International Journal of Innovative Computing, Information and Control
Volume **13**, Number **3**, June **2017**
ICIC International ©2017 ISSN 1349-4198
pp. 767–782

FINITE-TIME H_∞ OUTPUT TRACKING CONTROL FOR A CLASS OF SWITCHED NEUTRAL SYSTEMS WITH MODE-DEPENDENT AVERAGE DWELL TIME METHOD

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Received November 2016; revised March 2017

Abstract. *This paper addresses the finite-time H_∞ output tracking control problem for a class of switched neutral systems. First, the finite-time stability (FTS) and the finite-time H_∞ problem for the augmented systems are investigated. By using the mode-dependent average dwell time (MDADT) method, sufficient conditions for finite-time boundedness and finite-time H_∞ performance of the augmented systems are derived. Second, based on the sufficient conditions derived in finite-time H_∞ performance analysis, and a state feedback controller is designed which makes the closed-loop output tracking systems be finite-time boundedness with some H_∞ performance level. Finally, a numerical example is given to illustrate our results.*
Keywords: Finite-time stability, Finite-time H_∞ output tracking control, Switched neutral systems, Mode-dependent average dwell time method

1. **Introduction.** Tracking control concerns the problem of designing of controllers that enables the target output of a system follows a reference signal. It has been extensively applied in aerospace control systems, robot control systems, signal processing systems and other practical systems. Many results have been reported on this issue, see [1-3], and the references therein. In [1], reliable robust flight tracking control problem for aircraft system is studied. Based on the multi objective robust performance analysis, a controller including tracking error integral action is developed to against actuator faults and control surface impairment. Based on an optimal control approach, robust tracking controls for uncertain linear systems are studied in [2]. The output tracking control of switched systems with time-varying delay under asynchronous switching is investigated in [3] and a new Lyapunov function dependent on the controllers' switching signal is constructed, which can effectively counteract the difficulty of controller design to achieve tracking objective under asynchronous switching.

To the best of our knowledge, tracking control problem has received increasing attention in the last few years. A considerable amount of results have been reported in the literature. However, most of the results in this field concern the tracking control problem over an infinite-time interval. In this sense it appears reasonable to research the finite-time H_∞ tracking control problem which concerns the stability and the H_∞ output tracking performance of the system over a fixed finite-time interval.

In order to better study the finite-time H_∞ tracking control problem, it is necessary to have a grasp of the overall information of finite-time stability (FTS) and finite-time H_∞ control problem. FTS, which was proposed by Dorato in 1961 [4], is a practical stability concept of considering the transient stability performance of a system. Due to its good robustness and anti-disturbance performance, FTS problem has attracted widespread attention in the past several decades. Achievements emerged for different systems endlessly in the light of the development of Lyapunov theory and linear matrix inequalities (LMIs) theory, see [5-7] and the references therein. The H_∞ control problem concerns the problem of the suppression of disturbance and uncertainties in particular systems. Finite-time H_∞ control problem concerns both the FTS and the H_∞ control problem defined over a finite-time interval. It is natural that great efforts have been devoted to the study of finite-time H_∞ control problem, see [8-10].

On the other hand, switched systems, which consist of a family of subsystems and a switching signal governing the switching among them, have been extensively studied in the past several years, see [11-15]. Neutral systems are a special class of time-delay systems appearing in many dynamic systems, which depends on both the delays of state and the state derivative. Some practical processes can be modelled as neutral systems, such as distributed networks, heat exchanges, and steam processes. Switched neutral systems have attracted special attention during the past decade. Some useful results have been reported in the literature (see, e.g., [8,16] and the references therein), primarily on the investigation of stability. Dwell time (DT) method is a powerful tool in system analysis and control synthesis of switched systems (see [17,18]), which, then, is extended to a more flexibility and availability method, average dwell time (ADT) method (see [19,20]). Recently, the so-called mode-dependent average dwell time switching (MDADT) is proposed in [21]. An important feature of a switching signal with MDADT property is that each mode has its own average dwell time. Meanwhile, compared with the computation of minimal ADT, MDADT gives rise to more flexible and less conservative results. During the past several years, some results on stability and stabilization have been reported for switched systems with MDADT switching, see [22,23].

Based on the above discussion, finite-time H_∞ output tracking control for switched neutral systems is worth discussing. Sufficient conditions established by the MDADT method that guarantee the stability and the H_∞ output tracking performance for the switched neutral systems over a fixed finite-time interval have not been reported in literature yet. This motivates the main purpose of our research. In this paper, sufficient conditions for finite-time boundedness and finite-time H_∞ performance of the switched neutral systems are derived by using the MDADT method. A state feedback controller which makes the closed-loop output tracking systems be finite-time boundedness with some H_∞ performance level is designed.

The rest of this paper is organized as follows. In Section 2, some assumptions, lemmas and definitions are provided. In Section 3, the main results of this paper are presented as some sufficient conditions and controllers. In Section 4, a numerical example was provided to illustrate the effectiveness of our results. Finally, in Section 5, concluding remarks are given.

Notations: Throughout this paper, the notations are standard. The "$*$" denotes the symmetric term in a symmetric matrix, $diag\{\dots\}$ a block-diagonal matrix and I the identity matrix. $\mathbb{R}^n$ represents the n dimensional Euclidean space, $\mathbb{R}^{n\times m}$ the set of all $n \times m$ real matrices. $P > 0$ means that P is real symmetric and positive definite, and $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) is the minimum (maximum) eigenvalue of matrix P.

2. **Problem Statement and Preliminaries.** Consider the following switched neutral system

$$\dot{x}(t) - D_{\sigma(t)}\dot{x}(t-\tau) = A_{\sigma(t)}x(t) + A_{d,\sigma(t)}x(t-d) + B_{\sigma(t)}u(t) + E_{\sigma(t)}w(t), \quad (1a)$$

$$y(t) = Cx(t) + Fw(t) + Gu(t), \quad (1b)$$

$$x(t_0+\theta)=\varphi(\theta), \quad \theta\in[-h,0], \tag{1c}$$

where $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^l$ is the control input vector, $w(t)\in\mathbb{R}^p$ is the exogenous disturbance vector which belongs to $L_2[t_0,T)$, $y(t)\in\mathbb{R}^q$ is the control output vector, and $\varphi(\theta)\in\mathbb{R}^n$ is the initial condition. M is the number of subsystems, $\sigma(t):\mathbb{R}^+\to\underline{M}=\{1,2,\cdots,\underline{M}\}$ is the switching signal which is a piecewise constant and right continuous function, the switching sequence denotes $\sum=\{(t_0,\sigma(t_0)),(t_1,\sigma(t_1)),\cdots,(t_k,\sigma(t_k)),\cdots\}$, $(k=1,2,\cdots,N_\sigma(t_0,T))$ where $N_\sigma(t_0,T)$ denotes the switching number of $\sigma(t)$ in the time interval (t_0,T). $h=\max\{\tau,d\}$. For any $p\in\underline{M}$, A_p, $A_{d,p}$, B_p, D_p, E_p, C, F and G are constant matrices of appropriate dimensions.

