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SLIDING MODE FAULT TOLERANT CONTROL WITH PRESCRIBED PERFORMANCE

Jicheng Gao, Qikun Shen, Pengfei Yang and Jianye Gong

College of Information Engineering
Yangzhou University
No. 196, Huayang West Road, Yangzhou 225127, P. R. China
qkshen@yzu.edu.cn

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Abstract. *This paper considered state tracking problem of a class of nonlinear systems with actuator failures and unmodeled dynamics. Based on neural network and Nussbaum function, an adaptive neural network-based fault tolerant control scheme is proposed to ensure the tracking performance to satisfy a given prescribed performance. The problem of unmodeled dynamics is handled by introducing a dynamic signal. Using the properties of Nussbaum function, the problem of the unknown system direction is solved. Theoretical analysis shows that the closed-loop system is semi-globally uniformly bounded. Simulation results illustrate the effectiveness of the scheme.*

Keywords: Tolerant control, Unmodeled fault, Performance function, Adaptive control

1. **Introduction.** In modern control systems, with the increasing complexity of the systems, the components are vulnerable to faults, and because of the frequent operation of the actuator, it is more fault-prone than others. In order to eliminate the faults, ensure the normal operation of the system, fault tolerant control (FTC) has become an important field of control research and obtained many achievements.

In [1], by designing observers, the active fault-tolerant control problem was addressed. In [2], an actuator fault model that integrated varying bias and gained faults is proposed, and sliding mode observers (SMOs) are designed for fault detection and isolation. In [3], for a class of strict-feedback non-linear systems, the corrective control law is reconstructed by generated fault information to compensate the fault effects. In [4], a novel discrete-time estimator is proposed for a discrete-time dynamic system with actuator and sensor faults.

At present, the faults considered in papers are gain faults or bias faults, and there are only a few results on unmodeled faults. However, in many cases the faults cannot be expressed in the affine form, but in the form of unmodeled. Based on this, the results of unmodeled fault tolerant control have important theoretical significance and application value.

In engineering system, there exist unmodeled dynamics which can make system oscillating and divergent. In [5], an adaptive control scheme is proposed for a class of pure-feedback nonlinear systems with unmodeled dynamics and unknown gain signs. In [6], K-filters were introduced when the states of system were not measured. In [7], by combining fuzzy systems with K-filters, an adaptive output feedback dynamic surface control was investigated. The prescribed performance control demands the convergence rate no less than a prescribed value, and both the steady state and the transient performance were discussed. In [10], the robust adaptive control for strict feedback nonlinear system with prescribed performance was proposed.

In this paper, based on prescribed performance function and radial basis function (RBF) neural networks, the adaptive FTC scheme is investigated for a class of uncertain nonlinear system with unmodeled dynamics and unmodeled actuator fault. Compared with existing literature, the FTC scheme not only considers the more common unmodeled faults in practical applications, but also guarantees the prescribed transient and steady state error within the proper bound.

2. **Problem Statement and Preliminaries.** Consider a class of nonlinear systems in the following form:

$$\begin{cases} \dot{z} = q(z, x) \\ \dot{x}_i = x_{i+1}, \quad i = 1, 2, \dots, n-1 \\ \dot{x}_n = f_0(x) + F(x, u) + \Delta(x, z, t) \\ y = x_1 \end{cases} \tag{1}$$

where $x = [x_1, x_2, \dots, x_n]^T \in R^n$ are the states, $u \in R$ and $y \in R$ are the system input and output, the nonlinear function $f_0(x)$ is unknown and smooth, the nonlinear function $F(x, u)$ is the unmodeled actuator fault, and $\Delta(x, z, t)$ is the unmodeled dynamics.

Define the tracking error $e(t)$ as follows:

$$e = \left[e, \dot{e}, \dots, e^{(n-1)}\right]^T = \left[x_1 - y_d, x_2 - \dot{y}_d, \dots, x_n - y_d^{(n-1)}\right]^T \tag{2}$$

The control objective is to design adaptive tolerant controller for system (1) such that $e(t)$ satisfies the prescribed transient and steady state performances, and all the signals in the closed-loop system are bounded.

