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STATE FEEDBACK CONTROL FOR UNCERTAIN NETWORKED CONTROL SYSTEMS WITH DISTURBANCES

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Abstract. *In this paper, a model for uncertain networked control systems with disturbances and short random delays is studied, and a state feedback controller is investigated. First, a model for uncertain networked control systems with disturbances is proposed. The uncertainty of transmission delay is translated into the uncertainty of coefficient matrix in this system, and the uncertainty parameter is described by a Poisson distribution function. Then, by using the Lyapunov stability theory and linear matrix inequality, the stability of this system with disturbances is proved. A numerical example with two initial conditions case is given where the optimal solution in different states by LMI toolbox of Matlab is solved. Simulation results show the rationality and applicability of the proposed method.*
Keywords: Uncertain networked control systems, State feedback controller, Linear matrix inequality, Poisson distribution

1. Introduction.

As is well known, networked control system (NCS) is a feedback control system which is a closed loop composition of communication networks. NCS is a control system which uses a network as a communication medium to connect the plant and the controller [1,2]. The systems are sensing and exchanging control information by sharing wired or wireless network communications. So the NCSs have more advantages as less connections, low cost, great flexibility compared with traditional direct point-to-point control systems [3-5]. Therefore, the NCSs are widely used in automobile manufacturing, robot, vehicles control systems, etc. With the development of networking communication technologies, NCSs are becoming more and more popular. In the past few years, more and more researchers have paid attention to the study of stability analysis, state feedback controller analysis and filter design of NCSs [6,7].

The most significant negative factors that affect NCSs performance are transmission delays, quantisation and data packet dropouts [8-10]. Most of the analysis about distributed control systems assumed that the probability of transmission delays is known, but as we all know, the transmission delays are random and different to certainty [11-13]. On the other hand, because of the changing uncertain coefficient matrix, it is usually impossible to study NCS thoroughly. Therefore, the study about transmission delays of uncertain networked control systems (UNCSs) has received considerable attention over the past years [14-16]. Wang et al. [17] addressed the issue of providing guaranteed transient performance and robustness for a class of adaptive architectures of UNCSs. By using the Lyapunov stability theory, the uncertainty parameter model of networked control systems with long time delays is analyzed, and the robust control law is proposed by Liu et al. [18]. An approach to design predictive control for UNCSs' robust distributed model with time delays was addressed by Zhang et al. [19]. A guaranteed cost control problem of UNCSs with random networked delay and packet dropouts was presented by Du et al. [20]. Xie et al. [21] considered a fault-tolerant controller for UNCSs with packet dropouts and actuator fault and adopted a more practical and general model of actuator gain faults. By using the Lyapunov-Razumikhin method, Huang and Nguang [22] studied the robust stabilization and disturbance attenuation for a class of UNCSs with random communication network-induced delays. The output feedback guaranteed cost control problem of UNCSs with random packet dropouts and transmission delay was investigated by Qiu et al. [23].

Poisson distribution is a discrete probability distribution widely used in statistics and probability theory. It is suited to describe the number of random events in the unit time [24,25]. So, the uncertainty parameter is described by a Poisson distribution function in this paper. The transmission delay's uncertainty is translated into the uncertainty of system's coefficient matrix, in the time varying model of the UNCSs. After that, by using the Lyapunov stability theory and linear matrix inequality, the stabilization problem of UNCSs is solved.

In this paper, motivated by the above-mentioned works on predictive control and guaranteed cost control of UNCS, we are mainly concerned with the problem of asymptotically stability and state feedback controller for a class of discrete-time UNCS. It involves transmission delays and external disturbances at the same time. Inspired by the performance of Poisson distribution, we investigate the uncertain coefficient matrix of UNCS through the discrete Poisson distribution. The analysis method presented here has the following contributions compared with the existing ones. 1) A discrete-time UNCSs model with transmission delays and external disturbances is proposed, what is more, the disturbances are described as Gaussian white noise. 2) A state feedback controller is derived and the discrete time UNCS is asymptotically stable while transmission delays have upper bound. 3) In the process of proof, a new variable $e(k)$ is introduced, which gives us a new idea to simplify our analysis. 4) By employing the mathematical method of discrete probability distribution, two uncertain coefficient matrices are described by a Poisson distribution function in the simulation.

This paper is organized as follows. The discrete-time UNCS model is investigated in Section 2. By exploring the linear matrix inequality, two theorems about state feedback controller design are presented in Section 3. The numerical example with two different initial conditions is discussed in Section 4. Section 5 concludes this paper.

2. **The Model of Discrete-Time System.** The NCSs structure is depicted as Figure 1. From Figure 1, we can see that sensor-to-controller and actuator-to-controller are connected by network. There are induced delays in networks. In this system, the sensor and actuator are time-triggered. In other words, sensor and actuator sample the data

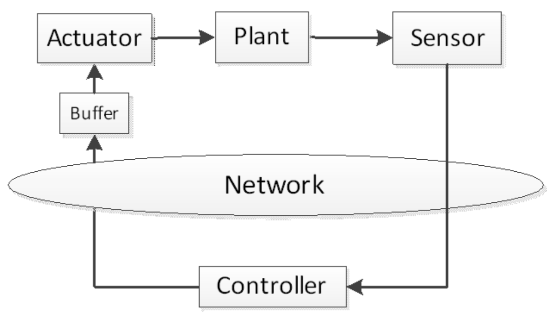

FIGURE 1. The structure of NCS

according to a sampling period. What is more, controller is event-triggered, when it accepts the data from sensor, the controller will begin some operations at once.

The discrete time system with time delays is

$$\begin{cases} x(k+1) = (A + \Delta A(k))x(k) + (B + \Delta B(k))u(k) + Cw(k) \\ y(k) = Lx(k) + C_1 v(k) \end{cases} \quad (1)$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^n$ is the control input vector, $y(k) \in R^n$ is the ideal control output vector, $w(k)$, $v(k)$ are the disturbance matrices, and both of the two disturbance matrices are mutually independent. A, B, C, C_1, L are the known appropriate dimensions matrices. $\Delta A(k)$, $\Delta B(k)$ are the time-varying unknown matrices with appropriate dimensions, satisfying the following forms:

$$\begin{bmatrix} \Delta A(k) & \Delta B(k) \end{bmatrix} = DF(k) \begin{bmatrix} H_1 & H_2 \end{bmatrix} \quad (2)$$

where D, H_1, H_2 are known appropriate dimensions matrices, $F(k)$ is an unknown matrix, but it is Lebesque measurable and satisfies $F^T(k)F(k) \leq I$.