Assumption 2.1. [23] *For a given time T, the external disturbance $w(t)$ satisfies*

$$\int_0^T w^T(t)w(t)dt\le\delta_1^2. \tag{2}$$

Definition 2.1. *For a switching signal $\sigma(t)$ and any $T>0$, let $N_{\sigma,p}(0,T)$ be the switching number that the pth subsystem is activated over the interval $[0,T]$ and $T_p(0,T)$ denotes the total running time of the pth subsystem over the interval $[0,T]$, $p\in\underline{M}$. We say that $\sigma(t)$ has a mode-dependent average dwell time τ_{ap} if there exist positive numbers N_{0p} (we call N_{0p} the mode-dependent chatter bounds here) and τ_{ap} such that*

$$N_{\sigma,p}(0,T)\le N_{0p}+\frac{T_p(0,T)}{\tau_{ap}}, \quad \forall T>0. \tag{3}$$

As commonly used in the literature, we choose $N_{0p}=0$ in the rest of this paper.

Definition 2.2. *(Finite-time stability and finite-time boundedness). Given three positive constants c_1, c_2 and T, a positive definite matrix R and a switching signal $\sigma(t)$, the switched neutral system (1) with $u(t)\equiv 0$ and $w(t)\equiv 0$ is said to be finite-time stable with respect to $(c_1,c_2,T,R,\sigma(t))$, if the following inequality holds:*

$$\sup_{-h\le s\le 0}\left\{x^T(s)Rx(s),\dot{x}^T(s)R\dot{x}(s)\right\}\le c_1\Rightarrow x^T(t)Rx(t)\le c_2, \quad \forall t\in[0,T]. \tag{4}$$

For $\forall w(t)$ satisfies Assumption 2.1, if condition (4) holds, we say the switched neutral system (1) with $u(t)\equiv 0$ is said to be finite-time bounded with respect to $(c_1,c_2,T,\delta_1^2,R,\sigma(t))$.

Definition 2.3. *(Finite-time H_∞ performance). Given positive constants c_1, c_2, T, d_1^2 and γ, a positive definite matrix R and a switching signal $\sigma(t)$, the switched neutral system (1) with $u(t)\equiv 0$ is said to be finite-time bounded with H_∞ disturbance attenuation level γ with respect to $(c_1,c_2,T,\delta_1^2,\gamma,R,\sigma(t))$, if the following inequality holds:*

$$(i)\ \sup_{-h\le s\le 0}\left\{x^T(s)Rx(s),\dot{x}^T(s)R\dot{x}(s)\right\}\le c_1\Rightarrow x^T(t)Rx(t)\le c_2, \quad \forall t\in[0,T],$$

$$\forall w(t):\int_0^T w^T(t)w(t)dt\le\delta_1^2. \tag{5}$$

(ii) *Under the zero-initial condition, the controlled output $y(t)$ satisfies*

$$\int_0^T y^T(t)y(t)dt\le\gamma^2\int_0^T w^T(t)w(t)dt. \tag{6}$$

Suppose the reference output signal is $r(t)$, and then the tracking error is

$$e(t)=y(t)-r(t). \tag{7}$$

Letting $z(t)=\int_0^t e(s)ds$, and choosing the controllers like the following

$$u(t)=K_{\sigma(t)}x(t)+L_{\sigma(t)}z(t), \tag{8}$$

we have the closed-loop output tracking control systems as follows

$$\dot{\eta}(t) = \left(\overline{A}_{\sigma(t)} + \overline{B}_{\sigma(t)}\overline{K}_{\sigma(t)}\right)\eta(t) + \overline{A}_{d,\sigma(t)}\eta(t-d) + \overline{D}_{\sigma(t)}\dot{\eta}(t-\tau) + \overline{E}_{\sigma(t)}\overline{w}(t), \tag{9a}$$

$$z(t) = \overline{C}\eta(t), \tag{9b}$$

where

$$\eta(t) = \begin{bmatrix} x^T(t) & z^T(t) \end{bmatrix}^T, \quad \overline{w}(t) = \begin{bmatrix} w^T(t) & r^T(t) \end{bmatrix}^T, \quad \overline{K}_{\sigma(t)} = \begin{bmatrix} K_{\sigma(t)} & L_{\sigma(t)} \end{bmatrix},$$

$$\overline{A}_{\sigma(t)} = \begin{bmatrix} A_{\sigma(t)} & 0 \\ C & 0 \end{bmatrix}, \quad \overline{B}_{\sigma(t)} = \begin{bmatrix} B_{\sigma(t)} \\ G \end{bmatrix}, \quad \overline{A}_{d,\sigma(t)} = \begin{bmatrix} A_{d,\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\overline{D}_{\sigma(t)} = \begin{bmatrix} D_{\sigma(t)} & 0 \\ 0 & 0 \end{bmatrix}, \quad \overline{E}_{\sigma(t)} = \begin{bmatrix} E_{\sigma(t)} & 0 \\ F & -I \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} 0 & I \end{bmatrix}.$$

The reference signal $r(t)$ plays a role of "exogenous disturbance" in the closed-loop output tracking control system, and then we made the following assumption.

Assumption 2.2. *For a given time, constant T, the reference output $r(t)$ satisfies*

$$\int_0^T r^T(t)r(t)dt \le \delta_2^2. \tag{10}$$

Definition 2.4. *(Finite-time H_∞ output tracking control). Given positive constants c_1, c_2, T, d^2 and γ, a positive definite matrix R and a switching signal $\sigma(t)$, the switched neutral system (1) is said to be finite-time stabilizable with H_∞ output tracking performance γ with respect to $(c_1, c_2, T, \delta^2, \gamma, R, \sigma(t))$, if there exists a controller $u(t)$ in the form of (9), such that the closed-loop system (10) satisfy:*

(i) $\sup_{-h \le s \le 0} \left\{\eta^T(s)R\eta(s), \dot{\eta}^T(s)R\dot{\eta}(s)\right\} \le c_1 \Rightarrow \eta^T(t)R\eta(t) \le c_2, \quad \forall t \in [0,T],$

$$\forall \overline{w}(t) : \int_0^T \overline{w}^T(t)\overline{w}(t)dt \le \delta^2. \tag{11}$$

(ii) *Under the zero-initial condition, the integral term of the tracking error $z(t)$ satisfies* $\int_0^T z^T(t)z(t)dt \le \gamma^2 \int_0^T \overline{w}^T(t)\overline{w}(t)dt.$ (12)

Lemma 2.1. [24] *For any symmetric and positive definite constant matrix $\Omega \in \mathbb{R}^{l \times l}$ and scalar function $0 \le r(t) \le r$, if there exists a vector function: $v : [0,r] \to \mathbb{R}^l$ such that integrals $\int_0^{r(t)} v^T(s)\Omega v(s)ds$ and $\int_0^{r(t)} v^T(s)ds$ are well defined, then the following inequality holds*

$$r\int_0^{r(t)} v^T(s)\Omega x(s)d\xi \ge \left(\int_0^{r(t)} v(s)ds\right)^T \Omega \left(\int_0^{r(t)} v(s)ds\right). \tag{13}$$

3. Main Results.

In this section, the finite-time H_∞ output tracking control problem for a class of switched neutral systems is studied, and it is necessary to study the finite-time stability and finite-time H_∞ performance first.