In this paper, by introducing RBF neural networks to approximate the unknown continuous function $h(\xi) = W^{*T}\psi(\xi) + \omega(\xi)$, where $\xi \in \Omega_\xi$ is the input, $\omega(\xi)$ is the approximation error, $\psi(\xi) = (\psi_1(\xi), \dots, \psi_l(\xi))^T \in R^l$ is a known smooth vector function and $l > 1$ represents the neural networks node number. The basis function $\psi_j(\xi)$ is chosen as following form $\psi_j(\xi) = \exp\left(-\|\xi - \mu_j\|^2 / \phi_j^2\right)$, $j = 1, 2, \dots, l$, where μ_j and ϕ_j denote, respectively, the center of the receptive field and the width of the Gaussian function. The ideal weight $W^* = (w_1, \dots, w_l)^T$ is defined as follows: $W^* = \arg\min\limits_{\hat{W} \in R^l} \left[\sup\limits_{\xi \in \Omega_\xi} \left|h(\xi) - \hat{W}^T\psi(\xi)\right|\right]$.

To design controller, some necessary assumptions are introduced as follows.

Assumption 2.1. *For any $x \in R^n$, $u \in R$, the function $f(x, u)$ is differentiable with respect to u and there exist positive constants g_0, g_1 and g_2 such that*

$$g_0 \le |g(x, u_\lambda)| \le g_1, \quad |\dot{g}(x, u_\lambda)| \le g_2 \tag{3}$$

where $g(x, u_\lambda) = \left[\partial f(x, u)/\partial u\right]\big|_{u=u_\lambda}$ and $u_\lambda \in [0, u]$.

Assumption 2.2. *There exist unknown nonnegative continuous functions $\varphi_1(\cdot)$ and unknown increasing continuous functions $\varphi_2(\cdot)$ such that $|\Delta(x, z, t)| \le \varphi_1(||x||) + \varphi_2(||z||)$.*

Assumption 2.3. *The subsystem $\dot{z} = q(z, x)$ is said to be exponentially input-state-practically stable (exp-ISpS), if there exists a Lyapunov function $V(t)$ such that*

$$\alpha_1(z) \le V(z) \le \alpha_2(z), \quad \frac{\partial V(z)}{\partial z} q(z, x) \le -cV(z) + \gamma(||x||) + d \tag{4}$$

where $\alpha_1(\cdot)$, $\alpha_2(\cdot)$, $\gamma(\cdot)$ are functions of class k_∞ and $\gamma(\cdot)$ is known, and c and d are known positive constants.

Assumption 2.4. *The desired signal y_d and its time derivatives $y_d^{(i)}$ $(i = 1, 2, \ldots, n)$ are continuously bounded.*

Assumption 2.5. *The RBF neural networks approximation error $\omega(\xi)$ satisfies $|\omega(\xi)| \leq \omega^*$ with ω^* being a positive constant.*

Lemma 2.1. [8]. *The function V is an exp-ISpS Lyapunov function for subsystem $\dot{z} = q(z, x)$, i.e., (5) and (6) hold, then for any constant $\bar{c} \in (0, c)$, in initial instant $t_0 > 0$, any initial state $z_0 = z(t_0)$, $\upsilon > 0$ and $\bar{\gamma}(||x||) > \gamma(||x||)$, there exists a finite $T_0 = \left[\left(V(z_0)/\upsilon_0\right) e^{(c-\bar{c})t_0}\right]/(c - \bar{c}) \geq 0$, a nonnegative function $D(t_0, t)$, define dynamic signal $\dot{\upsilon} = -\bar{c}\upsilon + \bar{\gamma}(||x||) + d$, $\upsilon(t_0) = \upsilon_0$ for $t \geq t_0 + T_0$, and there exist $D(t_0, t) = 0$ such that $V(z) \leq \upsilon(t) + D(t_0, t)$.*

Lemma 2.2. [9]. *For any real continuous function $f(x, y)$, there exist positive smooth scalar functions $\phi_1(x) \geq 0$ and $\phi_2(y) \geq 0$ such that the following inequality holds: $|f(x, y)| \leq \phi_1(x) + \phi_2(y)$, where $x \in R^m$, $y \in R^n$.*