Design the state feedback controller as:

$$u(k) = Kx(k) \quad (3)$$

where K is the gain matrix needed to be designed.

Combining (1) and (3) yields:

$$x(k+1) = (A + \Delta A(k))\, x(k) + (B + \Delta B(k))Kx(k) + Cw(k) \quad (4)$$

Let $\tau(k)$ denote the signal transmission delays at time instant k. d_M denotes the largest allowable delay of $\tau(k)$, $d_M = \max(\tau(k))$, and then we can obtain:

$$0 \leq \tau(k) \leq d(k) \leq d_M \leq T$$

where $d(k) = [\tau(k)]$. $[\tau(k)]$ denotes that it will always round down $\tau(k)$ to the nearest whole unit. T is the sampling periods.

Thinking $e(k) = x(k) - x(k - d(k))$, and combining with (4), obtain:

$$x(k+1) = (A+\Delta A(k))x(k)+(B+\Delta B(k))Kx(k-d(k))+(B+\Delta B(k))Ke(k)+Cw(k) \quad (5)$$

So, for all positive integers k, we have

$$e^T(k)\Omega e(k) \leq \mu x^T(k)\Omega x(k) \quad (6)$$

where Ω is a positive weighting matrix, and $\mu \in [0, 1)$.

Proof: Because $0 \leq \tau(k) \leq d(k) \leq T$, we have $|x(k) - x(k - d(k))| \leq |x(k)|$.

So, for some $\mu \in [0, 1)$ and positive weighting matrix Ω, we have:

$$e^T(k)\Omega e(k) \leq \mu x^T(k)\Omega x(k)$$

Thus, the result is proved.

In order to analyze our model better, the following remark is needed.

Remark 2.1. $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ *simplifies the symmetric matrix* $\begin{bmatrix} A & B^T \\ B & C \end{bmatrix}$. *In other words, we use* $*$ *to represent a term that is induced by symmetry, in the block symmetric matrice.*

3. **State Feedback Controller Design.** In order to analyze the UNCSs performance, some lemmas and theorems are given in the next.

Lemma 3.1. [26]: *Let $\sum_1$, $\sum_2$ and $\sum_3$ be real matrices of appropriate dimensions, with $\sum_1 = \sum_1^T$, the $\sum_1 + \sum_2 \Delta(k) \sum_3 + \sum_3^T \Delta^T(k) \sum_2^T < 0$ holds for all $\Delta(k)$ satisfying $\Delta(k)\Delta^T(k) \leq I$ if and only if for some $\varepsilon > 0$,*

$$\sum\nolimits_1 + \varepsilon^{-1} \sum\nolimits_2 \sum\nolimits_2^T + \varepsilon \sum\nolimits_3^T \sum\nolimits_3 < 0 \tag{7}$$

Lemma 3.2. [27]: **(Schur-complement of matrix):**

Let $Z_1 = Z_1^T$, $0 < Z_2 = Z_2^T$ and Z_3 be real matrices of appropriate dimensions, and then $Z_1 + Z_3^T Z_2^{-1} Z_3 < 0$, if and only if $\begin{bmatrix} Z_1 & Z_3^T \\ Z_3 & -Z_2 \end{bmatrix} < 0$ or $\begin{bmatrix} -Z_2 & Z_3 \\ Z_3^T & Z_1 \end{bmatrix} < 0$.

From [28], we can consider a system without uncertain factor firstly, then to analyze the performances of the systems which have certain parameters, and finally we will analyze the UNCSs performance.

Then, considering the system (1) as a system with certain factor, Equation (5) can be rewritten as:

$$x(k+1) = \mathcal{A}x(k) + \mathcal{B}Kx(k - d(k)) + \mathcal{B}Ke(k) + Cw(k) \tag{8}$$

where $\mathcal{A} = A + \Delta A(k)$, $\mathcal{B} = B + \Delta B(k)$, considering there are appropriate dimensions matrices $\mathcal{A}$, $\mathcal{B}$, without uncertain factors $\Delta A(k)$, $\Delta B(k)$.

Firstly, let us analyze the stability about the disturbance systems without considering the parameter uncertainties.

Theorem 3.1. *For given parameter $\mu > 0$, d_M and state feedback gain K, system (1) without considering parameter uncertainties is asymptotically stable if there exist real matrices $P > 0$, $Q > 0$, $R > 0$, $M > 0$, $\Omega > 0$ and X, Y with appropriate dimensions and $\varepsilon > 0$ satisfying that*

$$\begin{bmatrix} \begin{matrix} P^{-1}(Q - P \\ +X + X^T \\ +d_M M)P^{-1} \end{matrix} & * & * & * & * & * & * \\ \begin{matrix} P^{-1}(Y^T \\ -X^T)P^{-1} \end{matrix} & \mu P^{-1}\Omega P^{-1} & * & * & * & * & * \\ -P^{-1}Y^T P^{-1} & 0 & -P^{-1}QP^{-1} & * & * & * & * \\ 0 & 0 & 0 & -P^{-1}\Omega P^{-1} & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ \mathcal{A}P^{-1} & \mathcal{B}KP^{-1} & 0 & \mathcal{B}KP^{-1} & CP^{-1} & -P^{-1} & * \\ \begin{matrix} \sqrt{d_M}(\mathcal{A}P^{-1} \\ -P^{-1}) \end{matrix} & \sqrt{d_M}\mathcal{B}KP^{-1} & 0 & \sqrt{d_M}\mathcal{B}KP^{-1} & \sqrt{d_M}CP^{-1} & 0 & -R^{-1} \end{bmatrix} < 0$$

$$\begin{bmatrix} M & Y \\ Y^T & R \end{bmatrix} \geq 0, \quad \begin{bmatrix} M & X \\ X^T & R \end{bmatrix} \geq 0$$