Theorem 3.1. *Given positive constants c_1, c_2, T and d_1^2, if there exist symmetric positive definite matrices P_p, Q_p, Y_p, S_p and W_p, positive constants α_p, $\lambda_{1,p}$, $\lambda_{2,p}$, $\lambda_{3,p}$, $\lambda_{4,p}$ and*

λ_5, $\forall p \in \underline{M}$, *such that:*

$$\begin{bmatrix} \Xi & P_pA_{d,p} + d^{-1}e^{-\alpha_p h}S_p & P_pD_p & P_pE_p & A_p^TY_P + hA_p^TS_P \\ * & -e^{-\alpha_p d}Q_p - d^{-1}e^{-\alpha_p h}S_p & 0 & 0 & A_{d,p}^TY_P + hA_{d,p}^TS_P \\ * & * & -e^{-\alpha_p \tau}Y_p & 0 & D_p^TY_P + hD_p^TS_P \\ * & * & * & -W_p & E_p^TY_P + hE_p^TS_P \\ * & * & * & * & -Y_p - hS_p \end{bmatrix} < 0, \tag{14}$$

$$R < P_p < \lambda_{1,p}R, \tag{15}$$

$$0 < Q_p < \lambda_{2,p}R, \tag{16}$$

$$0 < Y_p < \lambda_{3,p}R, \tag{17}$$

$$0 < S_p < \lambda_{4,p}R, \tag{18}$$

$$0 < W_p < \lambda_5 I, \tag{19}$$

$$e^{-\alpha_p MT}\left(\lambda_p c_1 + \lambda_5 \delta_1^2\right) < c_2, \tag{20}$$

then under a switching signal $\sigma(t)$ satisfying following MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{MT \ln \mu_p}{\ln c_2 e^{\alpha_p MT} - \ln\left(\lambda_p c_1 + \lambda_5 \delta_1^2\right)}, \tag{21}$$

the system (1) is finite-time bounded with respect to $(c_1, c_2, T, R, \delta_1^2, \sigma(t))$, where $\lambda_p = \lambda_{1,p} + \lambda_{2,p}he^{-\alpha_p h} + \lambda_{3,p}he^{-\alpha_p h} + \lambda_{4,p}h^2e^{-\alpha_p h}$, $\Xi = A_p^TP_p + P_pA_p + Q_p - d^{-1}e^{\alpha_p h}S_p + \alpha_pP_p$, $\mu_p > 1$ satisfying:

$$P_p < \mu_pP_q,\ Q_p < \mu_pQ_q,\ Y_p < \mu_pY_q,\ S_p < \mu_pS_q,\ \forall p, q \in \underline{M}. \tag{22}$$

Proof: See A1 in the Appendixes.

Corollary 3.1. *Given positive constants c_1, c_2, T, if there exist symmetric positive definite matrices P_p, Q_p, Y_p, and S_p, positive constants α_p, $\lambda_{1,p}$, $\lambda_{2,p}$, $\lambda_{3,p}$ and $\lambda_{4,p}$, $\forall p \in \underline{M}$, such that:*

$$\begin{bmatrix} \begin{matrix} A_p^TP_p + P_pA_p + Q_p \\ -d^{-1}e^{-\alpha_p h}S_p - \alpha_pP_p \end{matrix} & P_pA_{d,p} + d^{-1}e^{-\alpha_p h}S_p & P_pD_p & A_p^TY_P + hA_p^TS_p \\ * & -e^{\alpha_p d}Q_p - d^{-1}e^{-\alpha_p h}S_p & 0 & A_{d,p}^TY_P + hA_{d,p}^TS_p \\ * & * & -e^{\alpha_p \tau}Y_p & D_p^TY_P + hD_p^TS_p \\ * & * & * & -Y_p - hS_p \end{bmatrix} < 0, \tag{23}$$

$$R < P_p < \lambda_{1,p}R, \tag{24}$$

$$0 < Q_p < \lambda_{2,p}R, \tag{25}$$

$$0 < Y_p < \lambda_{3,p}R, \tag{26}$$

$$0 < S_p < \lambda_{4,p}R, \tag{27}$$

$$e^{\alpha_p MT}\lambda c_1 < c_2, \tag{28}$$

then under a switching signal $\sigma(t)$ satisfying following MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{MT \ln \mu_p}{\ln c_2 e^{-\alpha_p MT} - \ln \lambda_p c_1}, \tag{29}$$

the system (1) with $w(t) \equiv 0$ is finite-time stable with respect to $(c_1, c_2, T, R, \sigma(t))$, where $\lambda_p = \lambda_{1,p} + \lambda_{2,p}he^{\alpha_p h} + \lambda_{3,p}he^{\alpha_p h} + \lambda_{4,p}h^2e^{\alpha_p h}$, $\mu_p > 1$ satisfying (22).

Proof: The proof is similar to that of Theorem 3.1, so it is omitted here.

Theorem 3.2. *Given positive constants c_1, c_2, T, γ and d_1^2, if there exist symmetric positive definite matrices P_p, Q_p, Y_p and S_p, positive constants α_p, $\lambda_{1,p}$, $\lambda_{2,p}$, $\lambda_{3,p}$ and $\lambda_{4,p}$, $\forall p \in \underline{M}$, such that:*

$$\begin{bmatrix} \Xi & P_pA_{d,p}+d^{-1}e^{-\alpha_p h}S_p & P_pD_p & P_pE_p+C^TF & A_p^TY_p+hA_p^TS_p \\ * & -e^{\alpha_p d}Q_p-d^{-1}e^{-\alpha_p h}S_p & 0 & 0 & A_{d,p}^TY_p+hA_{d,p}^TS_p \\ * & * & -e^{\alpha_p\tau}Y_p & 0 & D_p^TY_p+hD_p^TS_p \\ * & * & * & -\gamma^2I+F^TF & E_p^TY_p+hE_p^TS_p \\ * & * & * & * & -Y_p-hS_p \end{bmatrix} < 0, \quad (30)$$

$$R < P_p < \lambda_{1,p}R, \quad (31)$$

$$0 < Q_p < \lambda_{2,p}R, \quad (32)$$

$$0 < Y_p < \lambda_{3,p}R, \quad (33)$$

$$0 < S_p < \lambda_{4,p}R, \quad (34)$$

$$e^{\alpha_p MT}\left(\lambda c_1+\gamma^2\delta_1^2\right) < c_2, \quad (35)$$

then under a switching signal $\sigma(t)$ satisfying following MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{MT\ln\mu_p}{\ln c_2e^{-\alpha_p MT}-\ln\left(\lambda_pc_1+\gamma^2\delta_1^2\right)}, \quad (36)$$

the system (1) is finite-time bounded with H_∞ performance γ^2 with respect to $(c_1, c_2, T, R, d_1^2, \sigma(t))$, where $\lambda_p = \lambda_{1,p}+\lambda_{2,p}he^{\alpha_p h}+\lambda_{3,p}he^{\alpha_p h}+\lambda_{4,p}h^2e^{\alpha_p h}$, $\Xi = A_p^TP_p+P_pA_p+Q_p+C^TC-d^{-1}e^{-\alpha_p h}S_p-\alpha_pP_p$, $\mu_p > 1$ satisfying (22).

Proof: See A2 in Appendixes.