Definition 2.1. *A continuous function $N(\varsigma) : R \to R$ is defined Nussbaum function such that 1) $\lim\limits_{x\to\infty} \sup \frac{1}{s} \int_0^s N(\varsigma) d\varsigma = +\infty$; 2) $\lim\limits_{x\to-\infty} \inf \frac{1}{s} \int_0^s N(\varsigma) d\varsigma = -\infty$.*

Lemma 2.3. [5]. *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $t \in [0, t_f)$ and $V(t) \geq 0$, and $N(\cdot)$ be a Nussbaum function, if the following inequality holds,*

$$V(t) \leq c_0 + e^{-c_1 t} \int_0^t g(\tau) N(\varsigma) \dot{\varsigma} e^{c_1 \tau} d\tau + e^{-c_1 t} \int_0^t \dot{\varsigma} e^{c_1 t} d\tau, \quad \forall t \in [0, t_f) \tag{5}$$

where $c_1 > 0$ and c_0 is a suitable constant, $g(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I = [l^-, l^+]$ with $0 \notin I$, and then $V(t)$, $\varsigma(t)$ and $\int_0^t g(\tau) N(\varsigma) \dot{\varsigma} d\tau$ must be bounded on $[0, t_f)$.

3. **Performance Function and Error Transformation.**

3.1. **Performance function.**

Definition 3.1. *A continuous function $\rho(t)$: $R_+ \to R_+$ is defined a performance function if $\rho(t)$ is decreasing and $\lim\limits_{x\to\infty} \rho(t) = \rho_\infty > 0$.*

In this paper, $\rho(t) = (\rho_0 - \rho_\infty)e^{-lt} + \rho_\infty$ is chosen as the prescribed performance function, where ρ_0, ρ_∞ and l are appropriately defined positive constants, l determines the convergence rate of $\rho(t)$, $\rho_0 = \rho(0)$ denotes the bound of the overshoot, and the parameter ρ_∞ represents the maximum allowable steady tracking error.

3.2. **Error transformation.**

Define the error transformation as follows

$$e(t) = \rho(t) S(\varepsilon) \tag{6}$$

where ε is the transformed error, and the function $S(\varepsilon)$ is smoothly increasing and satisfies

$$-\delta < S(\varepsilon) < 1,\ e(0) > 0; \quad -1 < S(\varepsilon) < \delta,\ e(0) < 0 \tag{7}$$

Owing to the properties of $S(\varepsilon)$ and $\rho(t)$, one has $\varepsilon = S^{-1}\left[e(t)/\rho(t)\right]$. If $\varepsilon(t) \in L_\infty$ with $t \in [0, \infty)$, then $e(t)$ satisfies the prescribed performance, and owing to the properties of performance function, tracking error is confined to $\Omega_e = \{e \in R : |e(t)| \leq \rho_\infty\}$.

4. Adaptive Tolerant Controller Design and Stability Analysis.

4.1. Adaptive tolerant controller design. Define the transformed error as follows: $\bar{\varepsilon} = \left[\varepsilon, \dot{\varepsilon}, \ldots, \varepsilon^{(n-1)}\right]^T = [\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n]^T$. According to [11], the filtered error is defined as $s = [\Lambda^T \quad 1]\left[\varepsilon \quad \dot{\varepsilon} \quad \cdots \quad \varepsilon^{(n-1)}\right]^T$, where $\Lambda^T = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_{n-1}]$ is appropriately chosen such that $s^{n-1} + \lambda_{n-1}s^{n-2} + \cdots + \lambda_1$ is Hurwitz.

