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A NOVEL ONLINE BLACK-BOX IDENTIFICATION METHOD FOR UNKNOWN NONLINEAR SYSTEM VIA DIFFERENTIAL NEURAL NETWORK

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Abstract. *In this paper, two differential neural networks (DNN)-based adaptive identifiers for unknown nonlinear systems are proposed. The first proposed identifier is with the single layer DNN and the second one is with the multilayer DNN. Lyapunov approach is used to develop the online updating laws for the dynamic linear matrix and the weights of the proposed two DNN identifiers. Moreover, robust properties of the proposed two DNN identifiers are proved by means of passivity approach, and the commonly used robust modification methods such as dead-zone, e-modification or σ-modification are not needed. Simulation results of an engine idling system demonstrate that the proposed identifier with the multilayer DNN is more accurate than the proposed identifier with the single layer DNN, and both of them illustrate improved performance compared to the conventional neural network-based identifier based on the assumption that the linear matrix is known a priori.*
Keywords: Black box, Nonlinear identifier, Unknown nonlinear systems, Differential neural network, Robust property

1. **Introduction.** Nonlinearity and model uncertainty for most of the practical systems present great challenge for the controller design. Therefore, it becomes necessary to identify the system model before system control can be considered. Several conventional nonlinear identification methods have been proposed during the past decades. However, most of these studies rely on a prior information of the system model. This assumption is hardly satisfied for most practical systems when acquiring an exact model is quite difficult or even impossible. Engine idling process is a good example of such kind of systems. Engine idling operating condition accounted for 1/3 of the total run time in the city conditions and about 30% of the fuel consumed in this process. In order to reduce the fuel consumption, the idle speed should be as low as possible, but too low speed will cause engine stalling. Meanwhile, the presence of external disturbances, such as the air conditioning, the power steering, the change of environmental temperature and the quality of fuel and other factors make the engine idling system exhibit higher nonlinear, time delay and uncertainty characteristics. These properties bring great challenge for modeling the engine idling system and lead to severe difficulty for the controller design [1,2,30,31].

Neural network with the superiority of mapping the complex nonlinear system has been used to identify and control the engine idling system [3-9]. Design and implementation of neural network-based controller for SI engine can be found in [3]. A self-adaptive radial basis function network (RBF)-based control method for the engine idle speed control is presented in [4] and an RBF network model-based observer for idle speed control of ignition in SI engine can be found in [5]. A differential dynamic neural network (DNN) identifier with the application to the engine idling system is introduced in [6]. However, the linear matrix is assumed to be known which is inaccurate for the model uncertain engine idling system. In [7], a DNN algorithm with decoupled extended Kalman filter (DEKF) training method is presented to solve the control problem in engine idling system. Jagannathan and Lewis develop a DNN based nonlinear identifier in [8] and design the neural network-based output feedback control of the engines with high EGR levels in [9]. In addition, an excellent summarization of the identification and control of the engine system via DNN in discrete time domain can be found in [10].

Compared to the feed-forward neural network, recent results show that DNN seems to be more effective to identify and control high degrees complex nonlinear system for its incomparable advantage [11-13,32,33]. Since DNN incorporate feedback to the structure and have more powerful mapping ability, it can successfully overcome the drawback of feed-forward neural network that the static approximation was used to model the dynamic behavior which inevitably leads to the disadvantage of rapid expansion of network structure and slow convergence speed. However, there is no unified approach that can be followed to investigate in DNN due to lack of a unified model. In view of this, a standard DNN model consisting of a linear dynamic matrix and two nonlinear active functions is developed in [14,15], where the conclusions were drawn that the developed standard DNN model can map any complex nonlinear system in arbitrary degree of accuracy. Identification by standard DNN using the sliding mode updating laws can be found in [16] and the controller design based on the DNN identification model is investigated in [17,18]. When part of the system state cannot be available, a model free sliding observer based standard DNN identifier is shown in [19]. Nonlinearity in parameter DNN identifier is studied in [20,21]. Furthermore, standard DNN identification method is extended to discrete-time domain in [22].

Summarizing all of the aforementioned standard DNN identifier design, the dynamic linear matrix needed to be known a priori which is impossible for the real unknown nonlinear system. It should be pointed out that the identification error is influenced by the dynamic linear matrix. Big eigenvalues of dynamic linear matrix may speed up the convergence process, but at the same time, the identification accuracy is affected, so high accuracy is hard to achieve by trial-and-error method to select the dynamic linear matrix offline. In our previous research studies, we have designed the multi-time scales dynamic neural network identifier for a kind of structure uncertain singularly perturbed nonlinear system [23,24]. A dead-zone function is introduced in the updating laws to solve the weights drift problem caused by the structure uncertainty. However, it is well known that the dead-zone function makes the updating laws become complex and reduce the learning speed, which also makes them impossible for practical implementation. Moreover, single layer structure for our previous identifier design limited the ability to solve many practical systems with arbitrary degrees of nonlinearity and complexity. Motivated by these issues, this paper develops an adaptive standard DNN-based identification method for unknown nonlinear systems. The main contributions of this paper can be organized as follows.

1) Lyapunov approach is used to develop the online updating laws of the proposed two identifiers with the single layer DNN and multilayer DNN, respectively. The proposed two DNN identifiers do not need any information of the plant model, which makes them more convenient for practical implementation. In particular, the online updating law of

the linear matrix achieves the improved performance compared to the general neural network-based identifier design which rests on a prior known linear matrix.

2) By means of passivity approach, we prove that the proposed two DNN identifiers are robust with respect to bounded disturbance and the commonly used robust modification methods (e.g., dead-zone [23,24], e-modification [25] or σ-modification [26]) are not needed. Hence, the higher learning speed can be achieved by the simplified updating laws.

3) Online adaptation and robustness properties of the proposed identification method make it very convenient for operating in practical application. The simulation results of an engine idling system demonstrate the improved performance of the proposed identifier than the conventional neural network identification method.

The reminder of this paper is organized as follows. Section 2 discusses the identifier design with the single layer DNN without any prior knowledge about the system dynamics. In Section 3, the results are extended to the case of the identifier design with the multilayer DNN which makes it applicable to many complex nonlinear systems. The robustness properties of the proposed two DNN identifiers are introduced in Section 4 by using the passivity approach. We succeed in proving that the proposed updating laws for the identifiers with the single layer DNN and the multilayer DNN are robust with respect to any bounded uncertainties without using the conventional robust modification methods, such as dead-zone, e-modification or σ-modification. The identification performance is evaluated and demonstrated in Section 5 by simulation carried out on an engine idling system. Section 6 provides brief conclusions of this paper.