Proof: Construct the Lyapunov-Krasovskii function as:

$$V(k) = V_1 + V_2 + V_3 \tag{9}$$

where

$$
\begin{aligned}
V_1 &= x^T(k)Px(k) \\
V_2 &= \sum_{s=k-d_M}^{k-1} x^T(s)Qx(s) \\
V_3 &= \sum_{s=-d_M}^{-1} \sum_{l=k+s}^{k-1} \delta^T(l)R\delta(l), \quad \delta(k) = x(k+1) - x(k)
\end{aligned} \tag{10}
$$

where P, Q, R are positive-definite matrices with appropriate dimensions. $\delta(k)$ denotes the difference between two adjacent inputs. Taking the derivative of the Lyapunov function (11) along the solution of Equation (8), the increment of $V(k)$ is given by:

$$
\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \tag{11}
$$

where

$$
\begin{aligned}
\Delta V_1 &= x^T(k+1)Px(k+1) - x^T(k)Px(k) \\
&= [\mathcal{A}x(k) + \mathcal{B}Kx(k-d(k)) + \mathcal{B}Ke(k) + Cw(k)]^T P[\mathcal{A}x(k) \\
&\quad + \mathcal{B}Kx(k-d(k)) + \mathcal{B}Ke(k) + Cw(k)] - x^T(k)Px(k) \\
\Delta V_2(k) &= x^T(k)Qx(k) - x^T(k-d_M)Qx(k-d_M) \\
\Delta V_3(k) &= d_M\delta^T(k)R\delta(k) - \sum_{l=k-d_M}^{k-1} \delta^T(l)R\delta(l)
\end{aligned}
$$

Combining $e(k) = x(k) - x(k-d(k))$, $e^T(k)\Omega e(k) \le \mu x^T(k)\Omega x(k)$, $\sum_{j=k-d(k)}^{k-1} \delta(j) = \sum_{j=k-d(k)}^{k-1} (x(j+1) - x(j)) = x(k) - x(k-d(k))$ and $\sum_{j=k-d_M}^{k-1} \delta(j) = \sum_{j=k-d_M}^{k-1} (x(j+1) - x(j)) = x(k) - x(k-d_M)$, we can obtain that

$$
\begin{aligned}
\Delta V(k) &= \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) \\
&< \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + \mu x^T(k)\Omega x(k) - e^T(k)\Omega e(k) \\
&\quad + 2\xi^T(k)X\left[x(k) - x(k-d(k)) - \sum_{j=k-d(k)}^{k-1} \delta(j)\right] \\
&\quad + 2\xi^T(k)Y\left[x(k) - x(k-d_M) - \sum_{j=k-d_M}^{k} \delta(j)\right] \\
&\quad + d_M\xi^T(k)M\xi(k) - \xi^T(k)M\xi(k)
\end{aligned} \tag{12}
$$

This structure ensures the stability of the discrete time-delay system (8), where, $M > 0$, X and Y have appropriate dimensions, and

$$
\xi^T(k) = \begin{bmatrix} x^T(k) & x^T(k-d(k)) & x^T(k-d_M) & e^T(k) & w^T(k) \end{bmatrix}
$$

Denote the augmented state $\varsigma(k,l)$ as $\varsigma(k,l) = \begin{bmatrix} \xi(k) \\ \delta(l) \end{bmatrix}$.

Then Equation (12) can be simplified as:

$$\Delta V(k) < \xi^T(k) \left\{ \begin{bmatrix} Q - P + d_M M & * & * & * \\ 0 & \mu\Omega & * & * \\ 0 & 0 & -Q & * \\ 0 & 0 & 0 & -\Omega \end{bmatrix} + \begin{bmatrix} X & Y - X & -Y & 0_{5n\times 2n} \end{bmatrix} \right.$$
$$+ \begin{bmatrix} X^T \\ Y^T - X^T \\ -Y^T \\ 0_{2n\times 5n} \end{bmatrix} + \begin{bmatrix} P\mathcal{A} & P\mathcal{B}K & 0 & P\mathcal{B}K & PC \end{bmatrix}^{-T} P^{-1}$$
$$\begin{bmatrix} P\mathcal{A} & P\mathcal{B}K & 0 & P\mathcal{B}K & PC \end{bmatrix} + d_M \begin{bmatrix} R\mathcal{A} - R & R\mathcal{B}K & 0 & R\mathcal{B}K & RC \end{bmatrix}^{-T}$$
$$\left. R^{-1} \begin{bmatrix} R\mathcal{A} - R & R\mathcal{B}K & 0 & R\mathcal{B}K & RC \end{bmatrix} \right\} \xi(k)$$
$$- \sum_{l=k-d_M}^{k} \varsigma^T(k,l) \begin{bmatrix} M & Y \\ Y^T & R \end{bmatrix} \varsigma(k,l) - \sum_{l=k-d_M}^{k} \varsigma^T(k,l) \begin{bmatrix} M & X \\ X^T & R \end{bmatrix} \varsigma(k,l) \tag{13}$$

Next, Equation (13) can be simplified as:

$$\Delta V(k) \le \xi^T(k) \left\{ \Pi + \Gamma + \Gamma^T + \Psi_{21}^{-T} P^{-1} \Psi_{21} + d_M \Psi_{31}^{-T} R^{-1} \Psi_{31} \right\} \xi(k)$$
$$- \sum_{l=k-d_M}^{k} \varsigma^T(k,l) \begin{bmatrix} M & Y \\ Y^T & R \end{bmatrix} \varsigma(k,l) - \sum_{l=k-d_M}^{k} \varsigma^T(k,l) \begin{bmatrix} M & X \\ X^T & R \end{bmatrix} \varsigma(k,l)$$

where

$$\Pi = \begin{bmatrix} Q - P + d_M M & * & * & * & * \\ 0 & \mu\Omega & * & * & * \\ 0 & 0 & -Q & * & * \\ 0 & 0 & 0 & -\Omega & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} X & Y - X & -Y & 0_{5n\times 2n} \end{bmatrix}$$

$$\Psi_{21} = \begin{bmatrix} P\mathcal{A} & P\mathcal{B}K & 0 & P\mathcal{B}K & PC \end{bmatrix}, \quad \Psi_{31} = \begin{bmatrix} R\mathcal{A} - R & R\mathcal{B}K & 0 & R\mathcal{B}K & RC \end{bmatrix}$$

So, if $\Delta V(k) \le 0$, then we have

$$\Pi + \Gamma + \Gamma^T + \Psi_{21}^{-T} P^{-1} \Psi_{21} + d_M \Psi_{31}^{-T} R^{-1} \Psi_{31} < 0 \tag{14}$$

and

$$\begin{bmatrix} M & Y \\ Y^T & R \end{bmatrix} \ge 0, \quad \begin{bmatrix} M & X \\ X^T & R \end{bmatrix} \ge 0 \tag{15}$$

are all satisfied.