Differentiating, the filtered error dynamics may be written as $\dot{s} = \lambda_1\dot{\varepsilon}_1 + \lambda_2\dot{\varepsilon}_2 + \cdots + \dot{\varepsilon}_n$ from (2), differentiating $e_1(t)$ with respect to time t, one has

$$\dot{e}_1 = \dot{\rho}(t)S(\varepsilon) + \rho(t)\frac{\partial S}{\partial \varepsilon}\dot{\varepsilon} = \beta_1(\rho, \dot{\rho}, \varepsilon) + S_\rho\dot{\varepsilon} \tag{8}$$

where $S_\rho = \rho(t)\left(\partial S/\partial\varepsilon\right)$. From (2) and (8), one has

$$\dot{e}_2 = \frac{\partial\beta_1}{\partial\rho}\dot{\rho} + \frac{\partial\beta_1}{\partial\dot{\rho}}\ddot{\rho} + \frac{\partial\beta_1}{\partial\varepsilon}\dot{\varepsilon} + \left(\frac{\partial S_\rho}{\partial\rho}\dot{\rho} + \frac{\partial S_\rho}{\partial\varepsilon}\dot{\varepsilon}\right)\dot{\varepsilon} + S_\rho\ddot{\varepsilon} = \beta_2\left(\rho, \dot{\rho}, \ddot{\rho}, \varepsilon, \dot{\varepsilon}\right) + S_\rho\ddot{\varepsilon} \tag{9}$$

Similarly, one has $\dot{e}_i = \beta_i\left(\rho, \dot{\rho}, \ldots, \rho^{(i)}, \varepsilon, \ldots, \varepsilon^{(i-1)}\right) + S_\rho\varepsilon^{(i)}$, $i = 1, 2, \ldots, n$. In this paper, for notational simplicity, let β_i denote $\beta_i\left(\rho, \dot{\rho}, \ldots, \rho^{(i)}, \varepsilon, \ldots, \varepsilon^{(i-1)}\right)$ $(i = 1, 2, \ldots, n)$.

From above equality, one has $\dot{\varepsilon}_i = (\dot{e}_i - \beta_i)\big/S_\rho$, $i = 1, 2, \ldots, n$; furthermore, we have

$$\dot{s} = \sum_{i=1}^{n-1}\lambda_i\dot{\varepsilon}_i + \dot{\varepsilon}_n = \sum_{i=1}^{n-1}\lambda_i e_{i+1}\Big/S_\rho - \sum_{i=1}^{n-1}\lambda_i\beta_i\Big/S_\rho - \beta_n/S_\rho - y_d^{(n)}/S_\rho + \dot{x}_n/S_\rho \tag{10}$$

Furthermore, one has

$$\dot{s} = \beta + \gamma\left[F(x, u) + \Delta(x, z, t)\right] \tag{11}$$

where $\beta = \sum_{i=1}^{n-1}\lambda_i e_{i+1}/S_\rho - \sum_{i=1}^{n-1}\lambda_i\beta_i/S_\rho - \beta_n/S_\rho - y_d^{(n)}/S_\rho$, $\gamma = 1/S_\rho > 0$.

According to Assumption 2.1 and mean value theorem, there exist $u_\lambda \in [0, u]$ with $\forall t > 0$, such that $f(x, u) = f(x, 0) + \left[\partial f(x, u_\lambda)/\partial u\right]u$.

Let $g(x, u_\lambda) = \partial f(x, u_\lambda)/\partial u$, and then (11) can be expressed as follows:

$$\dot{s} = \beta + \gamma\left[f_0(x) + f(x, 0) + g(x, u_\lambda)u + \Delta(x, z, t)\right] \tag{12}$$

Define the following Lyapunov function:

$$V_s = \frac{1}{2|g(x, u_\lambda)|}s^2 \tag{13}$$

where from Assumption 2.1, one has $g_0 < |g(x, u_\lambda)| < g_1$. Differentiating with V_s respect to time t, one has

$$\begin{aligned} \dot{V}_s &= -\frac{|g(x, u_\lambda)|}{2g(x, u_\lambda)}\frac{\dot{g}(x, u_\lambda)}{g^2(x, u_\lambda)}s^2 + \frac{1}{|g(x, u_\lambda)|}s\dot{s} \\ &\le \frac{|\dot{g}(x, u_\lambda)|}{2g^2(x, u_\lambda)}s^2 + \frac{1}{|g(x, u_\lambda)|}s\beta + \frac{1}{|g(x, u_\lambda)|}s\gamma\left[f_0(x) + f(x, 0) + g(x, u_\lambda)u + \Delta(x, z, t)\right] \end{aligned} \tag{14}$$