2. **Identifier with the Single Layer DNN.** The general nonlinear system can be described by the following state space equation

$$\dot{x} = f(x, u) \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^p$ is the control input vector, and $f(x, u)$: $R^n \times R^p \to R^n$ is the unknown nonlinear vector function.

The following single layer DNN is used to identify the nonlinear system (1)

$$\dot{\hat{x}} = A\hat{x} + W_1\sigma(\hat{x}) + W_2\phi(\hat{x})\delta(u) \tag{2}$$

where $\hat{x} \in R^n$, $A \in \Re^{n\times n}$, $W_1, W_2 \in \Re^{n\times n}$ are the identification state, the linear matrix and the weights of the DNN, respectively. $\sigma(\hat{x}) = [\sigma(\hat{x}_1)\cdots\sigma(\hat{x}_n)]^T \in \Re^{n\times 1}$, $\phi(\hat{x}) = diag[\phi(\hat{x}_1)\cdots\phi(\hat{x}_n)]^T \in \Re^{n\times n}$. The differentiable input-output function $\delta(\cdot) : \Re^p \to \Re^n$ is assumed to be bounded $\|\delta(u)\|^2 \le \bar{u}$. The activation functions $\sigma(\cdot)$, $\phi(\cdot)$ are generally selected as sigmoid function i.e., $\sigma(\cdot) = \frac{a_1}{\left(1+e^{-b_1(\hat{x})}\right)-c_1}$, $\phi(\cdot) = \frac{a_2}{\left(1+e^{-b_2(\hat{x})}\right)-c_2}$.

As evidenced by [15], there definitely exist nominal constant values of the weights W_1^*, W_2^* and nominal constant Hurwitz matrix A^* such that the nonlinear system (1) can be described by the following DNN model

$$\dot{x} = A^*x + W_1^*\sigma(x) + W_2^*\phi(x)\delta(u) + \xi \tag{3}$$

where W_1^*, W_2^* are assumed to be the bounded unknown idea matrices, i.e., $W_1^*\Lambda_1^{-1}W_1^{*T} \le \overline{W}_1$, $W_2^*\Lambda_2^{-1}W_2^{*T} \le \overline{W}_2$, where Λ_1^{-1}, Λ_2^{-1} are the positive definite symmetric matrices, $\overline{W}_1$, $\overline{W}_2$ are prior known matrices, and ξ is the modeling error.

The identification error is defined as

$$e = x - \hat{x} \tag{4}$$

Then from (2) and (3), one can obtain the error dynamics equation

$$\dot{e} = A^*e + \tilde{A}\hat{x} + \widetilde{W}_1\sigma(\hat{x}) + \widetilde{W}_2\phi(\hat{x})\delta(u) + W_1^*\tilde{\sigma} + W_2^*\tilde{\phi}\delta(u) + \xi \tag{5}$$

where $\widetilde{W}_1 = W_1^* - W_1$, $\widetilde{W}_2 = W_2^* - W_2$, $\tilde{A} = A^* - A$. The error of the active functions $\tilde{\sigma} = \sigma(x) - \sigma(\hat{x})$, $\tilde{\phi} = \phi(x) - \phi(\hat{x})$ satisfy the following Lipchitz properties

$$\tilde{\sigma}^T \Lambda_\sigma \tilde{\sigma} \leq e^T D_\sigma e \quad \left(\tilde{\phi}\delta(u)\right)^T \Lambda_\phi \left(\tilde{\phi}\delta(u)\right) \leq \bar{u} e^T D_\phi e \tag{6}$$

where Λ_σ, Λ_ϕ, D_σ, D_ϕ are positive definite matrices.

Lemma 2.1. [27] *$A \in \Re^{n\times n}$ is a Hurwitz matrix, $R, Q \in \Re^{n\times n}$, $R = R^T > 0$, $Q = Q^T > 0$. If $\left(A, R^{1/2}\right)$ is controllable, $\left(A, Q^{1/2}\right)$ is observable, and $A^T R^{-1} A - Q \geq \frac{1}{4}\left(A^T R^{-1} - R^{-1} A\right) R \left(A^T R^{-1} - R^{-1} A\right)^T$ is satisfied, the algebraic Riccati equation $A^T P + PA + PRP + Q = 0$ has a unique positive definite solution $P = P^T > 0$.*

Assumption 2.1. *Define $R = \overline{W}_1 + \overline{W}_2$, $Q = D_\sigma + \bar{u} D_\phi + Q_o$, one can select proper Q_o, hence Q, such that the conditions in Lemma 2.1 are satisfied, and there exists matrix P satisfying the Riccati equation $A^T P + PA + PRP + Q = 0$.*

Remark 2.1. *Assumption 2.1 is presented here just for the subsequent satiability analysis. Note that in Assumption 2.1, R is related to the upper bounds of the weight matrices which are assumed to be bounded, Q can be freely selected because of Q_o, hence the conditions as depicted in Assumption 2.1 can be easily satisfied. Practical implementation of the identification algorithm is free of R and Q (or Q_o), and we only need to select the learning rate as illustrated in the following theorem.*

Theorem 2.1. *By properly designing the Lyapunov function, we obtain the updating laws as*

$$\begin{cases} \dot{A} = \eta_1 e \hat{x}^T \\ \dot{W}_1 = \eta_2 e \sigma^T(\hat{x}) \\ \dot{W}_2 = \eta_3 \phi(\hat{x}) \delta(u) e^T \end{cases} \tag{7}$$

where η_1, η_2, η_3 represent the learning rate of the DNN identifier. The identifier (2) with the updating laws (7) can guarantee the following stable properties: $e \in L_\infty \in L_2$, $W_{1,2} \in L_\infty$, $A \in L_\infty$ and $\lim_{t\to\infty} e = 0$, $\lim_{t\to\infty} \dot{W}_{1,2} = 0$.