Using Schur-complement with Equation (14), it is translated into:

$$\begin{bmatrix} \Pi + \Gamma + \Gamma^T & * & * \\ \Psi_{21} & -P & * \\ \sqrt{d_M}\Psi_{31} & 0 & -R \end{bmatrix} < 0 \tag{16}$$

Then, define a matrix as,

$$G=\begin{bmatrix} G_1 & & \\ & P^{-1} & \\ & & R^{-1}\end{bmatrix},\ \text{where } G_1=\begin{bmatrix} P^{-1} & * & * & * & * \\ 0 & P^{-1} & * & * & * \\ 0 & 0 & P^{-1} & * & * \\ 0 & 0 & 0 & P^{-1} & * \\ 0 & 0 & 0 & 0 & P^{-1}\end{bmatrix}.$$

From the above matrix inequality (16), we have

$$\begin{bmatrix} G_1 & & \\ & P^{-1} & \\ & & R^{-1}\end{bmatrix}\cdot\begin{bmatrix} \Pi+\Gamma+\Gamma^T & * & * \\ \Psi_{21} & -P & * \\ \sqrt{d_M}\Psi_{31} & 0 & -R\end{bmatrix}\cdot\begin{bmatrix} G_1 & & \\ & P^{-1} & \\ & & R^{-1}\end{bmatrix}<0$$

Then, it can be rewritten as

$$\begin{bmatrix} \begin{matrix}P^{-1}(Q-P\\+X+X^T\\+d_MM)P^{-1}\end{matrix} & * & * & * & * & * & * \\ \begin{matrix}P^{-1}(Y^T\\-X^T)P^{-1}\end{matrix} & \mu P^{-1}\Omega P^{-1} & * & * & * & * & * \\ -P^{-1}Y^TP^{-1} & 0 & -P^{-1}QP^{-1} & * & * & * & * \\ 0 & 0 & 0 & -P^{-1}\Omega P^{-1} & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ \mathcal{A}P^{-1} & \mathcal{B}KP^{-1} & 0 & \mathcal{B}KP^{-1} & \mathcal{C}P^{-1} & -P^{-1} & * \\ \begin{matrix}\sqrt{d_M}(\mathcal{A}P^{-1}\\-P^{-1})\end{matrix} & \sqrt{d_M}\mathcal{B}KP^{-1} & 0 & \sqrt{d_M}\mathcal{B}KP^{-1} & \sqrt{d_M}\mathcal{C}P^{-1} & 0 & -R^{-1}\end{bmatrix}<0 \tag{17}$$

From all above, Equation (17) is an LMI. After finding the LMI solutions, Equation (14) can be rewritten as $\Delta V\leq-\varepsilon\left\|\xi(k)\right\|^2$, and $\varepsilon>0$ can be any small.

Finally, from the LMI of Equation (17) with P, Q, R, M, Ω, we can get $\Delta V\leq-\varepsilon\left\|\xi(k)\right\|^2$ with $\varepsilon>0$. Therefore, the system (8) without considering parameter uncertainties is asymptotically stable. Thus, Theorem 3.1 is proved. Theorem 3.1 provides sufficient condition for state feedback controller design with closed loop UNCS. Then, we will analyze the performances of UNCS (1), and the state feedback controller gains with UNCS will be designed.

Theorem 3.2. *Consider the UNCS system (1). For given parameters $\mu>0$ and d_M, there exists a state-feedback controller $u(k)=Kx(k)$, the UNCS is asymptotically stable if there exist matrices $\widetilde{Q}>0$, $S_i>0$ $(i=1,2)$, $\widetilde{M}>0$, $\widetilde{\Omega}>0$, $\widetilde{X}$, $\widetilde{Y}$, $\widetilde{K}$ with appropriate dimensions and a scalar $\varepsilon>0$ satisfying the following LMIs:*

$$\begin{bmatrix} \widetilde{\Pi}+\widetilde{\Gamma}+\widetilde{\Gamma}^T & * & * & * \\ \Xi_{21} & -S_1+\varepsilon DD^T & * & * \\ \Xi_{31} & 0 & -S_2+\varepsilon d_MDD^T & * \\ \Xi_{41} & 0 & 0 & -\varepsilon I\end{bmatrix}<0 \tag{18}$$

$$\begin{bmatrix} \widetilde{M} & \widetilde{Y} \\ \widetilde{Y}^T & 2S_1-S_2\end{bmatrix}\geq0 \qquad \begin{bmatrix} \widetilde{M} & \widetilde{X} \\ \widetilde{X}^T & 2S_1-S_2\end{bmatrix}\geq0 \tag{19}$$

where

$$\widetilde{\Pi} = \begin{bmatrix} \widetilde{Q} - S_1 + d_M\widetilde{M} & * & * & * & * \\ 0 & \mu\widetilde{\Omega} & * & * & * \\ 0 & 0 & -\widetilde{Q} & * & * \\ 0 & 0 & 0 & -\widetilde{\Omega} & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{\Gamma} = \begin{bmatrix} \widetilde{X} & \widetilde{Y} - \widetilde{X} & -\widetilde{Y} & 0_{5n\times 2n} \end{bmatrix}, \quad \Xi_{21} = \begin{bmatrix} AS_1 & B\widetilde{K} & 0 & B\widetilde{K} & CS_1 \end{bmatrix}$$

$$\Xi_{31} = \sqrt{d_M}\begin{bmatrix} AS_1 - S_1 & B\widetilde{K} & 0 & B\widetilde{K} & CS_1 \end{bmatrix}, \quad \Xi_{41} = \begin{bmatrix} H_1S_1 & H_2\widetilde{K} & 0 & H_2\widetilde{K} & 0 \end{bmatrix}$$

Moreover, if the above conditions are feasible, a desired controller gain matrix is given by $K = \tilde{K}S_1^{-1}$.