Theorem 3.3. *Given positive constants c_1, c_2, T, γ and d_1^2, if there exist symmetric positive definite matrices $\overline{P}_p$, $\overline{Q}_p$, $\overline{Y}_p$, $\overline{S}_p$ and H_p, positive constants α_p, $\lambda_{1,p}$, $\lambda_{2,p}$, $\lambda_{3,p}$ and $\lambda_{4,p}$, $\forall p \in \underline{M}$, such that:*

$$\Sigma = \begin{bmatrix} \Xi & \overline{A}_{d,p}\overline{P}_p+d^{-1}e^{-\alpha_p h}\overline{S}_p & \overline{D}_p\overline{P}_p & \overline{E}_p & \overline{P}_p\overline{A}_p^T+\overline{\Pi}_p^T\overline{B}_p^T \\ * & -e^{\alpha_p d}\overline{Q}_p-d^{-1}e^{-\alpha_p h}\overline{S}_p & 0 & 0 & \overline{P}_p\overline{A}_{d,p}^T \\ * & * & -e^{\alpha_p\tau}\overline{Y}_p & 0 & \overline{P}_p\overline{D}_p^T \\ * & * & * & -\gamma^2I & \overline{E}_p^T \\ * & * & * & * & -H_p \end{bmatrix} < 0, \quad (37)$$

$$\overline{Y}_p+h\overline{S}_p+H_p-2\overline{P}_p < 0, \quad (38)$$

$$\lambda_{1,p}^{-1}R^{-1} < \overline{P}_p < R^{-1}, \quad (39)$$

$$0 < \overline{Q}_p < 2\lambda_{2,p}\overline{P}_p-\lambda_{2,p}R^{-1}, \quad (40)$$

$$0 < \overline{Y}_p < 2\lambda_{3,p}\overline{P}_p-\lambda_{3,p}R^{-1}, \quad (41)$$

$$0 < \overline{S}_p < 2\lambda_{4,p}\overline{P}_p-\lambda_{4,p}R^{-1}, \quad (42)$$

$$e^{\alpha_p MT}\left(\lambda c_1+\gamma^2\delta_1^2\right) < c_2, \quad (43)$$

then under a switching signal $\sigma(t)$ satisfying following MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{MT\ln\mu_p}{\ln c_2e^{-\alpha_p MT}-\ln\left(\lambda_pc_1+\gamma^2\delta^2\right)}, \quad (44)$$

the system (1) is finite-time stabilizable with H_∞ *output tracking performance* γ *with respect to* $(c_1, c_2, T, \delta^2, \gamma^2, R, \sigma(t))$*, where* $\lambda_p = \lambda_{1,p} + \lambda_{2,p}he^{\alpha_p h} + \lambda_{3,p}he^{\alpha_p h} + \lambda_{4,p}h^2e^{\alpha_p h}$, $\Xi = \overline{P}_p\overline{A}_p^T + \overline{A}_p\overline{P}_p + \overline{\Pi}_p^T\overline{B}_p^T + \overline{B}_p\overline{\Pi}_p + \overline{Q}_p + \overline{C}^T\overline{C} - d^{-1}e^{-\alpha_p h}\overline{S}_p - \alpha_p\overline{P}_p$, $\mu_p > 1$ *satisfying*

$$\overline{P}_p < \mu_p\overline{P}_q,\ \overline{Q}_p < \mu_p\overline{Q}_q,\ \overline{Y}_p < \mu_p\overline{Y}_q,\ \overline{S}_p < \mu_p\overline{S}_q,\ \forall p, q \in \underline{M}, \tag{45}$$

$\overline{A}_{\sigma(t)}$, $\overline{B}_{\sigma(t)}$, $\overline{A}_{d,\sigma(t)}$, $\overline{D}_{\sigma(t)}$, $\overline{E}_{\sigma(t)}$*, and* $\overline{C}$ *are the same as in (9).*

Furthermore, the state feedback controller is:

$$\overline{K}_p = \begin{bmatrix} K_p & L_p \end{bmatrix} = \overline{\Pi}_p\overline{P}_p^{-1}. \tag{46}$$

Proof: From Definition 2.4, the switched neutral system (1) is finite-time stabilizable with H_∞ output tracking performance γ with respect to $(c_1, c_2, T, \delta^2, \gamma, R, \sigma(t))$, if there exists a controller formed as (9) making the closed-loop output tracking control system (10) finite-time bounded with H_∞ performance γ^2 with respect to $(c_1, c_2, T, \delta, \gamma, R, \sigma(t))$, the sufficient conditions are that there exist symmetric positive definite matrices P_p, Q_p, Y_p and S_p, positive constants α_p, $\lambda_{1,p}$, $\lambda_{2,p}$, $\lambda_{3,p}$ and $\lambda_{4,p}$, $\forall p \in \underline{M}$, such that:

$$\begin{bmatrix} \Xi' & P_p\overline{A}_{d,p} + d^{-1}e^{-\alpha_p h}S_p & P_p\overline{D}_p & P_p\overline{E}_p & \begin{array}{c}\left(\overline{A}_p + \overline{B}_p\overline{K}_p\right)^T Y_p \\ +h\left(\overline{A}_p + \overline{B}_p\overline{K}_p\right)^T S_p\end{array} \\ * & -e^{\alpha_p d}Q_p - d^{-1}e^{-\alpha_p h}S_p & 0 & 0 & \overline{A}_{d,p}^T Y_p + h\overline{A}_{d,p}^T S_p \\ * & * & -e^{\alpha_p \tau}Y_p & 0 & \overline{D}_p^T Y_p + h\overline{D}_p^T S_p \\ * & * & * & -\gamma^2 I & \overline{E}_p^T Y_p + h\overline{E}_p^T S_p \\ * & * & * & * & -Y_p - hS_p \end{bmatrix} < 0, \tag{47}$$

$$R < P_p < \lambda_{1,p}R, \tag{48}$$

$$0 < Q_p < \lambda_{2,p}R, \tag{49}$$

$$0 < Y_p < \lambda_{3,p}R, \tag{50}$$

$$0 < S_p < \lambda_{4,p}R, \tag{51}$$

$$e^{\alpha_p MT}\left(\lambda c_1 + \gamma^2\delta^2\right) < c_2, \tag{52}$$

and then the switching signal $\sigma(t)$ satisfying following MDADT

$$\tau_{ap} \geq \tau_{ap}^* = \frac{MT \ln \mu_p}{\ln c_2 e^{-\alpha_p MT} - \ln\left(\lambda_p c_1 + \gamma^2\delta_1^2\right)} \tag{53}$$

where $\Xi' = \left(\overline{A}_p + \overline{B}_p\overline{K}_p\right)^T P_p + P_p\left(\overline{A}_p + \overline{B}_p\overline{K}_p\right) + Q_p + \overline{C}^T\overline{C} - d^{-1}e^{-\alpha_p h}S_p - \alpha_p P_p$, $\overline{A}_{\sigma(t)}$, $\overline{B}_{\sigma(t)}$, $\overline{A}_{d,\sigma(t)}$, $\overline{D}_{\sigma(t)}$, $\overline{E}_{\sigma(t)}$, and $\overline{C}$ are the same as in (9).

