,y)\rho(u)\Bigg) \\ &= r\Bigg(-\nu\left(f_1(t,x) - \hat{f}_1\left(x,\hat{\theta}_{f_1}\right)\right) + \nu\left(f_2(t,y) - \hat{f}_2\left(y,\hat{\theta}_{f_2}\right)\right) \\ &\quad + \nu^2\left(g_1(t,y) - \hat{g}_1\left(y,\hat{\theta}_{g_1}\right)\right)u_0 - k_0\nu^2 + \xi\nu^2 u_0 + u_r^2 u_0 \\ &\quad - u_r + \nu g(t,y)\rho(u) - |\nu|\hat{u}_s\left(y,\hat{\theta}_{u_s}\right)\Bigg) \\ &= r\Bigg(-k_0\nu^2 - \nu\tilde{\theta}^T_{f_1}\psi_{f_1}(x) + \nu\tilde{\theta}^T_{f_2}\psi(y) + \nu^2\tilde{\theta}^T_{g_1}u_0 - \varepsilon_{f_1}\nu + \varepsilon_{f_2}\nu + \varepsilon_{g_1}\nu^2 u_0 \\ &\quad + \xi\nu^2 u_0 + u_r^2 u_0 - u_r + \nu g(t,y)\rho(u) - |\nu|u_s + |\nu|\left(u_s - \hat{u}_s\left(y,\hat{\theta}_{u_s}\right)\right)\Bigg) \\ &= r\Bigg(-k_0\nu^2 - \nu\tilde{\theta}^T_{f_1}\psi_{f_1}(x) + \nu\tilde{\theta}^T_{f_2}\psi(y) + |\nu|\tilde{\theta}^T_{u_s}\psi_{u_s} \\ &\quad + \nu^2\tilde{\theta}^T_{g_1}\psi_{g_1}(y)u_0 + \nu g(t,y)\rho(u) - |\nu|u_s + u_r^2 u_0 - u_r + \Lambda\Bigg) \end{aligned} \tag{21}$$

where $\Lambda = -\varepsilon_{f_1}\nu + \varepsilon_{f_2}\nu + \varepsilon_{g_1}\nu^2 u_0 + \xi\nu^2 u_0 + |\nu|\varepsilon_{u_s}$.

According to Assumption 3.1, there exist unknown positive constants $\bar{\varepsilon}_{f_1}$, $\bar{\varepsilon}_{f_2}$, $\bar{\varepsilon}_{g_1}$ and $\bar{\varepsilon}_{u_s}$ such that $|\varepsilon_{f_1}| \leq \bar{\varepsilon}_{f_1}$, $|\varepsilon_{f_2}| \leq \bar{\varepsilon}_{f_2}$, $|\varepsilon_{g_1}| \leq \bar{\varepsilon}_{g_1}$ and $|\varepsilon_{u_s}| \leq \bar{\varepsilon}_{u_s}$, respectively. Let $\varepsilon_1 = \bar{\varepsilon}_{f_1} + \bar{\varepsilon}_{f_2} + \bar{\varepsilon}_{u_s}$, $\varepsilon_2 = \bar{\varepsilon}_{g_1}$, and we design u_r as follows:

$$\dot{u}_r = -ru_0u_r + r\left(1 - \frac{u_r}{u_r^2 + \varsigma^2}\Pi\right), \quad \dot{\varsigma} = -\frac{\varsigma r}{u_r^2 + \varsigma^2}\Pi, \tag{22}$$

where $u_r(0) \neq 0$, $\Pi = |\nu|\hat{\varepsilon}_1 + \nu^2\left(\hat{\varepsilon}_2 + \xi\right)|u_0|$, and $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$ are the estimates of ε_1 and ε_2, respectively. For unknown parameters $\hat{\theta}_{f_1}$, $\hat{\theta}_{f_2}$, $\hat{\theta}_{g_1}$, $\hat{\theta}_{u_s}$, $\hat{\varepsilon}_1$ and $\hat{\varepsilon}_2$, we choose the following adaptation laws:

$$\begin{aligned} &\dot{\hat{\theta}}_{f_1} = -\kappa_{f_1}r\nu\psi_{f_1}(x), \quad \dot{\hat{\theta}}_{f_2} = \kappa_{f_2}r\nu\psi_{f_2}(y), \quad \dot{\hat{\theta}}_{g_1} = \kappa_{g_1}r\nu^2u_0\psi_{g_1}(y), \\ &\dot{\hat{\theta}}_{u_s} = \kappa_{u_s}r\nu\psi_{u_s}(y), \quad \dot{\hat{\varepsilon}}_1 = \kappa_{\varepsilon_1}r|\nu|, \quad \dot{\hat{\varepsilon}}_2 = \kappa_{\varepsilon_2}r\nu^2|u_0|, \end{aligned} \tag{23}$$

where $\kappa_{f_1}, \kappa_{f_2}, \kappa_{g_1}$, κ_{u_s}, κ_{ε_1} and κ_{ε_2} are positive constants. So, we obtain the following theorem.