Proof: Replace matrices $\mathcal{A}$, $\mathcal{B}$ with $A + \Delta A$, $B + \Delta B$ in Equation (16). Because $\begin{bmatrix} \Delta A(k) & \Delta B(k) \end{bmatrix} = DF(k)\begin{bmatrix} H_1 & H_2 \end{bmatrix}$, we have $\Delta A = DF(k)H_1$ and $\Delta B = DF(k)H_2$, and then Equation (16) can be rewritten as:

$$\begin{bmatrix} \begin{matrix} Q - P + X \\ +X^T + d_M M \end{matrix} & * & * & * & * & * & * \\ Y^T - X^T & \mu\Omega & * & * & * & * & * \\ -Y^T & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\Omega & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ PA + PDF(k)H_1 & \begin{matrix} PBK \\ +PDF(k)H_2K \end{matrix} & 0 & \begin{matrix} PBK \\ +PDF(k)H_2K \end{matrix} & PC & -P & * \\ \begin{matrix} \sqrt{d_M}(RA \\ +RDF(k)H_1 - R) \end{matrix} & \begin{matrix} \sqrt{d_M}(RBK \\ +RDF(k)H_2K) \end{matrix} & 0 & \begin{matrix} \sqrt{d_M}(RBK \\ +RDF(k)H_2K) \end{matrix} & \sqrt{d_M}RC & 0 & -R \end{bmatrix} < 0 \tag{20}$$

Equation (20) can be rewritten as:

$$\begin{bmatrix} Q - P + X + X^T + d_M M & * & * & * & * & * & * \\ Y^T - X^T & \mu\Omega & * & * & * & * & * \\ -Y^T & 0 & -Q & * & * & * & * \\ 0 & 0 & 0 & -\Omega & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ PA & PBK & 0 & PBK & PC & -P & * \\ \sqrt{d_M}(RA - R) & \sqrt{d_M}RBK & 0 & \sqrt{d_M}RBK & \sqrt{d_M}RC & 0 & -R \end{bmatrix}$$
$$+ \begin{bmatrix} 0 & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * \\ 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * \\ PDF(k)H_1 & PDF(k)H_2K & 0 & PDF(k)H_2K & 0 & 0 & * \\ \sqrt{d_M}RDF(k)H_1 & \sqrt{d_M}RDF(k)H_2K & 0 & \sqrt{d_M}RDF(k)H_2K & 0 & 0 & 0 \end{bmatrix} < 0$$

Then, it can be simplified as:

$$\begin{bmatrix} \Pi+\Gamma+\Gamma^T & * & * \\ \Psi'_{21} & -P & * \\ \sqrt{d_M}\Psi'_{31} & 0 & -R \end{bmatrix} + \begin{bmatrix} 0 & * & * \\ \Psi''_{21} & 0 & * \\ \sqrt{d_M}\Psi''_{31} & 0 & 0 \end{bmatrix} < 0 \tag{21}$$

where

$$\Pi = \begin{bmatrix} Q-P+d_M M & * & * & * & * \\ 0 & \mu\Omega & * & * & * \\ 0 & 0 & -Q & * & * \\ 0 & 0 & 0 & -\Omega & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} X & Y-X & -Y & 0_{5n\times 2n} \end{bmatrix}$$

$$\Psi'_{21} = \begin{bmatrix} PA & PBK & 0 & PBK & PC \end{bmatrix},$$

$$\Psi'_{31} = \begin{bmatrix} RA-R & RBK & 0 & RBK & RC \end{bmatrix},$$

$$\Psi''_{21} = \begin{bmatrix} PDF(k)H_1 & PDF(k)H_2K & 0 & PDF(k)H_2K & 0 \end{bmatrix}$$

$$\Psi''_{31} = \begin{bmatrix} RDF(k)H_1 & RDF(k)H_2K & 0 & RDF(k)H_2K & 0 \end{bmatrix}$$

Let $\Sigma_1 = \begin{bmatrix} \Pi+\Gamma+\Gamma^T & * & * \\ \Psi'_{21} & -P & * \\ \sqrt{d_M}\Psi'_{31} & 0 & -R \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 0_{5n\times n} \\ PD \\ \sqrt{d_M}RD \end{bmatrix}$, $\Sigma_3 = [H_1 \quad H_2K \quad 0 \quad H_2K \quad 0_{n\times 3n}]$. Equation (21) can be further rewritten as:

$$\Sigma_1 + \Sigma_2 F(k)\Sigma_3 + \Sigma_3^T F^T(k)\Sigma_2^T < 0 \tag{22}$$

Combining it with Lemma 3.1, we have $\varepsilon > 0$ with

$$\Sigma_1 + \varepsilon\Sigma_2\Sigma_2^T + \varepsilon\Sigma_3^T\Sigma_3 < 0 \tag{23}$$

Let $J = \begin{bmatrix} J_1 & * & * \\ 0 & P^{-1} & * \\ 0 & 0 & R^{-1} \end{bmatrix}$, where $J_1 = \begin{bmatrix} P^{-1} & * & * & * & * \\ 0 & P^{-1} & * & * & * \\ 0 & 0 & P^{-1} & * & * \\ 0 & 0 & 0 & P^{-1} & * \\ 0 & 0 & 0 & 0 & P^{-1} \end{bmatrix}$.