Let $\overline{P}_p = P_p^{-1}$, $\overline{\Pi}_p = \overline{K}_p\overline{P}_p$, $\overline{S}_p = \overline{P}_pS_p\overline{P}_p$, $\overline{Q}_p = \overline{P}_pQ_p\overline{P}_p$, $\overline{Y}_p = \overline{P}_pY_p\overline{P}_p$, by pre- and post-multiplying (47) by $diag\{\overline{P}, \overline{P}, \overline{P}, I, (Y_p - hS_p)^{-1}\}$, we have

$$\overline{\Sigma} = \begin{bmatrix} \Xi & \overline{A}_{d,p}\overline{P}_p + d^{-1}e^{-\alpha_p h}\overline{S}_p & \overline{D}_p\overline{P}_p & \overline{E}_p & \overline{P}_p\overline{A}_p^T + \overline{\Pi}_p^T\overline{B}_p^T \\ * & -e^{\alpha_p d}\overline{Q}_p - d^{-1}e^{-\alpha_p h}\overline{S}_p & 0 & 0 & \overline{P}_p\overline{A}_{d,p}^T \\ * & * & -e^{\alpha_p\tau}\overline{Y}_p & 0 & \overline{P}_p\overline{D}_p^T \\ * & * & * & -\gamma^2 I & \overline{E}_p^T \\ * & * & * & * & -(Y + hS)^{-1} \end{bmatrix} < 0. \tag{54}$$

Note that $\left(H_p - \overline{P}_p\right) H_p^{-1} \left(H_p - \overline{P}_p\right) \geq 0$, and then we have

$$2\overline{P}_p - H_p \leq \overline{P}_p H_p^{-1} \overline{P}_p.$$

From (38), we have

$$\overline{Y}_p + h\overline{S}_p < 2\overline{P}_p - H_p \leq \overline{P}_p H_p^{-1} \overline{P}_p. \tag{55}$$

By pre- and post-multiplying the both sides of (55) by $P_p = \overline{P}_p^{-1}$, we have $-(Y_p + hS_p)^{-1} < -H_p$, which means that $\overline{\Sigma} < \Sigma$, and inequality (37) ensures that the inequality (47) holds.

Note that $\overline{P}_p = P_p^{-1}$, inequality (39) is equivalent to inequality (48).

Note that $\left(R^{-1} - \overline{P}_p\right) R \left(R^{-1} - \overline{P}_p\right) \geq 0$, and then we have

$$\overline{P}_p R \overline{P}_p \geq 2\overline{P}_p - R^{-1}. \tag{56}$$

Substituting (56) into (40), we have

$$0 < \overline{Q}_p < \lambda_{2,p} \overline{P}_p R \overline{P}_p. \tag{57}$$

By pre- and post-multiplying the both side of (57) by $P_p = \overline{P}_p^{-1}$, we have (49).

Following the same proof line, we have

$$0 < \overline{Y}_p < 2\lambda_{3,p}\overline{P}_p - \lambda_{3,p}R^{-1} \leq \lambda_{3,p}\overline{P}_p R \overline{P}_p, \quad 0 < \overline{S}_p < 2\lambda_{4,p}\overline{P}_p - \lambda_{4,p}R^{-1} \leq \lambda_{4,p}\overline{P}_p R \overline{P}_p.$$

Inequalities (40)-(42) ensure that inequalities (49)-(51) hold.

Hence, the proof is completed. □

Remark 3.1. *In the proof of Theorem 3.3, the replacement of* $2\overline{P}_p - R^{-1}$ *to* $\overline{P}_p R \overline{P}_p$ *and the introduction of inequality (38) may bring some conservatism to the design of the controller, but provide a simple method of solving LMIs using MATLAB.*

4. **Numerical Example.**

In this section, we present a numerical example to illustrate the proposed method.

Consider the continuous-time switched neutral system (1) composed of two subsystems in [8] with parameters as follows:

$$A_1 = \begin{bmatrix} 2 & 0 \\ 3 & 3 \end{bmatrix}, \ A_{d,1} = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}, \ B_1 = \begin{bmatrix} 3 & -3 \\ 0 & 4 \end{bmatrix}, \ D_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & -0.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}, \ A_2 = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}, \ A_{d,2} = \begin{bmatrix} -1 & 0 \\ -2 & 1 \end{bmatrix}, \ B_2 = \begin{bmatrix} 4 & -1 \\ 1 & 6 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.3 \end{bmatrix}, \ E_2 = \begin{bmatrix} -1 & 0 \\ 2 & 0.8 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 1 \end{bmatrix},$$

$$F = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \ G = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$

the reference signal and exogenous disturbance: $\begin{cases} r(t) = \cos 0.6t \\ w(t) = \begin{bmatrix} e^{-t} & e^{-t} \end{bmatrix}^T \end{cases}.$

Choosing $T = 14$, $d = 0.2$, $\tau = 0.3$, $\delta^2 = 2$, $\gamma^2 = 1$, $c_1 = 1$, $c_2 = 12$, $R = I$. Using LMI Toolbox to solve the matrix inequalities (37)-(43), we obtain the feasible solution with the following symmetric matrices and positive constants

$$\overline{P}_1 = \begin{bmatrix} 0.8897 & -0.0004 & 0.0010 & -0.0002 \\ -0.0004 & 0.8938 & 0.0003 & -0.0026 \\ 0.0010 & 0.0003 & 0.9014 & 0 \\ -0.0002 & -0.0026 & 0 & 0.9036 \end{bmatrix},$$

$$\overline{P}_2 = \begin{bmatrix} 0.9116 & -0.0079 & -0.0096 & -0.0082 \\ -0.0079 & 0.8875 & -0.0028 & -0.0127 \\ -0.0096 & -0.0028 & 0.9344 & 0.0037 \\ -0.0082 & -0.0127 & 0.0037 & 0.9415 \end{bmatrix},$$

$$\overline{Q}_1 = \begin{bmatrix} 4.3054 & -0.2438 & 0.0613 & -0.2980 \\ -0.2438 & 4.3538 & -0.0847 & -0.1994 \\ 0.0613 & -0.0847 & 3.7276 & -0.0182 \\ -0.2980 & -0.1994 & -0.0182 & 3.5812 \end{bmatrix},$$

$$\overline{Q}_2 = \begin{bmatrix} 7.6291 & -0.2020 & 0.2661 & 1.3255 \\ -0.2020 & 5.4841 & -0.7156 & -0.9772 \\ 0.2661 & -0.7156 & 8.5653 & -0.8561 \\ 1.3255 & -0.9772 & -0.8561 & 6.7489 \end{bmatrix},$$

$$\overline{Y}_1 = \begin{bmatrix} 0.4215 & -0.0366 & -0.0007 & -0.0025 \\ -0.0366 & 0.3685 & -0.0057 & 0.0016 \\ -0.0007 & -0.0057 & 0.3719 & -0.0013 \\ -0.0025 & 0.0016 & -0.0013 & 0.3513 \end{bmatrix},$$

$$\overline{Y}_2 = \begin{bmatrix} 0.3724 & -0.0072 & 0.0236 & 0.0513 \\ -0.0072 & 0.2183 & -0.0294 & -0.0362 \\ 0.0236 & -0.0294 & 0.3361 & -0.0356 \\ 0.0513 & -0.0362 & -0.0356 & 0.2505 \end{bmatrix},$$

$$\overline{S}_1 = \begin{bmatrix} 3.4211 & -0.0151 & -0.0640 & 0.3224 \\ -0.0151 & 3.1849 & 0.0654 & 0.3090 \\ -0.0640 & 0.0654 & 4.7161 & 0.0069 \\ 0.3224 & 0.3090 & 0.0069 & 4.5652 \end{bmatrix},$$