According to Assumption 2.1 and Lemma 2.1, one has $|\Delta(x, z, t)| \le \varphi_1(||x||) + \varphi_2(||z||)$, $||z|| \le \alpha_1^{-1}(\upsilon(t) + D(t_0, t))$. From Assumption 2.3, we obtain $\varphi_2(||z||) \le \varphi_2 \circ \alpha_1^{-1}(\upsilon(t) + D(t_0, t))$, where $\varphi_2 \circ \alpha_1^{-1}(\cdot) = \varphi_2\left(\alpha_1^{-1}(\cdot)\right)$, because $\varphi_2 \circ \alpha_1^{-1}(\cdot)$ is increasing smooth function, according to Lemma 2.3, one has $\varphi_2(||z||) \le \phi_1(\upsilon(t)) + \phi_2(D(t_0, t))$.

Using Young's inequality, we have $\varphi_1(||x||) \le \varphi_1^2(||x||) + 1/4$, $\phi_1(\upsilon(t)) \le \phi_1^2(\upsilon(t)) + 1/4$, $\phi_2(D(t_0, t)) \le \phi_2^2(D(t_0, t)) + 1/4$. In addition, because $D(t_0, t)$ and $\phi_2(\cdot)$ are nonnegative smooth functions, let $\phi_2(D(t_0, t)) \le p^*$ with $p^* > 0$.

According to the above three inequalities and Assumption 2.1, one has

$$\begin{aligned}\dot{V}_s \leq & \frac{g_2}{2g^2(x,u_\lambda)}s^2 + \frac{s\beta}{|g(x,u_\lambda)|} + \frac{s\gamma}{|g(x,u_\lambda)|}g(x,u_\lambda)u + \frac{s\gamma}{|g(x,u_\lambda)|}[f_0(x) + f(x,0)] \\ & + \frac{1}{|g(x,u_\lambda)|}|s\gamma|\left[\varphi_1^2(||x||) + \phi_1^2(\upsilon(t))\right] + \frac{1}{g_0}|s|\gamma\left(p^{*2} + \frac{3}{4}\right)\end{aligned} \tag{15}$$

By introducing inequality $|x| - x\tanh\left(\frac{x}{\delta}\right) \leq 0.2785\delta$, $\forall x \in R$, $\delta > 0$, one has

$$|s\gamma|\varphi_1^2(||x||) \leq s\gamma\varphi_1^2(||x||)\tanh\left(s\gamma\varphi_1^2(||x||)/\delta_1\right) + 0.2785\delta_1$$

$$|s\gamma|\phi_1^2(\upsilon(t)) \leq s\gamma\phi_1^2(\upsilon(t))\tanh\left(s\gamma\phi_1^2(\upsilon(t))/\delta_2\right) + 0.2785\delta_2$$

where $\delta_1 > 0$, $\delta_2 > 0$, and let

$$\begin{aligned}h(\xi) = & \frac{g_2}{2\gamma g^2(x,u_\lambda)}s + \frac{\beta}{\gamma|g(x,u_\lambda)|} + \frac{1}{|g(x,u_\lambda)|}\Big[f_0(x) + f(x,0) \\ & + \varphi_1^2(||x||)\tanh\left(s\gamma\varphi_1^2(||x||)/\delta_1\right) + \phi_1^2(\upsilon(t))\tanh\left(s\gamma\phi_1^2(\upsilon(t))/\delta_2\right)\Big]\end{aligned} \tag{16}$$

where $\xi = [x^T, \beta, s, \upsilon]^T$. Furthermore, (15) can be expressed as follows:

$$\dot{V}_s \leq s\gamma h(\xi) + s\gamma\frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}u + \frac{1}{g_0}|s|\gamma p^* + \frac{1}{g_0}|s|\gamma\left(p^{*2} + \frac{3}{4}\right) + 0.2785(\delta_1 + \delta_2) \tag{17}$$