Then, pre- and post-multiplying Equation (23) with J, we can obtain that $J\Sigma_1 J + \varepsilon J\Sigma_2\Sigma_2^T J + \varepsilon J\Sigma_3^T\Sigma_3 J < 0$, and because $J = J^T$, Equation (23) can be further rewritten as:

$$J\Sigma_1 J + \varepsilon J\Sigma_2 (J\Sigma_2)^T + \varepsilon (J\Sigma_3)^T J\Sigma_3 < 0$$

Combining it with Lemma 3.2, we have:

$$\begin{bmatrix} J\Sigma_1 J + \varepsilon J\Sigma_2 (J\Sigma_2)^T & (J\Sigma_3)^T \\ J\Sigma_3 & -\varepsilon I \end{bmatrix} < 0 \tag{24}$$

and

$$\begin{bmatrix} J_1 M J_1 & (P^{-1}YP^{-1})^T \\ P^{-1}YP^{-1} & P^{-1}RP^{-1} \end{bmatrix} \geq 0, \quad \begin{bmatrix} J_1 M J_1 & (P^{-1}XP^{-1})^T \\ P^{-1}XP^{-1} & P^{-1}RP^{-1} \end{bmatrix} \geq 0 \tag{25}$$

According to the above analysis, one can conclude that:

$$\begin{bmatrix} P^{-1}(Q-P+X+X^T+d_M M)P^{-1} & * & * & * & * & * & * & * \\ P^{-1}(Y^T-X^T)P^{-1} & \mu P^{-1}\Omega P^{-1} & * & * & * & * & * & * \\ -P^{-1}Y^T P^{-1} & 0 & -P^{-1}QP^{-1} & * & * & * & * & * \\ 0 & 0 & 0 & -P^{-1}\Omega P^{-1} & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * \\ AP^{-1} & BKP^{-1} & 0 & BKP^{-1} & CP^{-1} & -P^{-1}+\varepsilon DD^T & * & * \\ \sqrt{d_M}(AP^{-1}-P^{-1}) & \sqrt{d_M}BKP^{-1} & 0 & \sqrt{d_M}BKP^{-1} & \sqrt{d_M}CP^{-1} & 0 & -R^{-1}+\varepsilon d_M DD^T & * \\ H_1P^{-1} & H_2KP^{-1} & 0 & H_2KP^{-1} & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{26}$$

Defining some new variables $S_1 = P^{-1}$, $S_2 = R^{-1}$, $\tilde{Q} = P^{-1}QP^{-1}$, $\tilde{R} = P^{-1}RP^{-1}$, $\tilde{\Omega} = P^{-1}\Omega P^{-1}$, $\tilde{M} = J_1 M J_1$, $\tilde{\Gamma} = J_1 \Gamma J_1$, $\tilde{K} = KP^{-1}$, $\tilde{X} = P^{-1}XP^{-1}$, $\tilde{Y} = P^{-1}YP^{-1}$, Equation (26) can be simplified as:

$$\begin{bmatrix} \tilde{Q}-S_1+\tilde{X}+\tilde{X}^T+d_M\tilde{M} & * & * & * & * & * & * & * \\ \tilde{Y}^T-\tilde{X}^T & \mu\tilde{\Omega} & * & * & * & * & * & * \\ -\tilde{Y}^T & 0 & -\tilde{Q} & * & * & * & * & * \\ 0 & 0 & 0 & -\tilde{\Omega} & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & * & * & * \\ AS_1 & B\tilde{K} & 0 & B\tilde{K} & CS_1 & -S_1+\varepsilon DD^T & * & * \\ \sqrt{d_M}(AS_1-S_1) & \sqrt{d_M}B\tilde{K} & 0 & \sqrt{d_M}B\tilde{K} & \sqrt{d_M}CS_1 & 0 & -S_2+\varepsilon d_M DD^T & * \\ H_1S_1 & H_2\tilde{K} & 0 & H_2\tilde{K} & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{27}$$

Then, Equation (27) can be rewritten as:

$$\begin{bmatrix} \tilde{\Pi}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & * \\ \Xi_{21} & -S_1+\varepsilon DD^T & * & * \\ \Xi_{31} & 0 & -S_2+\varepsilon d_M DD^T & * \\ \Xi_{41} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0$$

where

$$\tilde{\Pi} = \begin{bmatrix} \tilde{Q}-S_1+d_M\tilde{M} & * & * & * & * \\ 0 & \mu\tilde{\Omega} & * & * & * \\ 0 & 0 & -\tilde{Q} & * & * \\ 0 & 0 & 0 & -\tilde{\Omega} & * \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma} = \begin{bmatrix} \tilde{X} & \tilde{Y}-\tilde{X} & -\tilde{Y} & 0_{5n\times 2n} \end{bmatrix}, \quad \Xi_{21} = \begin{bmatrix} AS_1 & B\tilde{K} & 0 & B\tilde{K} & CS_1 \end{bmatrix},$$

$$\Xi_{31} = \sqrt{d_M}\begin{bmatrix} AS_1-S_1 & B\tilde{K} & 0 & B\tilde{K} & CS_1 \end{bmatrix}, \quad \Xi_{41} = \begin{bmatrix} H_1S_1 & H_2\tilde{K} & 0 & H_2\tilde{K} & 0 \end{bmatrix}$$

From Equation (25), one can conclude that the order principal minor determinant of $\begin{bmatrix} \tilde{M} & \tilde{Y} \\ \tilde{Y}^T & S_1RS_1 \end{bmatrix}$ and $\begin{bmatrix} \tilde{M} & \tilde{X} \\ \tilde{X}^T & S_1RS_1 \end{bmatrix}$ is greater than zero.

Because $(S_2 - S_1)R(S_2 - S_1) = S_2 - 2S_1 + S_1RS_1 \geq 0$, we have

$$\begin{bmatrix} \tilde{M} & \tilde{Y} \\ \tilde{Y}^T & 2S_1 - S_2 \end{bmatrix} \geq 0, \quad \begin{bmatrix} \tilde{M} & \tilde{X} \\ \tilde{X}^T & 2S_1 - S_2 \end{bmatrix} \geq 0$$

In conclusion, we can obtain that:

$$\begin{bmatrix} \tilde{\Pi} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * \\ \Xi_{21} & -S_1 + \varepsilon DD^T & * & * \\ \Xi_{31} & 0 & -S_2 + \varepsilon d_M DD^T & * \\ \Xi_{41} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \quad \begin{bmatrix} \tilde{M} & \tilde{Y} \\ \tilde{Y}^T & 2S_1 - S_2 \end{bmatrix} \geq 0,$$

$\begin{bmatrix} \tilde{M} & \tilde{X} \\ \tilde{X}^T & 2S_1 - S_2 \end{bmatrix} \geq 0$ are all satisfied. Here, Equation (27) is an LMI, and it can ensure our closed loop system (1) is asymptotically stable. For given constants $\mu > 0$ and d_M, the state feedback controller $u(k) = Kx(k)$ with control law $K = \tilde{K}S_1^{-1}$ is designed after finding the LMI solutions. Therefore, the proof of Theorem 3.2 is completed.