$$\overline{S}_2 = \begin{bmatrix} 4.3850 & 0.0334 & 0.9357 & 0.4207 \\ 0.0334 & 5.1524 & 0.1689 & 0.9941 \\ 0.9357 & 0.1689 & 3.5262 & -0.2740 \\ 0.4207 & 0.9941 & -0.2740 & 2.8576 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.5063 & 0.0154 & -0.0086 & 0.0042 \\ 0.0154 & 0.3365 & 0.0004 & -0.0202 \\ -0.0086 & 0.0004 & 0.2838 & 0.0306 \\ 0.0042 & -0.0202 & 0.0306 & 0.3302 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.4093 & -0.0111 & -0.2424 & -0.1778 \\ -0.0111 & 0.4205 & 0.0045 & -0.1700 \\ -0.2424 & 0.0045 & 0.6551 & 0.1157 \\ -0.1778 & -0.1700 & 0.1157 & 0.9315 \end{bmatrix},$$

$$\Pi_1 = \begin{bmatrix} -0.1457 & -0.0507 & -0.0402 & -0.0183 \\ -0.0487 & 0.0009 & -0.0197 & -0.0151 \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} -0.1686 & -0.2279 & -0.0111 & -0.0106 \\ 0.0219 & -0.1437 & -0.0184 & -0.0038 \end{bmatrix},$$

$$K_1 = \begin{bmatrix} -0.1640 & -0.0568 & -0.0446 & -0.0201 \\ -0.0548 & 0.0010 & -0.0218 & -0.0166 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.1875 & -0.2587 & -0.0145 & -0.0164 \\ 0.0224 & -0.1619 & -0.0200 & -0.0059 \end{bmatrix},$$

$\alpha_1 = 0.01,\ \alpha_2 = 0.014,\ \lambda_{1,1} = 0.8932,\ \lambda_{1,2} = 1.0552,\ \lambda_{2,1} = 3.7158,$

$$\lambda_{2,2} = 3.2295,\ \lambda_{3,1} = 0.3511,\ \lambda_{3,2} = 0.2542,\ \lambda_{4,1} = 3.3655,\ \lambda_{4,2} = 3.2231.$$

From (45), we have $\mu_1 = 1.2499$, $\mu_2 = 1.1814$, and then according (44), we have

$$\tau_{a1} \geq \tau_{a1}^* = \frac{MT \ln \mu_1}{\ln c_2 e^{\alpha_1 MT} - \ln(\lambda_1 c_1 + \delta_1^2)} = 4.7513,$$

$$\tau_{a2} \geq \tau_{a2}^* = \frac{MT \ln \mu_2}{\ln c_2 e^{\alpha_2 MT} - \ln(\lambda_2 c_1 + \delta_1^2)} = 3.1684.$$

According to Theorem 3.3, for a switching signal $\sigma(t)$ satisfying MDADT $\tau_{a1} = 5$s, $\tau_{a2} = 4$s, the switched neutral system (1) is finite-time stabilizable with H_∞ output tracking performance γ with respect to $(c_1, c_2, T, \delta^2, \gamma, R, \sigma(t))$. The state trajectories of the switched neutral systems (1) with switching signal $\sigma(t)$ is presented in Figure 1. The tracking performances for system (1) with the above reference signal and exogenous disturbance are given in Figure 2. From the simulation results, we can draw that the switched neutral systems (1) can track the reference signals with H_∞ output tracking performance γ within a finite time interval $T = 14$ by the designed controller.

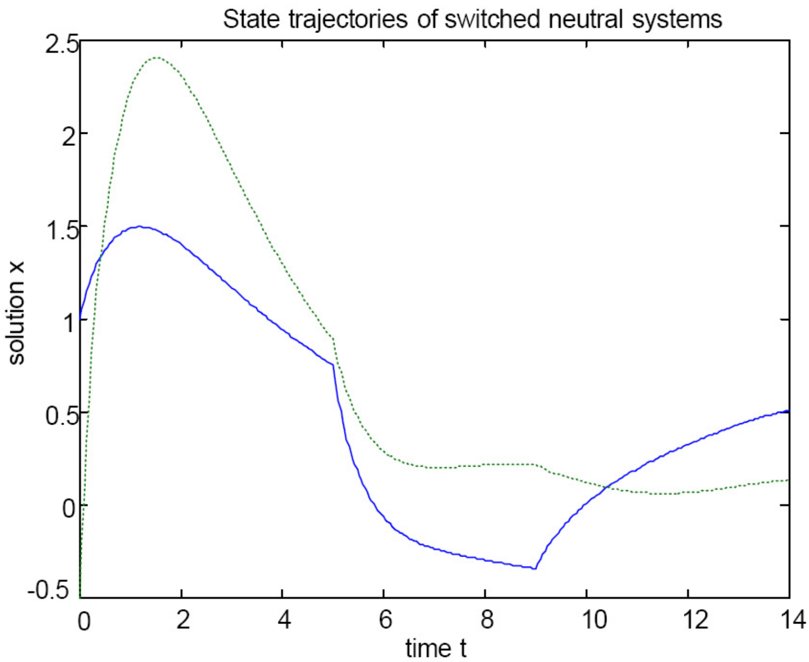

FIGURE 1. The state trajectories of the switched neutral systems with switching signal $\sigma(t)$

5. **Conclusions.** This paper addresses the finite-time H_∞ output tracking control problem for a class of switched neutral systems. First, the finite-time stability (FTS) and the finite-time H_∞ problem for the augmented systems are investigated. By using the mode-dependent average dwell time (MDADT) method, sufficient conditions for finite-time boundedness and finite-time H_∞ performance of the augmented systems are derived. Second, based on the sufficient conditions derived in finite-time H_∞ performance analysis, a state feedback controller is designed which makes the closed-loop output tracking systems be finite-time boundedness with some H_∞ performance level.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (No. 61473252 and No. 61633019). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

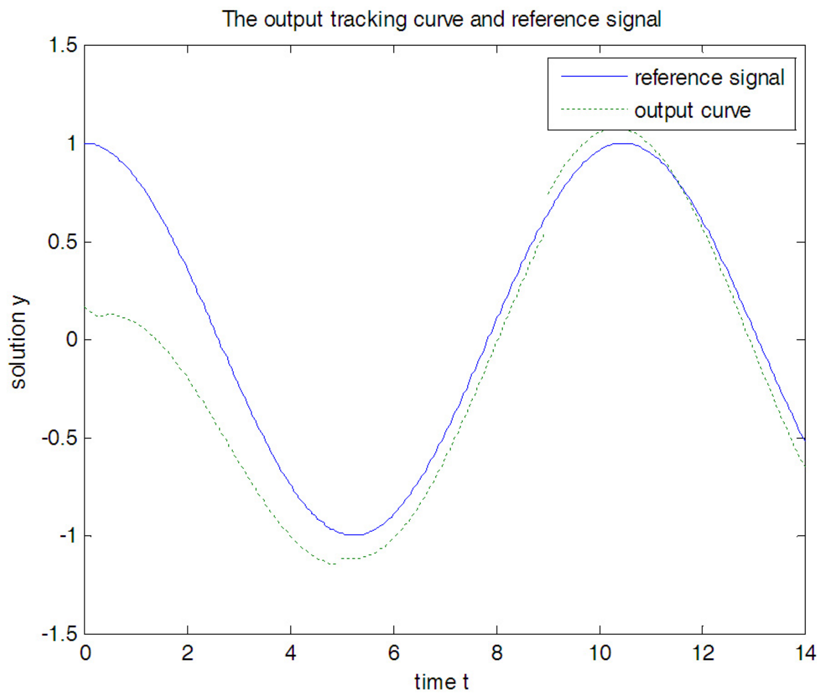

FIGURE 2. The output tracking curve and the reference signal

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Appendixes.

A1: Proof of Theorem 3.1.