According to neural networks and Assumption 2.5, one has

$$\dot{V}_s \leq s\gamma W^{*T}\psi(\xi) + s\gamma\frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}u + |s|\gamma\omega^* + \frac{1}{g_0}|s|\gamma\left(p^{*2} + \frac{3}{4}\right) + 0.2785(\delta_1 + \delta_2) \tag{18}$$

Let $b^* = \max\{\omega^*, (p^{*2} + 3/4)/g_0\}$, and (18) can be expressed as

$$\dot{V}_s \leq s\gamma W^{*T}\psi(\xi) + s\gamma\frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}u + b^*|s|\gamma + 0.2785(\delta_1 + \delta_2) \tag{19}$$

The control and adaptive laws are designed as follows:

$$u = N(\varsigma)\left[(k/2\gamma)s + \hat{W}^T\psi(\xi) + \mathrm{sgn}(s)\hat{b}\right] \tag{20}$$

$$\dot{\varsigma} = ks^2 + s\gamma\hat{W}^T\psi(\xi) + |s|\gamma\hat{b} \tag{21}$$

$$\dot{\hat{W}} = \eta_0 s\gamma\psi(\xi) + \eta_1\hat{W} \tag{22}$$

$$\dot{\hat{b}} = \sigma_0|s|\gamma + \sigma_1\hat{b} \tag{23}$$

where $k > 0$, $\eta_0 > 0$, $\eta_1 > 0$, $\sigma_0 > 0$, $\sigma_1 > 0$ are design parameters, $\hat{W}$ is the estimation value of W at time t, $\hat{b}$ is the estimation value of b at time t, and $\widetilde{W} = W^* - \hat{W}$, $\tilde{b} = b^* - \hat{b}$.

4.2. Stability analysis.

Theorem 4.1. *Consider the system (1) with unmodeled dynamics and actuator unmodeled fault, the control law (20) and adaptive laws (22) and (23) are employed, if Assumptions 2.1-2.5 hold, from error transformation (6), then all signals in the closed-loop system remain semi-globally uniformly ultimately bounded, and the tracking error is confined to a predefined residual set.*

Proof: Select the following Lyapunov function

$$V = V_s + \frac{1}{2\eta_0}\widetilde{W}^T\widetilde{W} + \frac{1}{2\sigma_0}\tilde{b}^2 \tag{24}$$

Differentiating with V respect to time t, from (17) one has

$$\dot{V} \le s\gamma W^{*T}\psi(\xi) + s\gamma\frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}u + b^*|s|\gamma - \frac{1}{\eta_0}\widetilde{W}^T\dot{\hat{W}} - \frac{1}{\sigma_0}\tilde{b}\dot{\hat{b}} + 0.2785(\delta_1+\delta_2) \tag{25}$$

Substituting control law and adaptive laws into (25), we obtain

$$\dot{V} \le s - \frac{ks^2}{2} + \frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}N(\varsigma)\dot{\varsigma} + \dot{\varsigma} - \frac{\eta_1}{\eta_0}\widetilde{W}^T\hat{W} - \frac{\sigma_1}{\sigma_0}\tilde{b}\hat{b} + 0.2785(\delta_1+\delta_2) \tag{26}$$

According to $\widetilde{W} = W^* - \hat{W}$, $\tilde{b} = b^* - \hat{b}$ and Young's inequality, we have

$$-\frac{\eta_1}{\eta_0}\widetilde{W}^T\hat{W} \le -\frac{\eta_1}{2\eta_0}||\widetilde{W}||^2 + \frac{\eta_1}{2\eta_0}||W^*||^2, \quad -\frac{\sigma_1}{\sigma_0}\tilde{b}\hat{b} \le -\frac{\sigma_1}{2\sigma_0}||\tilde{b}||^2 + \frac{\sigma_1}{2\sigma_0}||b^*||^2 \tag{27}$$

Applying (27) to (26), one has

$$\begin{aligned}\dot{V} \le & -\frac{ks^2}{2} + \frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}N(\varsigma)\dot{\varsigma} + \dot{\varsigma} - \frac{\eta_1}{2\eta_0}||\widetilde{W}||^2 - \frac{\sigma_1}{2\sigma_0}||\tilde{b}||^2 + \frac{\eta_1}{2\eta_0}||W^*||^2 \\ & + \frac{\sigma_1}{2\sigma_0}||b^*||^2 + 0.2785(\delta_1+\delta_2)\end{aligned} \tag{28}$$