Proof: Consider the Lyapunov function as follows

$$L = e^T P e + \eta_1^{-1} tr\left\{\tilde{A}^T P \tilde{A}\right\} + \eta_2^{-1} tr\left\{\widetilde{W}_1^T P \widetilde{W}_1\right\} + \eta_3^{-1} tr\left\{\widetilde{W}_2^T P \widetilde{W}_2\right\} \tag{8}$$

Then, the time derivative of L is obtained by using (5)

$$\begin{aligned} \dot{L} = & \; e^T \left(A^{*^T} P + PA^*\right) e + 2e^T P \tilde{A} \hat{x} + 2e^T P \widetilde{W}_1 \sigma(\hat{x}) + 2e^T P \widetilde{W}_2 \phi(\hat{x}) \delta(u) \\ & + 2e^T P W_1^* \tilde{\sigma} + 2e^T P W_2^* \tilde{\phi} \delta(u) + 2e^T P \xi + \frac{2}{\eta_1} tr\left\{\dot{\tilde{A}}^T P \tilde{A}\right\} \\ & + \frac{2}{\eta_2} tr\left\{\dot{\widetilde{W}}_1^T P \widetilde{W}_1\right\} + \frac{2}{\eta_3} tr\left\{\dot{\widetilde{W}}_2^T P \tilde{W}_2\right\} \end{aligned} \tag{9}$$

By using the updating laws (7) and taking the facts $\dot{\tilde{A}} = -\dot{A}$, $\dot{\widetilde{W}}_{1,2} = -\dot{W}_{1,2}$, $tr\left(yz^T\right) = z^T y$, for any $y, z \in R^{n\times 1}$ into consideration, (9) becomes

$$\begin{aligned} \dot{L} = & \; e^T \left(A^{*^T} P + PA^*\right) e + 2e^T P W_1^* \tilde{\sigma} + 2e^T P W_2^* \tilde{\phi} \delta(u) + 2e^T P \xi \\ & + 2tr\left\{\left(\hat{x} e^T + \frac{1}{\eta_1} \dot{\tilde{A}}^T\right) P \tilde{A}\right\} + 2tr\left\{\left(\sigma(\hat{x}) e^T + \frac{1}{\eta_2} \dot{\widetilde{W}}_1^T\right) P \widetilde{W}_1\right\} \\ & + 2tr\left\{\left(e(\phi(\hat{x})\delta(u))^T + \frac{1}{\eta_3} \dot{\widetilde{W}}_2^T\right) P \widetilde{W}_2\right\} \end{aligned}$$

$$= e^T \left(A^{*^T} P + PA^*\right) e + 2e^T PW_1^* \tilde{\sigma} + 2e^T PW_2^* \tilde{\phi}\delta(u) + 2e^T P\xi \tag{10}$$

From [27], one has the following matrix inequality

$$X^T Y + \left(X^T Y\right)^T \leq X^T \Lambda^{-1} X + Y^T \Lambda Y \tag{11}$$

where $X, Y \in R^{n\times k}$ are any matrices, and Λ is any positive definite matrix.

Then, from (6) and (11) we have

$$\begin{aligned} &2e^T PW_1^* \tilde{\sigma} \leq e^T PW_1^* \Lambda_\sigma^{-1} W_1^{*T} Pe + \tilde{\sigma}^T \Lambda_\sigma \tilde{\sigma} \leq e^T \left(P\overline{W}_1 P + D_\sigma\right) e \\ &2e^T PW_2^* \tilde{\phi}\delta(u) \leq e^T \left(P\overline{W}_2 P + \bar{u} D_\phi\right) e \end{aligned} \tag{12}$$

Substituting (12) into (10), one obtains

$$\begin{aligned} \dot{L} &= e^T \left(A^{*^T} P + PA^*\right) e + 2e^T PW_1^* \tilde{\sigma} + 2e^T PW_2^* \tilde{\phi}\delta(u) + 2e^T P\xi \\ &\leq e^T \left[A^{*^T} P + PA^* + P\left(\overline{W}_1 + \overline{W}_2\right) P + \left(D_\sigma + \bar{u} D_\phi + Q_o\right)\right] e + 2e^T P\xi \\ &\leq -e^T Q_o e + 2e^T P\xi \end{aligned} \tag{13}$$

If the modeling error is zero, i.e., $\xi = 0$, then from (13) we have

$$\dot{L} \leq -e^T Q_o e = -\|e\|_{Q_o}^2 \leq 0 \tag{14}$$

Thus, $e, W_{1,2}, A \in L_\infty$, so $\hat{x} = e + x$ is also bounded. Then, from the error dynamics (5), $\dot{e} \in L_\infty$ is achieved. Further, integrating $\dot{L}$ on both sides, one obtains

$$\int_0^\infty \|e\|_{Q_o}^2 = L(0) - L(\infty) < \infty \tag{15}$$

which implies that $e \in L_2 \cap L_\infty$; using Barbalat's Lemma in [28], we have $\lim_{t\to\infty} e = 0$. In view of the fact that u and $\sigma(\cdot)$, $\phi(\cdot)$ are bounded, thus $\lim_{t\to\infty} \dot{W}_{1,2} = 0$. Theorem 2.1 is proved.

3. **Identifier with the Multilayer DNN.** To take advantage of the full capability of universal approximation of DNN and increase the identification performance, the proposed single layer DNN identifier as described in Section 2 is further extended to the case of multilayer DNN architecture, such that

$$\dot{\hat{x}} = A\hat{x} + W_1 \sigma(V_1 x) + W_2 \phi(V_2 x)\delta(u) \tag{16}$$

where $\hat{x} \in R^n$ is the identification state, $A \in \Re^{n\times n}$ is the linear matrix, $W_1, W_2 \in \Re^{n\times m}$ are the weights in the output layers, $V_1, V_2 \in \Re^{m\times n}$ are the weights in the hidden layers, and $\sigma(\cdot)$ and $\phi(\cdot)$ are activation functions. For simple analysis, the differentiable input-output function $\delta(\cdot) : \Re^p \to \Re^n$ is assumed to be identify matrix I. $u = [u_1, u_2, \cdots, u_p, 0, \cdots, 0]^T \in \Re^m$ is the control input vector.

From the multilayer perception theory, we know that nonlinear system (1) can be identified by the following DNN

$$\dot{\hat{x}} = A^* \hat{x} + W_1^* \sigma\left(V_1^* x\right) + W_2^* \phi\left(V_2^* x\right) u + \xi \tag{17}$$

where W_1^*, W_2^*, V_1^*, V_2^* are unknown idea matrices, A^* is unknown idea linear matrix, and the vector functions ξ can be regarded as modeling error.