Choose the following piecewise Lyapunov function candidate of the form:

$$\begin{aligned} V(t) = {} & x^T(t)P_{\sigma(t)}x(t) + \int_{t-d}^{t} e^{-\alpha_{\sigma(t)}(t-s)}x^T(s)Q_{\sigma(t)}x(s)ds \\ & + \int_{t-\tau}^{t} e^{-\alpha_{\sigma(t)}(t-s)}\dot{x}^T(s)Y_{\sigma(t)}\dot{x}(s)ds \\ & + \int_{-h}^{0}\int_{t+\theta}^{t} e^{-\alpha_{\sigma(t)}(t-s)}\dot{x}^T(s)S_{\sigma(t)}\dot{x}(s)dsd\theta. \end{aligned} \tag{a1}$$

Suppose $\sigma(t_k) = p$, this means that the pth subsystem is activated in the time interval $t \in [t_k, t_{k+1})$. Taking the time-derivative of $V(t)$ along the trajectory of the pth subsystem, we have

$$\begin{aligned} \dot{V}(t) = {} & x^T(t)\left(A_p^T P_p + P_p A_p\right)x(t) + 2x^T(t-d)A_{d,p}^T P_p x(t) + 2\dot{x}^T(t-\tau)D_p^T P_p x(t) \\ & + 2w^T(t)E_p^T P_p x(t) - \alpha\int_{t-d}^{t} e^{-\alpha_p(t-s)}x^T(s)Q_p x(s)ds + x^T(t)Q_p x(t) \\ & - e^{-\alpha_p d}x^T(t-d)Q_p x(t-d) - \alpha\int_{t-\tau}^{t} e^{-\alpha_p(t-s)}\dot{x}^T(s)Y_p\dot{x}(s)ds + \dot{x}^T(t)Y_p\dot{x}(t) \\ & - e^{-\alpha_p\tau}\dot{x}^T(t-\tau)Y_p\dot{x}(t-\tau) - \alpha\int_{-h}^{0}\int_{t+\theta}^{t} e^{-\alpha_p(t-s)}\dot{x}^T(s)S_p\dot{x}(s)dsd\theta \\ & + h\dot{x}^T(t)S_p\dot{x}(t) - \int_{t-h}^{t} e^{-\alpha_p(t-s)}\dot{x}^T(s)S_p\dot{x}(s)ds. \end{aligned} \tag{a2}$$

$$\dot{x}^T(t)Y_p\dot{x}(t) + h\dot{x}^T(t)S_p\dot{x}(t) = \xi^T(t)\begin{bmatrix} A_p^T \\ A_{d,p}^T \\ D_p^T \\ E_p^T \end{bmatrix}(Y_p + hS_p)\begin{bmatrix} A_p^T \\ A_{d,p}^T \\ D_p^T \\ E_p^T \end{bmatrix}^T \xi(t),$$

where $\xi(t) = \begin{bmatrix} x^T(t) & x^T(t-d) & \dot{x}^T(t-\tau) & w^T(t) \end{bmatrix}$.

From Lemma 2.1, we have

$$\begin{aligned} -\int_{t-h}^{t} e^{-\alpha_p(t-s)}\dot{x}^T(s)S_p\dot{x}(s)ds &\leq -e^{-\alpha_p h}\int_{t-d}^{t}\dot{x}^T(s)S_p\dot{x}(s)ds \\ &\leq -d^{-1}e^{-\alpha_p h}\left[\int_{t-d}^{t}\dot{x}^T(s)ds\right]S_p\left[\int_{t-d}^{t}\dot{x}(s)ds\right] \\ &= -d^{-1}e^{-\alpha_p h}\left[x^T(t) - x^T(t-d)\right]S_p\left[x(t) - x(t-d)\right]. \end{aligned} \tag{a3}$$

Then, substituting (a3) into (a2), we get

$$\dot{V}(t) + \alpha_p V(t) - w^T(t)W_p w(t) \leq \xi^T(t)\Phi_p\xi(t),$$

where

$$\Phi_p = \begin{bmatrix} \Xi & P_pA_{d,p} + d^{-1}e^{-\alpha_p h}S_p & P_pD_p & P_pE_p \\ * & -e^{-\alpha_p d}Q_p - d^{-1}e^{-\alpha_p h}S_p & 0 & 0 \\ * & * & -e^{-\alpha_p \tau}Y_p & 0 \\ * & * & * & -W_p \end{bmatrix} + \begin{bmatrix} A_p^T \\ A_{d,p}^T \\ D_p^T \\ E_p^T \end{bmatrix}(Y_i + hS_i)\begin{bmatrix} A_p^T \\ A_{d,p}^T \\ D_p^T \\ E_p^T \end{bmatrix}^T,$$

$$\Xi = A_p^T P_p + P_pA_p + Q_p - d^{-1}e^{\alpha_p h}S_p + \alpha_p P_p.$$

In view of LMI (14), we get

$$\dot{V}(t) + \alpha_p V(t) - w^T(t)W_p w(t) < 0. \tag{a4}$$

It can be obtained by (a4) that, for $t \in [t_k, t_{k+1})$,

$$V(t) < e^{-\alpha_p(t-t_k)}V(t_k) + \int_{t_k}^{t} e^{-\alpha_p(t-s)}w^T(s)W_p w(s)ds.$$

Note that $k = N_\sigma(0,t) = \sum_{p=1}^{M} N_{\sigma,p}(0,t)$, and suppose that $\sigma(t_{k-1}) = q \in \underline{M}$, from (22), we can obtain

$$V(t_k) \leq \mu_p V(t_k^-).$$

$$\begin{aligned} V(t) &< e^{-\alpha_p\left(t-t_{N_\sigma(0,t)}\right)}V\left(t_{N_\sigma(0,t)}\right) + \int_{t_{N_\sigma(0,t)}}^{t} e^{-\alpha_p(t-s)}w^T(s)W_p w(s)ds \\ &\leq \mu_p e^{-\alpha_p\left(t-t_{N_\sigma(0,t)}\right)}V\left(t_{N_\sigma(0,t)}^-\right) + \int_{t_{N_\sigma(0,t)}}^{t} e^{-\alpha_p(t-s)}w^T(s)W_p w(s)ds \\ &< \mu_p e^{-\alpha_p\left(t-t_{N_\sigma(0,t)}\right)-\alpha_q\left(t_{N_\sigma(0,t)}-t_{N_\sigma(0,t)-1}\right)}V\left(t_{t_{N_\sigma(0,t)-1}}\right) \\ &\quad + \mu_p e^{-\alpha_p\left(t-t_{N_\sigma(0,t)}\right)-\alpha_q\left(t_{N_\sigma(0,t)}-t_{N_\sigma(0,t)-1}\right)}\int_{t_{N_\sigma(0,t)-1}}^{t_{N_\sigma(0,t)}} w^T(s)W_q w(s)ds \\ &\quad + \int_{t_{N_\sigma(0,t)}}^{t} e^{-\alpha_p(t-s)}w^T(s)W_p w(s)ds \end{aligned}$$

$$
\begin{aligned}
&\le \cdots \\
&< \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(0,t)} \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(0,t)\right\} V(0) \\
&\quad + \lambda_5 \int_0^t \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(s,t)} \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(s,t)\right\} w^T(s)w(s)ds \\
&= \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(0,t) + N_{\sigma,p}(0,t)\ln\mu_p\right\}\left(V(0) + \lambda_5 \int_0^t w^T(s)w(s)ds\right).
\end{aligned}
$$