Furthermore

$$\dot{V} \le -c_1 V + [g(x,u_\lambda)/|g(x,u_\lambda)|]\,N(\varsigma)\dot{\varsigma} + \dot{\varsigma} + c_2 \tag{29}$$

where $c_1 = \min\{kg_0, \eta_1, \sigma_1\}$, $c_2 = \eta_1/2\eta_0||W^*||^2 + \sigma_1/2\sigma_0||b^*||^2 + 0.2785(\delta_1+\delta_2)$.

Furthermore, one has

$$\begin{aligned}V(t) \le & \frac{c_2}{c_1} + \left[V(0) - \frac{c_2}{c_1}\right]e^{-c_1 t} + e^{-c_1 t}\int_0^t \frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}N(\varsigma)\dot{\varsigma}e^{c_1 t}d\tau + e^{-c_1 t}\int_0^t \dot{\varsigma}e^{c_1 t}d\tau \\ \le & \frac{c_2}{c_1} + V(0) + e^{-c_1 t}\int_0^t \left[\frac{g(x,u_\lambda)}{|g(x,u_\lambda)|}N(\varsigma) + 1\right]\dot{\varsigma}e^{c_1 t}d\tau\end{aligned} \tag{30}$$

According to Lemma 2.3, obviously, $V(t)$, ς, $\hat{W}$ and $\hat{b}$ are bounded in $[0, t_f)$. From [5], the conclusion is also right as $t_f = +\infty$, on the other hand $s^2/2g_1 \le V_s(t) \le V(t)$, and then we have $s \in L_\infty$. From (29) we obtain $\varepsilon(t)$ is bounded; furthermore we have $e(t)$ is bounded. Thus, it can be shown that all the signals are bounded. Furthermore, according to the properties of $S(\varepsilon)$ and $\rho(t)$, we have tracking error satisfies the prescribed performance.

5. **Simulation Results.** In this section, consider the dynamics of autonomous underwater vehicle (AUV), from the simulations to demonstrate the theoretical results. In this paper, we consider the FTC of steering control subsystem. As [12], the steering subsystem can be represented as

$$\begin{cases}\dot{\psi} = r \\ \dot{r} = f(r) + g\delta_r + h(t)\end{cases} \tag{31}$$

where ψ is the steering angle, r is the yaw velocity, g is a design parameter, δ_r is the rudder deflection, and $h(t)$ is a bounded signal, which denotes the modeling error or external disturbance. In order to investigate the problem of actuator unmodeled fault, the fault system may be written as

$$\begin{cases}\dot{\psi} = r \\ \dot{r} = 2r|r| + 0.5r + (1 - 0.5\sin(r))\delta_r + rz\end{cases} \tag{32}$$

where $f(r) = 2r|r| + 0.5r$, the unmodeled fault is $f(r, \delta_r) = (1 - 0.5\sin(r))\delta_r$, system unmodeled dynamic is rz, and δ_r is the system input. The reference signal $y_d(t)$ is chosen as $y_d(t) = 0.5\sin(2t)$, performance function $\rho(t)$ is chosen as $\rho(t) = (1-10^{-1})e^{(-3t)}+10^{-1}$, and the Nussbaum function is chosen as $N(\varsigma) = \varsigma^2\cos(\varsigma)$. The initial conditions: $\psi(0) = 0.1$, $r(0) = 0.5$. The design parameters are taken as $\sigma_0 = 0.5$, $\sigma_1 = 0.01$, $\eta_0 = 0.5$, $\eta_1 = 0.01$, $\lambda = 0.25$, $k = 10$. Simulation results are shown in Figures 1-4. We can observe that tracking errors eventually converge to a prescribed range as shown in Figure 2. Meanwhile, Figure 3 illustrates the boundedness of the control signal. Simulation results demonstrate the feasibility of the proposed method.