Theorem 3.1. *Consider the error system (7). Suppose that Assumptions 2.1-2.4 and 3.1 are satisfied. Then the control law (18), with robust controller u_r and the adaptation law (23) can ensure that all signals in the closed-loop system are bounded, and the error state*

$e(t)$ remains in a neighborhood of the origin within the prescribed performance bounds for all $t \geq 0$.

Proof: Consider a Lyapunov function as $V = V_1 + V_2$, where

$$V_1 = \frac{1}{2}\left\{\nu^2 + \frac{1}{\kappa_{f_1}}\tilde{\theta}_{f_1}^T\tilde{\theta}_{f_1} + \frac{1}{\kappa_{f_2}}\tilde{\theta}_{f_2}^T\tilde{\theta}_{f_2} + \frac{1}{\kappa_{g_1}}\tilde{\theta}_{g_1}^T\tilde{\theta}_{g_1} + \frac{1}{\kappa_{u_s}}\tilde{\theta}_{u_s}^T\tilde{\theta}_{u_s}\right\}, \tag{24}$$

and

$$V_2 = \frac{1}{2}\left\{u_r^2 + \varsigma^2 + \frac{1}{\kappa_{\varepsilon_1}}\tilde{\varepsilon}_1^2 + \frac{1}{\kappa_{\varepsilon_2}}\tilde{\varepsilon}_2^2\right\}, \tag{25}$$

where $\tilde{\varepsilon}_1 = \varepsilon_1 - \hat{\varepsilon}_1$, and $\tilde{\varepsilon}_2 = \varepsilon_2 - \hat{\varepsilon}_2$.

The time derivative of V_1 is given by

$$\dot{V}_1 = \nu\dot{\nu} - \left\{\frac{1}{\kappa_{f_1}}\tilde{\theta}_{f_1}^T\dot{\hat{\theta}}_{f_1} + \frac{1}{\kappa_{f_2}}\tilde{\theta}_{f_2}^T\dot{\hat{\theta}}_{f_2} + \frac{1}{\kappa_{g_2}}\tilde{\theta}_{g_2}^T\dot{\hat{\theta}}_{g_2} + \frac{1}{\kappa_{u_s}}\tilde{\theta}_{u_s}^T\dot{\hat{\theta}}_{u_s}\right\}. \tag{26}$$

Substituting (21) and (23) into (26), we have

$$\dot{V}_1 = -k_0 r\nu^2 + r\nu g(t,y)\rho(u) - r|\nu|u_s - ru_r + ru_0u_r^2 + r\Lambda. \tag{27}$$

Notice that

$$\begin{gathered}\Lambda \leq \varepsilon_1|\nu| + (\varepsilon_2 + \xi)\nu^2|u_0| = \tilde{\varepsilon}_1|\nu| + \tilde{\varepsilon}_2\nu^2|u_0| + \Pi,\\ r\nu g(t,y)\rho(u) \leq r|\nu||g(t,y)|d^* = r|\nu|u_s.\end{gathered} \tag{28}$$

Substituting (28) into (27), one can obtain

$$\dot{V}_1 \leq -k_0 r\nu^2 - ru_r + ru_0u_r^2 + r\tilde{\varepsilon}_1|\nu| + r\tilde{\varepsilon}_2\nu^2|u_0| + r\Pi. \tag{29}$$

The time derivative of V_2 is

$$\dot{V}_2 = u_r\dot{u}_r + \varsigma\dot{\varsigma} - \frac{1}{\kappa_{\varepsilon_1}}\tilde{\varepsilon}_1\dot{\hat{\varepsilon}}_1 - \frac{1}{\kappa_{\varepsilon_2}}\tilde{\varepsilon}_2\dot{\hat{\varepsilon}}_2. \tag{30}$$

Substituting (22) and (23) into (30), one gets

$$\dot{V}_2 = -ru_r^2u_0 + ru_r - \tilde{\varepsilon}_1 r|\nu| - \tilde{\varepsilon}_2 r\nu^2|u_0| - r\Pi. \tag{31}$$

Combining (29) and (31) gives

$$\dot{V}(t) \leq -k_0 r\nu^2. \tag{32}$$

Therefore, $V(t)$ is always negative, which implies that $z \in L_\infty$. Then, according to the properties of function $s(z)$, we know that $-\delta_{\min} < s(z) < \delta_{\max}$. Then, one can conclude that the objective of the error system (7) with prescribed error performance (9) is achieved. This completes the proof.