4. **Numerical Example.**

4.1. **Example 1.**

A numerical example with two initial conditions which can justify the effectiveness of the presented method is given in this section. Considering a generalized discrete UNCS with transmission delays:

$$x(k+1) = \left(\begin{bmatrix} 0.8 & 0.2 & 0.4 \\ 0 & 0.6 & 0.8 \\ 0 & 0 & 0.4 \end{bmatrix} + \Delta A \right) x(k) + \left(\begin{bmatrix} -1.0 & 0.5 & 0 \\ -2.0 & -1.0 & 0 \\ -1.0 & 0.1 & 0.1 \end{bmatrix} + \Delta B \right) u(k)$$
$$+ \begin{bmatrix} 0.002 & 0.01 & 0 \\ 0 & 0.01 & 0.001 \\ 0.01 & 0.02 & 0.02 \end{bmatrix} w(k)$$

$$y(k) = \begin{bmatrix} 2.0 & 1.0 & 0 \\ 0 & 1.1 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} x(k) + \begin{bmatrix} 0.002 & 0.01 & 0 \\ 0 & 0.01 & 0.001 \\ 0.01 & 0.02 & 0.02 \end{bmatrix} v(k)$$

Choose as $\mu = 0.4$, $d_M = 3$, the disturbance matrices $w(k)$, $v(k)$ are described as Gaussian white noise. Because $\Delta A = DF(k)H_1$, $\Delta B = DF(k)H_2$, we choose:

$$D = \begin{bmatrix} 0.1 & 0.1 & 0 \\ 0 & 0.1 & 0.1 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.8 & 1.1 & 0.6 \\ 0.1 & 0.1 & 0.1 \\ 0 & 0 & 0.1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 1.0 & 0.1 & 0.5 \\ 0.1 & 0.3 & 0 \\ 0.2 & 0.3 & 0.2 \end{bmatrix}$$

and $F(k)$ is a discrete probability distribution function obeying Poisson distribution, $F(k) = \frac{\lambda^k}{k!}e^{-\lambda}$ where $\lambda = 0.9$. $F(k)$ denotes the number of random events in the unit time (space) at time instant k.

Then, by using the LMI toolbox to solve the LMI Equation (19) and Equation (27) in Theorem 3.2, the corresponding feedback gain K is obtained

$$K = \begin{bmatrix} -0.009 & -0.0133 & -0.0271 \\ 0.0432 & 0.0568 & 0.2177 \\ -0.1508 & -0.2270 & -0.3798 \end{bmatrix} \tag{28}$$

The simulation time is chosen as $k \in [0, 100]$, and we can choose two different initial conditions $X(0)$ to describe the proposed method.

The initial condition is assumed to be $X(0) = [\ -0.9 \quad 1.9 \quad 1.5\ ]^T$, and the variations of state responses $x(k)$ and control input $u(k)$ are shown as Figure 2 and Figure 3.

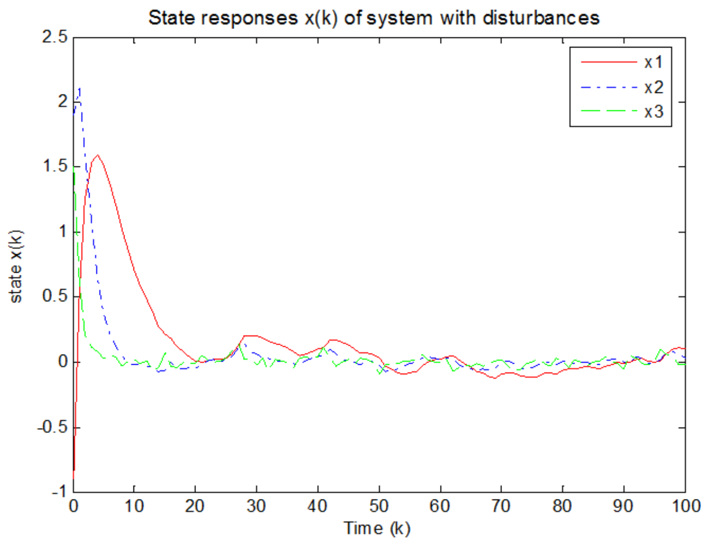

FIGURE 2. The different situations of state $x(k)$ with $X(0) = [-0.9 \quad 1.9 \quad 1.5]^T$

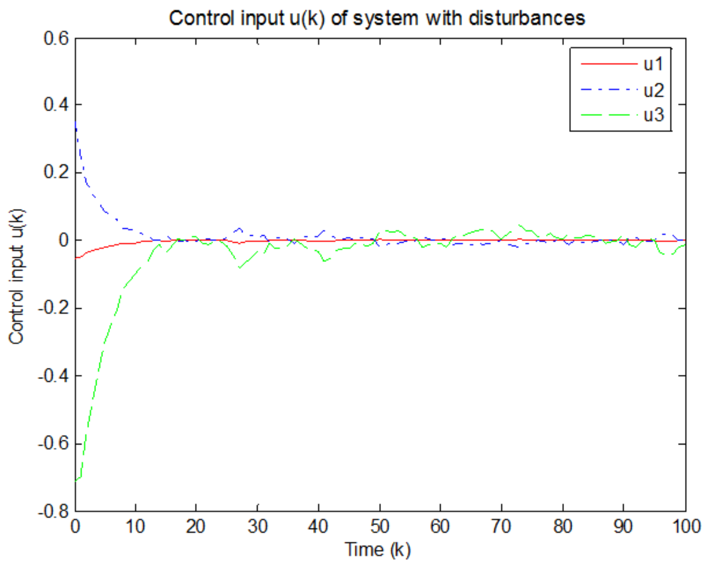

FIGURE 3. The different situations of control input $u(k)$ with $X(0) = [-0.9 \quad 1.9 \quad 1.5]^T$

The state responses of the UNCS with disturbances are shown in Figure 2. The control input responses are described in Figure 3 with the state feedback gain as in (28). It can be concluded that the system responses are expected with the initial condition $X(0) = [\ -0.9 \quad 1.9 \quad 1.5\ ]^T$.