From (16) and (17), one can obtain the error dynamics equation

$$\dot{e} = A^* e + \tilde{A}\hat{x} + W_1^* \tilde{\sigma} + \widetilde{W}_1 \sigma(V_1 x) + W_2^* \tilde{\phi} u + \widetilde{W}_2 \phi(V_2 x) u + \xi \tag{18}$$

where $\tilde{A} = A^* - A$, $\widetilde{W}_{1,2} = W_{1,2}^* - W_{1,2}$, $\tilde{\sigma} = \sigma\left(V_1^* x\right) - \sigma(V_1 x)$, $\tilde{\phi} = \phi\left(V_2^* x\right) - \phi(V_2 x)$.

By using Taylor's formula, one has

$$\tilde{\sigma} = D_\sigma \widetilde{V}_1 x + O\left(\widetilde{V}_1 x\right)^2$$
$$\tilde{\phi} u = \sum_{q=1}^{m} D_{\phi_q} \widetilde{V}_2 x u_q + O\left(\widetilde{V}_2 x\right)^2 u = D_\phi \widetilde{V}_2 x + O\left(\widetilde{V}_2 x\right)^2 u$$
$$D_\sigma = \frac{\partial \sigma(V_1 x)}{\partial (V_1 x)} \quad D_\phi = \sum_{q=1}^{m} \frac{\partial \phi_q(V_2 x)}{\partial (V_2 x)} u_q \tag{19}$$

where $\widetilde{V}_{1,2} = V_{1,2}^* - V_{1,2}$, $q = 1, \cdots, m$.

From (18) and (19), we have

$$\begin{aligned} \dot{e} = {} & A^* e + \tilde{A}\hat{x} + W_1 D_\sigma \widetilde{V}_1 x - \widetilde{W}_1 D_\sigma V_1 x + \widetilde{W}_1 \sigma(V_1 x) + W_2 D_\phi \widetilde{V}_2 x \\ & - \widetilde{W}_2 D_\phi V_2 x + \widetilde{W}_2 \phi(V_2 x) u + \xi \end{aligned} \tag{20}$$

where the disturbance term is

$$\xi = \widetilde{W}_1 D_\sigma V_1^* x + \widetilde{W}_2 D_\phi V_2^* x + W_1^* O\left(\widetilde{V}_1 x\right)^2 + W_2^* O\left(\widetilde{V}_2 x\right)^2 u + \xi \tag{21}$$

Assumption 3.1. *The idea weights values are bounded as* $\|W_1^*\|_F \le \overline{W}_1$, $\|W_2^*\|_F \le \overline{W}_2$, $\|V_1^*\|_F \le \overline{V}_1$ *and* $\|V_2^*\|_F \le \overline{V}_2$. *We further define the compact form of the idea weights values as* $Z_l^* = diag\,[W_l^*, V_l^*]$, $l = 1, 2$, *and then we have* $\|Z_1^*\|_F \le \overline{Z}_1$, $\|Z_2^*\|_F \le \overline{Z}_2$, *where* $\overline{W}_{1,2}$, $\overline{V}_{1,2}$, $\overline{Z}_{1,2}$ *are known boundary, and* $\|\bullet\|_F$ *is the Frobenius norm.*

From [28], we know that the higher-order terms of the sigmoid activation functions in the Taylor series are bounded as

$$\left\| O\left(\widetilde{V}_1 x\right)^2 \right\| \le C_1 + C_2 \left\| \widetilde{V}_1 \right\|_F \|x\|, \quad \left\| O\left(\widetilde{V}_2 x\right)^2 u \right\| \le C_3 \|u\| + C_4 \left\| \widetilde{V}_2 \right\|_F \|x\| \|u\| \tag{22}$$

where C_1, C_2, C_3, C_4 are positive constants.

From Assumption 3.1, the disturbance term (22) can be further expressed as

$$\|\xi\| \le C_5 + C_6 \left\| \widetilde{Z}_1 \right\|_F + C_7 \left\| \widetilde{Z}_2 \right\|_F \tag{23}$$

where C_5, C_6, C_7 are positive constants, $\widetilde{Z}_{1,2} = Z_{1,2}^* - Z_{1,2}$.

Online adaptive laws are derived by considering the Lyapunov function as

$$\begin{aligned} L = {} & e^T P e + tr\left\{\widetilde{W}_1^T \lambda_1^{-1} \widetilde{W}_1\right\} + tr\left\{\widetilde{W}_2^T \lambda_2^{-1} \widetilde{W}_2\right\} + tr\left\{\widetilde{V}_1^T \lambda_3^{-1} \widetilde{V}_1\right\} \\ & + tr\left\{\widetilde{V}_2^T \lambda_4^{-1} \widetilde{V}_2\right\} + \lambda_A^{-1} tr\left\{\tilde{A}^T P \tilde{A}\right\} \end{aligned} \tag{24}$$

Then from (18), the time derivative of L is obtained as

$$\begin{aligned} \dot{L} = {} & -e^T Q e + 2e^T P \tilde{A}\hat{x} + 2e^T P \left[W_1 D_\sigma \widetilde{V}_1 x - \widetilde{W}_1 D_\sigma V_1 x + W_2 D_\phi \widetilde{V}_2 x - \widetilde{W}_2 D_\phi V_2 x\right] \\ & + 2e^T P\left[\widetilde{W}_1 \sigma(V_1 x) + \widetilde{W}_2 \phi(V_2 x) u\right] + 2e^T P \xi + 2tr\left\{\dot{\widetilde{W}}_1^T \lambda_1^{-1} \widetilde{W}_1\right\} + 2tr\left\{\dot{\widetilde{W}}_2^T \lambda_2^{-1} \widetilde{W}_2\right\} \\ & + 2tr\left\{\dot{\widetilde{V}}_1^T \lambda_3^{-1} \widetilde{V}_1\right\} + 2tr\left\{\dot{\widetilde{V}}_2^T \lambda_4^{-1} \widetilde{V}_2\right\} + 2\lambda_A^{-1} tr\left\{\dot{\tilde{A}}^T P \tilde{A}\right\} \end{aligned} \tag{25}$$