Remark 3.2. *Compared with the results in [9], the unknown dead-zone inputs are considered in the paper. Meanwhile, the singular problem will avoid by using the control law (18).*

4. **Numerical Example.** In this section, the numerical simulations are performed to verify and demonstrate the effectiveness of the proposed control scheme. Firstly, we employ the method of [15] to control error system (7). Firstly, we define a sliding surface:

$$\sigma = k_0e_1 + e_2, \tag{33}$$

where k_0 is a designed positive constant. Let $H(t,w) = f_2(t,y) - f_1(t,x) + g(t,y)\rho(u(t))$, $w = [x^T, y^T]^T$, and we need to assume that $g_1(t,y) > \varrho$ for all t and y, where ϱ is a positive constant. So, the error system (7) can be rewritten as follows

$$\begin{cases}\dot{e}_1 = e_2\\ \dot{e}_2 = H(t,w) + g_1(t,y)u(t).\end{cases} \tag{34}$$

The control law is designed as

$$
\begin{aligned}
&u = u_1 + u_2,\\
&u_1 = -\hat{g}_1(y,\theta_{g_1})\left[\epsilon + \hat{g}_1(y,\theta_{g_1})^2\right]^{-1}\left[k_1 e_2 + \hat{H}(w,\theta_H) + k_0\mathrm{sign}(\sigma)\right],\\
&u_2 = -\frac{\left(\varepsilon_H + \varepsilon_{g_1}|u_1| + \left[\epsilon + \hat{g}_1(y,\theta_{g_1})^2\right]^{-1}\left[k_1 e_2 + \hat{H}(w,\theta_H) + k_1\mathrm{sign}(\sigma)\right]\right)}{\varrho}\mathrm{sign}(\sigma),\\
&\dot{\theta}_H = \kappa_H\psi_H(w)\sigma, \quad \dot{\theta}_{g_1} = \kappa_{g_1}\psi_{g_1}(y)u_1\sigma,
\end{aligned} \tag{35}
$$

where $\hat{H}(w,\theta_H) = \theta_H^T\psi_H(w)$, and $\hat{g}_1(w,\theta_{g_1}) = \theta_{g_1}^T\psi_{g_1}(y)$. And there exist ε_H and ε_{g_1} such that $\left|H(t,w) - \hat{H}(w,\theta_H^*)\right| \leq \varepsilon_H$ and $\left|g_1(t,y) - \hat{g}_1\left(y,\theta_{g1}^*\right)\right| \leq \varepsilon_{g_1}$, respectively. In all the simulation process, the initial values of the chaotic system are $[x_1(0),x_2(0)]^T = [-1,1]^T$, $[y_1(0),y_2(0)]^T = [1,-1]^T$, $f = 35.7$, $\alpha^2 = 100$, $\beta = 1$, $c_1 = 0.5$, $c_2 = 0.05$, $\omega = 2$, $g(t,x) = 2 - \sin(x_1)$, $k_0 = k_1 = \varepsilon_H = \varepsilon_{g_1}$, $\kappa_H = \kappa_{g_1} = 2$, $\epsilon = 0.05$, $\varrho = 0.5$, $m = 7$, $b_1 = 1.25$, $b_2 = 0.25$, $\delta_{\max} = \delta_{\min} = 1$, $\mu(t) = y(t) = 3.14e^{-1.71t} + 0.05$. We define seven Gaussian membership functions uniformly distributed on the interval $[-7,7]$. And we choose the initial values of parameters of the fuzzy systems as $\theta_H = \theta_{g_1} = 0.1$. Applying the control method (35), the simulation results are shown in Figures 3 and 4, where Figure 3 expresses the curves of the error states e_1 and e_2; Figure 4 expresses the curves of the controller $\Gamma(u(t))$. From the simulation results in Figures 3 and 4, we know that the tracking error $e(t)$ violates the prescribed error bound $y(t)$ and cannot achieve the good performances in the beginning stage. Moreover, the chatter phenomenon has not been eliminated in Figure 4.