4.2. **Example 2.** In this part, we put a contrast example which has the same condition with Example 1. We described the NCS with a certain model.

Consider a generalized discrete time-delay system with certain coefficient.

$$x(k+1) = \left(\begin{bmatrix} 0.8 & 0.2 & 0.4 \\ 0 & 0.6 & 0.8 \\ 0 & 0 & 0.4 \end{bmatrix} \right) x(k) + \left(\begin{bmatrix} -1.0 & 0.5 & 0 \\ -2.0 & -1.0 & 0 \\ -1.0 & 0.1 & 0.1 \end{bmatrix} \right) u(k)$$

$$+ \begin{bmatrix} 0.002 & 0.01 & 0 \\ 0 & 0.01 & 0.001 \\ 0.01 & 0.02 & 0.02 \end{bmatrix} w(k)$$

$$y(k) = \begin{bmatrix} 2.0 & 1.0 & 0 \\ 0 & 1.1 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} x(k) + \begin{bmatrix} 0.002 & 0.01 & 0 \\ 0 & 0.01 & 0.001 \\ 0.01 & 0.02 & 0.02 \end{bmatrix} v(k)$$

Then, by using the LMI toolbox to solve the LMI Equation (15) and Equation (17) in Theorem 3.1, and the initial condition $X(0) = [\ -0.9 \quad 1.9 \quad 1.5\ ]^T$, the variation of state $x(k)$ and control input $u(k)$ can be obtained as Figure 4 and Figure 5.

The state responses and control input response of the NCSs with disturbances are plotted in Figure 4 and Figure 5, respectively. From the simulation results, it can be concluded that the system becomes stable slowly and has more turbulence in the control input response than the turbulence produced in our method.

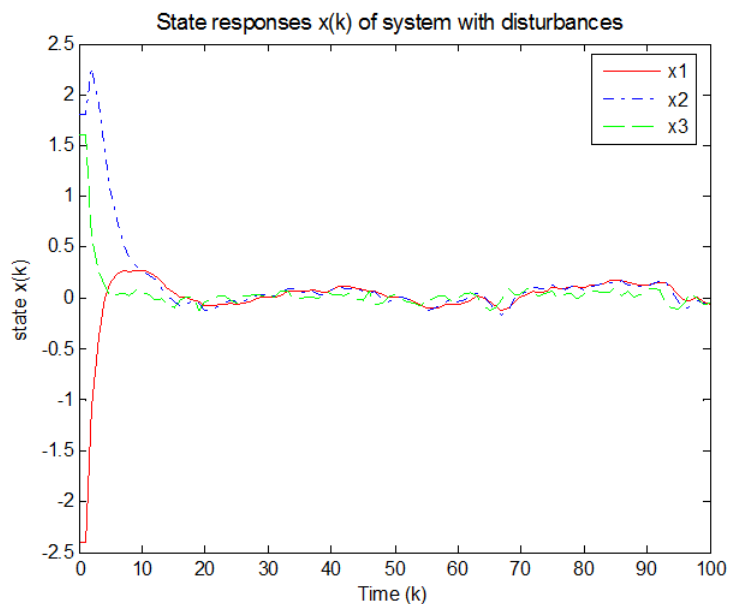

FIGURE 4. The NCS of state $x(k)$ with $X(0) = [-0.9 \quad 1.9 \quad 1.5]^T$

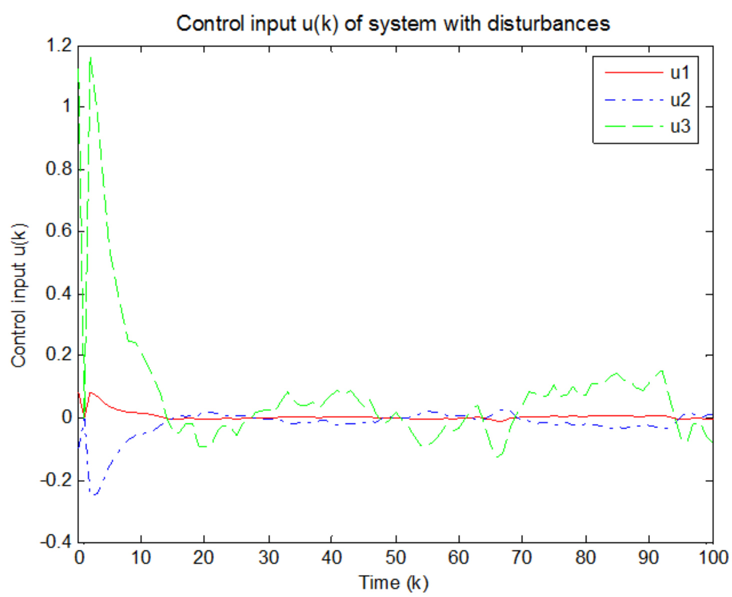

FIGURE 5. The NCS of control input $u(k)$ with $X(0) = [-0.9 \quad 1.9 \quad 1.5]^T$

Therefore, this study discusses the different states with $k \in [0, 100]$. On the whole, the system with the proposed method is asymptotically stable. From the contrast example, our new method can make the system become stable quickly and ensure the control input response steadier. So the results presented in our paper provided evidence for our new ways.

5. **Conclusion.** The model of UNCSs with short random delays and disturbances is investigated in this paper. First, a model for uncertain networked control systems with disturbances is introduced. The transmission delay's uncertainty is translated into the uncertainty in the UNCSs model. By using the Lyapunov stability theory, the stability of the model without uncertain factors is verified, and then the stability of the model with uncertain factors is testified further. Next, by using the LMI method, the state feedback controller is designed. It should be pointed out that the unknown matrix is described as the Poisson distribution function in the numerical simulation. Finally, the numerical example verified that this system is asymptotically stable, demonstrating the validity and availability of the presented method.

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