Theorem 3.1. *Consider the nonlinear system (1) and the identification model (16), the updating laws for the DNN are proposed as*

$$\dot{A} = s\left(\lambda_A e \hat{x}^T\right)$$

$$\begin{aligned}
\dot{W}_1 &= s\left\{\lambda_1 Pe\sigma^T(V_1x) - \lambda_1 Pe(V_1x)^T D_\sigma - k\lambda_1\|e\|W_1\right\} \\
\dot{W}_2 &= s\left\{\lambda_2 Pe[\phi(V_2x)u]^T - \lambda_2 Pe(V_2x)^T D_\phi - k\lambda_2\|e\|W_2\right\} \\
\dot{V}_1 &= s\left\{\lambda_3 (W_1D_\sigma)^T Pex - k\lambda_3\|e\|V_1\right\} \\
\dot{V}_2 &= s\left\{\lambda_4 (W_2D_\phi)^T Pex - k\lambda_4\|e\|V_2\right\} \\
&\quad \textit{with } s = \begin{cases} 1 & \textit{if } \|e\| > \varpi \\ 0 & \textit{if } \|e\| \le \varpi \end{cases}
\end{aligned} \tag{26}$$

where $\lambda_{1,2,3,4}$ are positive definite matrices, λ_A, k are positive constant, and ϖ is specified as $\varpi = \left(2^{-1}\left(kC_8^2 + kC_9^2\right) + 2\|P\|C_1\right)/\lambda_{\min}(Q)$. The above learning laws (26) can guarantee the following stability properties, i.e., $e, A, W_{1,2}, V_{1,2} \in L_\infty$, $\lim_{t\to\infty} \dot{\widetilde{W}}_{1,2} = 0$, $\lim_{t\to\infty} \dot{\widetilde{V}}_{1,2} = 0$.

Proof: Equation (25) can be further rewritten as follows

$$\dot{L} = L_A + L_{W_1} + L_{W_2} + L_{V_1} + L_{V_2} - e^TQe + 2e^TP\xi \tag{27}$$

where

$$\begin{aligned}
L_A &= 2\lambda_A^{-1} tr\left\{\dot{\tilde{A}}^T P\tilde{A}\right\} + 2e^TP\tilde{A}\hat{x} \\
L_{W_1} &= 2tr\left\{\dot{\widetilde{W}}_1^T \lambda_1^{-1}\widetilde{W}_1\right\} + 2e^TP\widetilde{W}_1\sigma(V_1x) - 2e^TP\widetilde{W}_1D_\sigma V_1x \\
L_{W_2} &= 2tr\left\{\dot{\widetilde{W}}_2^T \lambda_2^{-1}\widetilde{W}_2\right\} + 2e^TP\widetilde{W}_2\phi(V_2x)u - 2e^TP\widetilde{W}_2D_\phi V_2x \\
L_{V_1} &= 2tr\left\{\dot{\widetilde{V}}_1^T \lambda_3^{-1}\widetilde{V}_1\right\} + 2e^TPW_1D_\sigma\widetilde{V}_1x \\
L_{V_2} &= 2tr\left\{\dot{\widetilde{V}}_2^T \lambda_4^{-1}\widetilde{V}_2\right\} + 2e^TPW_2D_\phi\widetilde{V}_2x
\end{aligned}$$

By using the updating laws (26), we have

$$\begin{aligned}
\dot{L} &= -e^TQe + 2k\|e\|tr\left(W_1^* - \widetilde{W}_1\right)^T\widetilde{W}_1 + 2k\|e\|tr\left(V_1^* - \widetilde{V}_1\right)^T\widetilde{V}_1 \\
&\quad + 2k\|e\|tr\left(W_2^* - \widetilde{W}_2\right)^T\widetilde{W}_2 + 2k\|e\|tr\left(V_2^* - \widetilde{V}_2\right)^T\widetilde{V}_2 + 2e^TP\xi \\
&= -e^TQ_xe + 2k\|e\|tr\left(Z_1^* - \widetilde{Z}_1\right)^T\widetilde{Z}_1 + 2k\|e\|tr\left(Z_2^* - \widetilde{Z}_2\right)^T\widetilde{Z}_2 + 2e^TP\xi
\end{aligned} \tag{28}$$

By using the fact $tr\left(Z^* - \widetilde{Z}\right)^T\widetilde{Z} = \left\langle Z^*, \widetilde{Z}\right\rangle_F - \left\|\widetilde{Z}\right\|_F^2 \le \left\|\widetilde{Z}\right\|_F\|Z^*\|_F - \left\|\widetilde{Z}\right\|_F^2$ and Equation (23), one obtains

$$\begin{aligned}
\dot{L} &\le -\lambda_{\min}(Q)\|e\|^2 + 2k\|e\|\left\|\widetilde{Z}_1\right\|_F\left(\overline{Z}_1 - \left\|\widetilde{Z}_1\right\|_F\right) + 2k\|e\|\left\|\widetilde{Z}_2\right\|_F\left(\overline{Z}_2 - \left\|\widetilde{Z}_2\right\|_F\right) \\
&\quad + 2\|e\|\|P\|\|\xi\| \\
&\le -\|e\|\left[\lambda_{\min}(Q)\|e\| + 2k\left\|\widetilde{Z}_1\right\|_F\left(\left\|\widetilde{Z}_1\right\|_F - \overline{Z}_1\right) - 2\|P\|\left(C_5 + C_6\left\|\widetilde{Z}_1\right\|_F\right)\right. \\
&\quad \left. + 2k\left\|\widetilde{Z}_2\right\|_F\left(\left\|\widetilde{Z}_2\right\|_F - \overline{Z}_2\right) - 2C_7\|P\|\left\|\widetilde{Z}_2\right\|_F\right]
\end{aligned} \tag{29}$$

Thus, $\dot{L}$ is negative as long as the term in the bracket of (29) is positive. Define $C_8 = \overline{Z}_1 + \frac{\|P\|C_6}{k}$, $C_9 = \overline{Z}_3 + \frac{\|P\|C_7}{k}$, and then the term in the bracket of (29) becomes

$$\begin{aligned} &\lambda_{\min}(Q)\|e\| + 2k\left\|\widetilde{Z}_1\right\|_F\left(\left\|\widetilde{Z}_1\right\|_F - C_8\right) - 2\|P\|C_1 + 2k\left\|\widetilde{Z}_2\right\|_F\left(\left\|\widetilde{Z}_2\right\|_F - C_9\right) \\ = {}& 2k\left(\left\|\widetilde{Z}_1\right\|_F - C_8/2\right)^2 - kC_8^2/2 + \lambda_{\min}(Q)\|e\| - 2\|P\|C_1 + 2k\left(\left\|\widetilde{Z}_2\right\|_F - C_9/2\right)^2 \\ & - kC_9^2/2 \end{aligned} \tag{30}$$