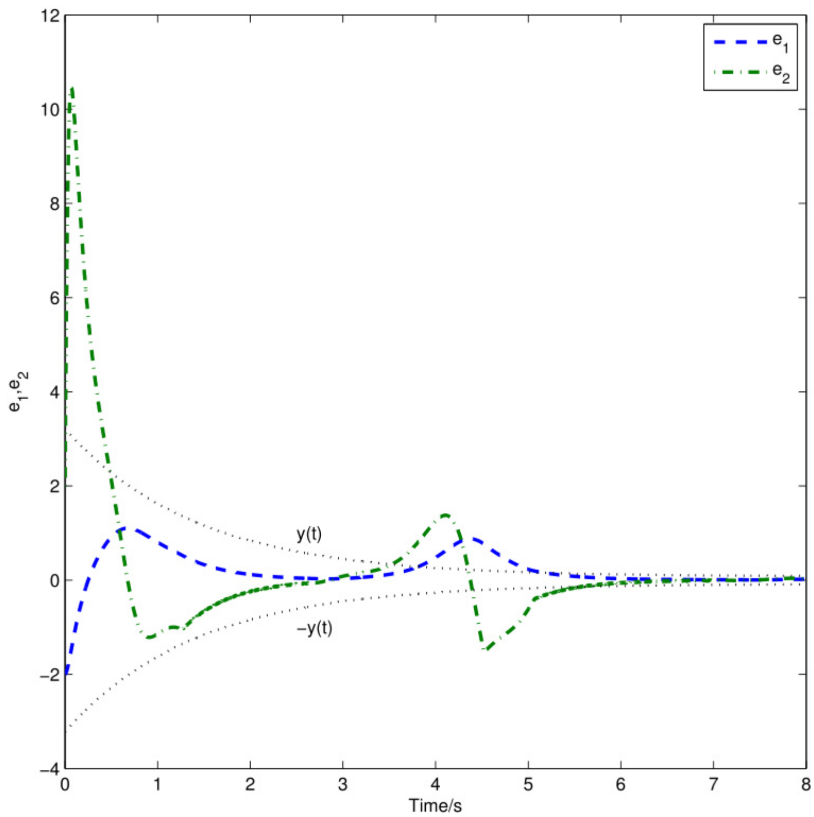

FIGURE 3. The curves of $e(t)$ under the method of (35)

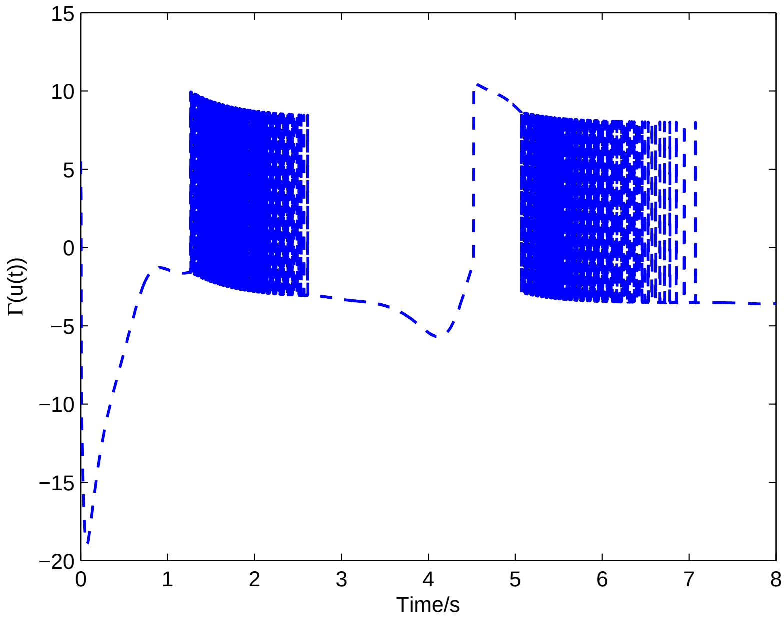

FIGURE 4. The curves of $\Gamma(u(t))$ under the method of (35)