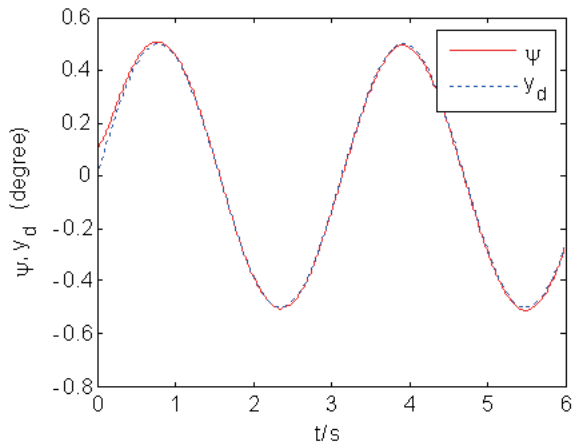

FIGURE 1. Output and desired trajectory

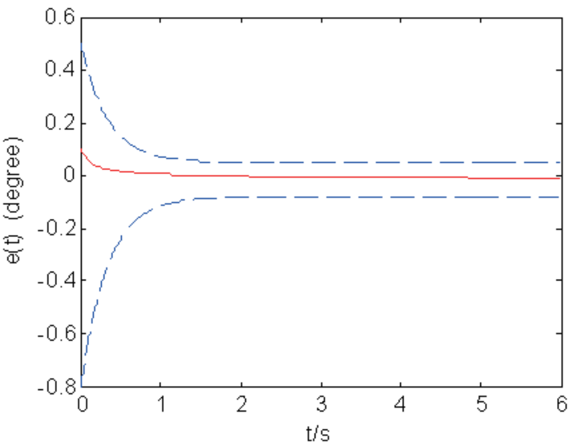

FIGURE 2. Tracking error

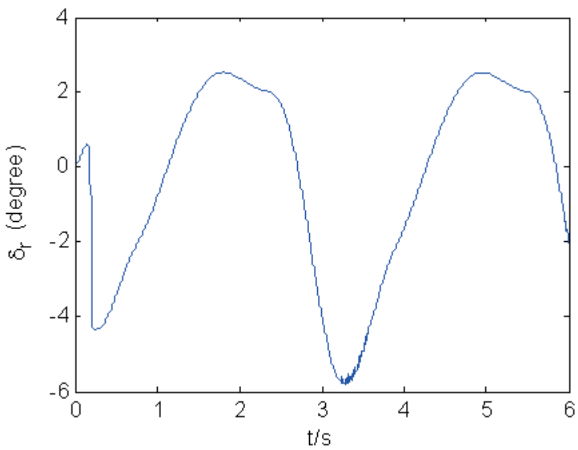

FIGURE 3. Control signal

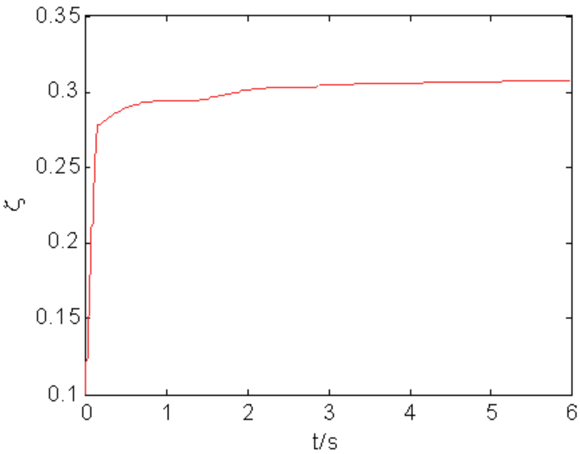

FIGURE 4. Nussbaum parameter

6. **Conclusions.** In this paper, for a class of nonlinear systems with actuator failures and unmodeled dynamics, considering the unmodeled fault, by applying prescribed performance function, sliding mode control, radial basis function neural networks and Nussbaum-type gain technique, an adaptive fault tolerant control scheme is designed. Theoretical analysis shows that the closed-loop system is semi-globally uniformly bounded, and the tracking error is confined to a predefined residual set. However, in this paper, the system

structure we consider is a lower triangular form. When the system structure is more complex, the design of fault tolerant controller is more challenging and the problem deserves further research.

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