The above term is guaranteed to be positive as long as either

$$\|e\| > \frac{2^{-1}\left(kC_8^2 + kC_9^2\right) + 2\|P\|C_1}{\lambda_{\min}(Q)} = \varpi \text{ or } \begin{array}{l} \left\|\widetilde{Z}_1\right\|_F > C_8/2 + \sqrt{\left(C_8^2 + C_9^2\right)/4 + \|P\|C_1/k} \\ \left\|\widetilde{Z}_2\right\|_F > C_9/2 + \sqrt{\left(C_8^2 + C_9^2\right)/4 + \|P\|C_1/k} \end{array} \tag{31}$$

Thus, $\dot{L}$ is negative outside a compact set. Then UUB of $\|e\|$, $\left\|\widetilde{Z}_1\right\|_F$, $\left\|\widetilde{Z}_2\right\|_F$ are established by using the standard Lyapunov theorem extension in [28], which implies $e, W_{1,2}, V_{1,2}, A \in L_\infty$. Furthermore, $\hat{x} = x - e$ is also bounded. Then from Equation (18), we can draw the conclusion that $\dot{e} \in L_\infty$ with the assumption that error and disturbance are bounded. Since the control input u and $\sigma(\cdot)$, $\phi(\cdot)$ are bounded, it is concluded that $\lim\limits_{t\to\infty} \dot{\widetilde{W}}_{1,2} = 0$, $\lim\limits_{t\to\infty} \dot{\widetilde{V}}_{1,2} = 0$.

Schematic representation of the proposed DNN identifier is illustrated as Figure 1.

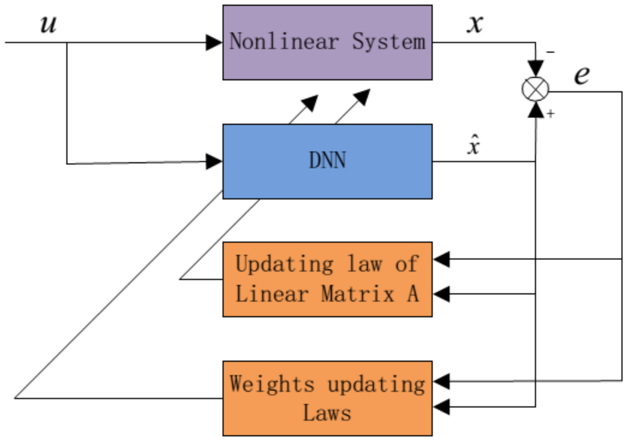

Figure 1. Schematic representation of the proposed DNN identifier

A summary of the two identifiers are listed as follows.

1) Construct the single layer DNN as (2) or the hidden layer DNN as (16) to identify the unknown nonlinear system (1). The initial values of A and W_1, W_2, V_1, V_2 are selected as any random small values. $\sigma(\cdot)$ and $\phi(\cdot)$ are usually selected as the sigmoidal function $\sigma(\cdot) = a/(1 + e^{-bx}) - c$, where a, b, c are design parameters which are determined in terms of trial-and-error methods.
2) Select the proper learning rate (through λ_A and $\lambda_{1,2,3,4}$) in (7) and (26) for different practical nonlinear systems. It is well known that larger learning rates can lead to faster convergence but extra care should be taken to avoid overshoot. There is no preliminary offline learning phase. The weights W_1, W_2, V_1, V_2 and linear part matrix A are tuned online according to (7) and (26) for the single layer DNN and hidden layer DNN, respectively.

3) Online identification process. One can measure the system state x from the plant and obtain the neural networks state $\hat{x}$ from Equation (2) corresponding to the single layer DNN or from Equation (16) corresponding to the hidden layer DNN. Based on the identification error $e = x - \hat{x}$, the weights of single layer DNN and hidden layer DNN are updated online according to Equation (7) and Equation (26), respectively.

4. **Robust Properties of the Proposed Identifiers.** It is well known that NN identification is in sense of black-box identification, and robust property needs to be considered when confronted with uncertainty. By using the passive theory, robust properties of the proposed two identifiers are analyzed in this section.

Lemma 4.1. [28] *A system with input $u(t)$ and output $y(t)$ is said to be passive if it verifies an equality of the so-called "power form"*

$$\dot{L}(t) = y^T u - g(t)$$

and satisfies the following inequality

$$\int_0^T y^T(\tau)u(\tau)d\tau \geq \int_0^T g(\tau)d\tau - \gamma^2$$

for all $T \geq 0$ and some $\gamma \geq 0$, where $g(t)$ is positive semi-definite function of the state.

Rewrite the error Equation (13) as

$$\dot{L} \leq -e^T Q_o e + 2e^T P\xi \tag{32}$$

From Lemma 4.1, if we choose the input as ξ and the output as $2Pe$, then the proposed DNN identifier is state strictly passive (SSP). The SSP property of the weight tuning subsystem can guarantee the boundedness of the internal state in terms of the power delivered to each block. Hence, the PE condition is not needed to ensure the boundedness of the weights and the proposed DNN identifier is robust according to the definition in [28].

Theorem 4.1. *The proposed updating laws (7) and (26) can also keep the DNN identifier robust to the bounded modeling error and disturbance, i.e., $e \in L_\infty$, $W_{1,2} \in L_\infty$, $M \in L_\infty$.*

Proof: From the matrix inequality (11) we have

$$2e^T P\xi \leq e^T P\Lambda_\xi Pe + \xi^T \Lambda_\xi^{-1}\xi$$

Then, (16) can be expressed as

$$\dot{L} \leq -e^T Q_o e + 2e^T P\xi \leq -\lambda_{\min}(Q_o)\|e\|^2 + e^T P\Lambda_\xi Pe + \xi^T \Lambda_\xi^{-1}\xi \leq -k_1(\|e\|) + k_2(\|\xi\|) \tag{33}$$

where $k_1(\|e\|) = [\lambda_{\min}(Q_o) - \lambda_{\max}(P\Lambda_\xi P)]\|e\|^2$, $k_2(\|\xi\|) = \lambda_{\max}(\Lambda_\xi^{-1})\|\xi\|^2$. One can select the small enough positive matrix Λ_ξ such that $\lambda_{\min}(Q_o) \geq \lambda_{\max}(P\Lambda_\xi P)$ is satisfied. Then from (17) and using the input-to-state stability (ISS) definition in [29], one can draw the conclusion that the error dynamics of the DNN identifier (5) are ISS. ISS property of the DNN identifier can ensure the boundedness of the whole learning process for the bounded input. i.e., $e \in L_\infty$, $W_{1,2} \in L_\infty$, $A \in L_\infty$. The same conclusions hold for the updating law (26) by using the similar analysis.