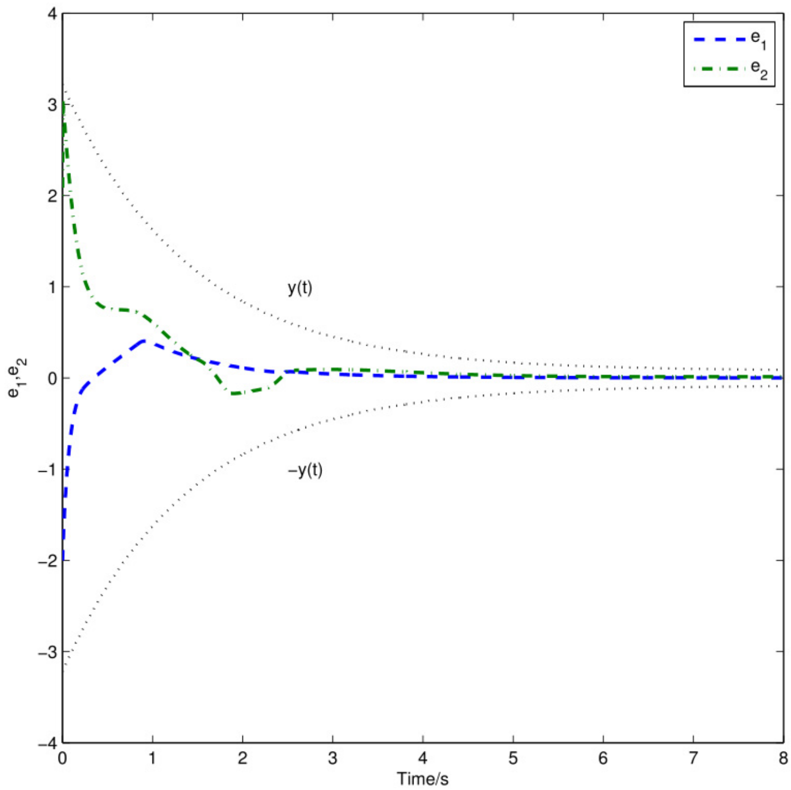

FIGURE 5. The curves of $e(t)$ under the presented method (18)

Now, by using the presented control scheme (18), the simulation results are shown in Figure 5 and Figure 6. From simulation results, we know that the presented control method can guarantee that all the variables are bounded. Moreover, the error $e(t)$ remains

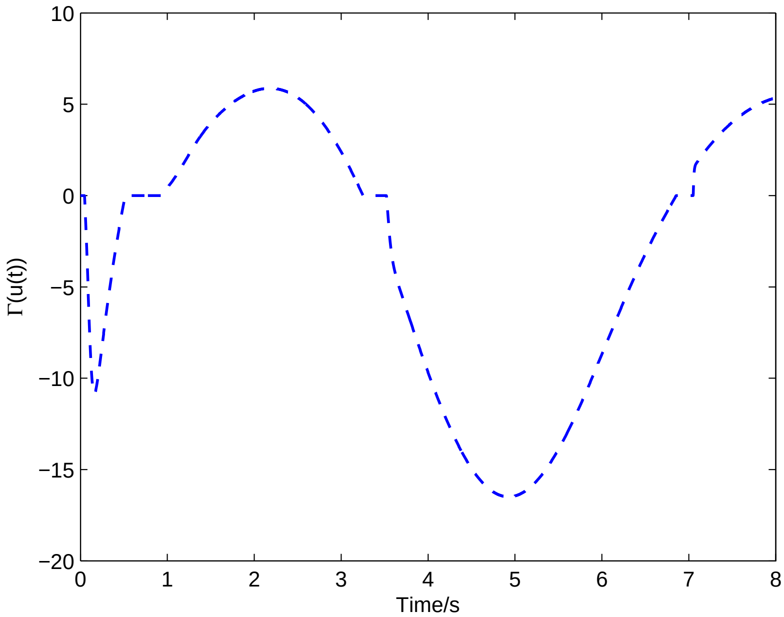

FIGURE 6. The curves of $\Gamma(u(t))$ under the presented method (18)

within the prescribed performance bounds for all the time. And the chatter phenomenon is eliminated.

All the aforementioned results clearly show that the presented PPF-based control method (18) can obtain better regulation performance, i.e., $e(t)$ can be retained within the PPF bound, and achieves faster convergence performance compared to the method (35). The simulation results show that the proposed prescribed transient and steady-state performances are achieved. Thus, the numerical simulations verify theoretical analysis.

5. Conclusions.

For the problem of the synchronization of two uncertain gyros with unknown control gain and unknown dead-zone input, the adaptive fuzzy prescribed performance control scheme has been considered. By using functions, we transform the error system into an equivalent one, and apply the fuzzy logic systems to identifying the unknown nonlinear functions. It is sufficient to guarantee the boundedness of all the variables in the closed-loop system. Simulation results have shown the effectiveness of the proposed scheme. How to design the periodically intermittent adaptive control scheme for the finite-time synchronization between two uncertain gyros systems is our next research direction.

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