Remark 4.1. *It should be pointed out that ISS property of the two DNN identifiers can ensure that the entire learning process remains bounded for the bounded input. Then from (3), we know that bounded state and input of the system can guarantee the boundedness of the modeling error ξ. Therefore, the boundedness of the modeling error is not needed for the tuning laws of the proposed two DNN identifiers.*

Remark 4.2. *It should be noted that model uncertainties may lead to the parameter drift problem in adaptive control community. Some conventional robust modification methods, such as dead-zone, e-modification or σ-modification, have been used to make identification error stable. Unlike the conventional robust adaptive laws, in this paper, the passivity approach has been successfully used to prove that the proposed two DNN identifiers are robust with respect to bounded uncertainties.*

5. A Case Research: Application to an Engine System.

To illustrate the usefulness and flexibility of the proposed identifiers, model identification of engine idling system is discussed in this section. Engine idling system is a typical system with the characteristic of complex nonlinearity, time-varying and structure uncertainty. The two inputs and two outputs dynamic engine model in [15] is utilized here

$$\begin{cases} \dot{P} = k_p(\dot{m}_{ai} - \dot{m}_{ao}) \\ \dot{N} = k_N(T_i - T_L) \\ \dot{m}_{ai} = (1 + k_{m1}\theta + k_{m2}\theta^2)\, g(P) \\ \dot{m}_{ao} = -k_{m3}N - k_{mA}P + k_{m5}NP + k_{m6}NP^2 \end{cases} \tag{34}$$

An engine model with 1.6-L four-cylinder fuel injected is selected as the simulation parameters

$$g(P) = \begin{cases} 1 & P \le 50.6625 \\ 0.0197\sqrt{101.325P - P^2} & P \ge 50.6625 \end{cases}$$

$$T_i = -39.22 + 325024m_{ao} - 0.0112\delta^2 + 0.635\delta + \frac{2\pi}{60}\left(0.0216 + 6.75 \times 10^{-4}\delta\right)N - \left(\frac{2\pi}{60}\right)^2 1.02 \times 10^{-4}N^2$$

$$T_L = \left(\frac{N}{263.17}\right)^2 + T_d; \quad m_{ao} = \dot{m}_{ao}(t-\tau)/(120N); \quad k_P = 42.4; \quad k_N = 54.26$$

$$k_{m1} = 0.907; \quad k_{m2} = 0.0998; \quad k_{m3} = 5.968 \times 10^{-4}; \quad k_{m4} = 5.341 \times 10^{-4}$$

$$k_{m5} = 1.757 \times 10^{-6}; \quad \tau = 45/N$$

where manifold press P (kPa) and engine speed N (r/min) are the system outputs. Throttle angle θ and the spark advance δ are the system inputs. $\dot{m}_{ai}$, $\dot{m}_{ao}$ indicate the mass air that flows into and out of the mainfold. m_{ao} is the air mass in the cylinder. Uncertain disturbance load torque is expressed as T_d. Engine output torque is T_i. Load torque is T_L. $g(P)$ is the manifold pressure function. τ is the time delay.

The aforementioned engine model can be expressed as a general nonlinear system

$$\dot{x}(t) = f(x, u, t) \tag{35}$$

where $x(t) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} P \\ N \end{pmatrix}$, $u = (\theta, \delta)^T$ and $f(x, u, t) = \begin{pmatrix} f_1(x, u, t) \\ f_2(x, u, t) \end{pmatrix}$ is the unknown nonlinear function.

The designed two DNN identifiers with the single layer structure (2) corresponding to the updating laws (7) and the multilayer structure (16) corresponding to the updating laws (26) are used to identify the engine model (35), and parameters are selected as follows.

For the single layer identifier: $a = 2$, $b = 2$, $c = 0.5$, $\eta_1 = 1$, $\eta_2 = \eta_3 = 200$.

For the multilayer layer identifier: $a = 2$, $b = 2$, $c = 0.5$, $\lambda_A = -1000$, $k = 0.05$, $\lambda_1 = -200I$, $\lambda_2 = -100I$, $\lambda_3 = \lambda_4 = -0.05I$.

Input $u = (\theta, \delta)^T$ is selected as $\delta = 30\sin(t/2)$ and θ is a saw tooth wave with amplitude 10 and frequency 0.5. T_d is selected as a square wave with amplitude 20, frequency 0.25. The state initial values are selected as $\begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix} = \begin{pmatrix} 10 \\ 500 \end{pmatrix}$.

The proposed two identifiers are compared to [15] which does not consider the on-line updating law for the linear matrix (i.e., the linear matrix A is assumed to be known a priori). Comparative identification results are illustrated as in Figures 2-5. It can be seen that the proposed identifier with the multilayer DNN is more accurate than the proposed identifier with the single layer DNN, and both of them demonstrate better performance than [15]. The reason is that the constant value of linear matrix A in [15] will influence the accuracy of the identification results due to the time varying linear matrix A in

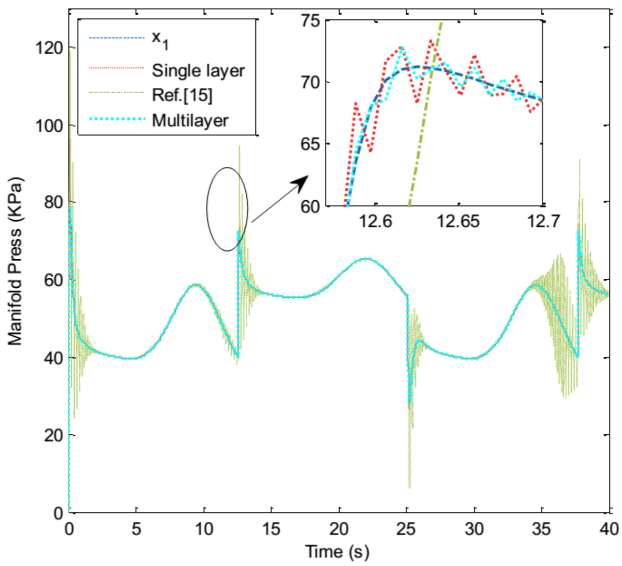

FIGURE 2. Comparative identification result of manifold press

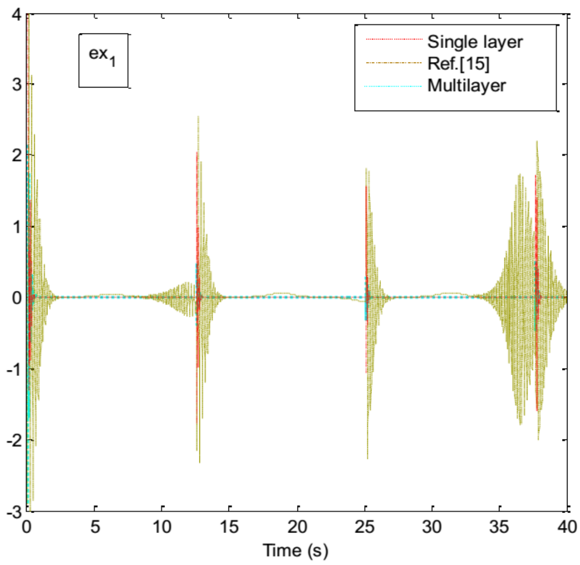

FIGURE 3. Comparative identification error of manifold press

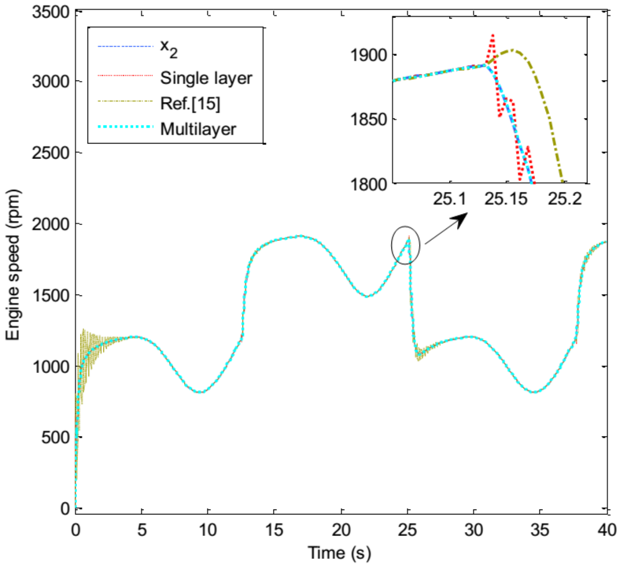

FIGURE 4. Comparative identification result of engine speed

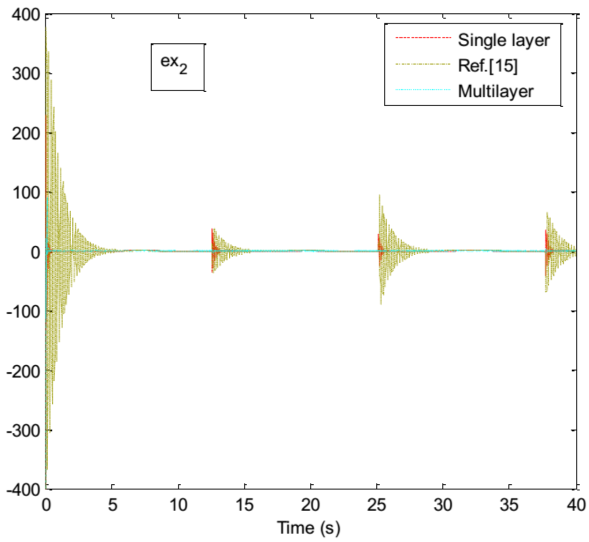

FIGURE 5. Comparative identification error of engine speed

the practical system. However, the linear matrix A is updated online for the proposed identifiers, which greatly improves the accuracy of the identification results.

To further make a comparison, the performance index: root mean square (RMS) is adopted:

$$RMS = \sqrt{\left(\sum_{i=1}^{n} e^2(i)\right) \Big/ n}$$

where n is the number of the simulation steps, and $e(i)$ is the difference between the state variables in model and system at the ith step. The RMS values of the aforementioned identification process are illustrated in Table 1. The RMS values of the proposed identifier with the multilayer DNN are smaller than that of the proposed identifier with the single layer DNN, and both of them smaller than [15], which further demonstrates the better performance of the proposed two DNN identifiers.

TABLE 1. The RMS values for the identification error

	x_1	x_2
Single layer	0.00032	0.64917
Multilayer	0.00012	0.00179
Ref. [15]	0.05467	4.58098

Remark 5.1. *From the whole learning process, one can see that the proposed two DNN identifiers are actually a kind of black-box identification methods, which is just based on the state and input of the nonlinear system, and no precise plant model is needed. Moreover, convergence speed can be adjusted by setting different learning rates and robust property can be guaranteed during the entire learning process. These advantages make them more convenient for practical application.*

6. **Conclusions.** The problem of designing a black-box identification method for unknown nonlinear systems using both single layer DNN and multilayer DNN has been investigated in this paper. To the best of our knowledge, it is the first time that the online updating laws are designed for both the weights and the linear part matrix A, unlike the general neural network-based identification method, where a strong assumption about the linear part matrix A was posed as a known Hurwitz matrix which was sometimes unrealistic for the black-box nonlinear system. Moreover, by means of the passivity approach, we succeed in proving that the proposed updating laws of the identifiers with both single layer DNN and multilayer DNN are robust with respect to any bounded uncertainties without using the conventional robust modification methods, such as dead-zone, e-modification or σ-modification. Simulation results of an engine idling system demonstrate that the proposed identifier with the multilayer DNN is more accurate than the proposed identifier with the single layer DNN, and both of them illustrate improved performance compared to the general neural network identifier without considering the updating law of the linear matrix. Further study can be carried out on the control strategies based on the identification results.

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