Then, for $\forall t \in [0, T]$,

$$
V(t) \le \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(0,T) + N_{\sigma,p}(0,T)\ln\mu_p\right\}\left(V(0) + \lambda_5 \int_0^T w^T(s)w(s)ds\right). \quad \text{(a5)}
$$

From Definition 2.2, we have

$$
V(t) < \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(0,T) + \frac{T_p(0,T)}{\tau_{ap}}\ln\mu_p\right\}\left(V(0) + \lambda_5 \int_0^T w^T(s)w(s)ds\right). \quad \text{(a6)}
$$

Let $\tilde{P} = R^{-1/2}PR^{-1/2}$, $\tilde{Q} = R^{-1/2}QR^{-1/2}$, $\tilde{Y} = R^{-1/2}YR^{-1/2}$ and $\tilde{S} = R^{-1/2}SR^{-1/2}$, and we have

$$
x^T(0)P_{\sigma(0)}x(0) \le \lambda_{\max}\left(\tilde{P}_{\sigma(0)}\right)x^T(0)Rx(0), \quad \text{(a7)}
$$

$$
x^T(0)Q_{\sigma(0)}x(0) \le \lambda_{\max}\left(\tilde{Q}_{\sigma(0)}\right)x^T(0)Rx(0), \quad \text{(a8)}
$$

$$
x^T(0)Y_{\sigma(0)}x(0) \le \lambda_{\max}\left(\tilde{Y}_{\sigma(0)}\right)x^T(0)Rx(0), \quad \text{(a9)}
$$

$$
x^T(0)S_{\sigma(0)}x(0) \le \lambda_{\max}\left(\tilde{S}_{\sigma(0)}\right)x^T(0)Rx(0), \quad \text{(a10)}
$$

$$
x^T(t)Rx(t) \le \lambda_{\min}^{-1}\left(\tilde{P}_{\sigma(t)}\right)x^T(t)P_{\sigma(t)}x(t). \quad \text{(a11)}
$$

From (a7)-(a10), we have

$$
\begin{aligned}
V(0) &= x^T(0)P_{\sigma(0)}x(0) + \int_{-d}^{0} e^{-\alpha_{\sigma(0)}s}x^T(s)Q_{\sigma(0)}x(s)ds \\
&\quad + \int_{-\tau}^{0} e^{-\alpha_{\sigma(0)}s}\dot{x}^T(s)Y_{\sigma(0)}\dot{x}(s)ds + \int_{-h}^{0}\int_{\theta}^{0} e^{-\alpha_{\sigma(0)}s}\dot{x}^T(s)S_{\sigma(0)}\dot{x}(s)dsd\theta \\
&\le \lambda_{\max}\left(\tilde{P}_{\sigma(0)}\right)x^T(0)Rx(0) + \lambda_{\max}\left(\tilde{Q}_{\sigma(0)}\right)de^{-\alpha_{\sigma(0)}d}\sup_{-d\le s\le 0} x^T(s)Rx(s) \\
&\quad + \lambda_{\max}\left(\tilde{Y}_{\sigma(0)}\right)\tau e^{-\alpha_{\sigma(0)}\tau}\sup_{-\tau\le s\le 0}\dot{x}^T(s)R\dot{x}(s) \\
&\quad + \lambda_{\max}\left(\tilde{S}_{\sigma(0)}\right)h^2 e^{-\alpha_{\sigma(0)}h}\sup_{-\tau\le s\le 0}\dot{x}^T(s)R\dot{x}(s) \\
&\le \Big(\lambda_{\max}\left(\tilde{P}_{\sigma(0)}\right) + \lambda_{\max}\left(\tilde{Q}_{\sigma(0)}\right)de^{-\alpha_{\sigma(0)}d} + \lambda_{\max}\left(\tilde{Y}_{\sigma(0)}\right)\tau e^{-\alpha_{\sigma(0)}\tau} \\
&\quad + \lambda_{\max}\left(\tilde{S}_{\sigma(0)}\right)h^2 e^{-\alpha_{\sigma(0)}h}\Big)c_1.
\end{aligned} \quad \text{(a12)}
$$

Let $\Lambda = \frac{\lambda_{\max}\left(\tilde{P}_{\sigma(0)}\right)}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)} + \frac{\lambda_{\max}\left(\tilde{Q}_{\sigma(0)}\right)}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)}he^{-\alpha_{\sigma(0)}h} + \frac{\lambda_{\max}\left(\tilde{Y}_{\sigma(0)}\right)}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)}he^{-\alpha_{\sigma(0)}h} + \frac{\lambda_{\max}\left(\tilde{S}_{\sigma(0)}\right)}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)}h^2 e^{-\alpha_{\sigma(0)}h}$.

From (a6), (a11) and (a12), we have

$$\begin{aligned} x^T(t)Rx(t) &< \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(0,T) + \frac{T_p(0,T)}{\tau_{ap}}\ln\mu_p\right\}\left(\Lambda c_1 + \frac{\lambda_5}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)}\delta_1^2\right) \\ &\le \exp\left\{\sum_{p=1}^{M} -\alpha_p T_p(0,T) + \frac{T_p(0,T)}{\tau_{ap}^*}\ln\mu_p\right\}\left(\Lambda c_1 + \frac{\lambda_5}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)}\delta_1^2\right). \end{aligned} \tag{a13}$$

From (15)-(19), we have

$$\begin{aligned} &\Lambda \le \lambda_{1,\sigma(0)} + \lambda_{2,\sigma(0)} h e^{-\alpha_{\sigma(0)}h} + \lambda_{3,\sigma(0)} h e^{-\alpha_{\sigma(0)}h} + \lambda_{4,\sigma(0)} h^2 e^{-\alpha_{\sigma(0)}h} \le \lambda_p, \\ &\frac{\lambda_5}{\lambda_{\min}\left(\tilde{P}_{\sigma(0)}\right)} \le \lambda_5. \end{aligned} \tag{a14}$$

Then, substituting (21), (a14) into (a13), and according to (20), we have

$$x^T(t)Rx(t) < c_2$$

This completes the proof. □

A2: Proof of Theorem 3.2.

Choosing the same Lyapunov-Krasovskii functional as in the proof of Theorem 3.1, after some mathematical manipulation, for $t \in [t_k, t_{k+1})$, $\sigma(t_k) = p$, we can get

$$\dot{V}(t) - \alpha_p V(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) \le \xi^T(t)\Psi_p\xi(t), \tag{a15}$$

where

$$\begin{aligned} \Psi_p = &\begin{bmatrix} \Xi & P_pA_{d,p} + d^{-1}e^{-\alpha_p h}S_p & P_pD_p & P_pE_p + C^TF \\ * & -e^{\alpha_p d}Q_p - d^{-1}e^{-\alpha_p h}S_p & 0 & 0 \\ * & * & -e^{\alpha_p \tau}Y_p & 0 \\ * & * & * & -\gamma^2 I + F^TF \end{bmatrix} \\ &+ \begin{bmatrix} A_p^T \\ A_{d,p}^T \\ D_p^T \\ E_p^T \end{bmatrix} (Y_i + hS_i) \begin{bmatrix} A_p^T \\ A_{d,p}^T \\ D_p^T \\ E_p^T \end{bmatrix}^T, \\ \Xi = {} & A_p^TP_p + P_pA_p + Q_p + C^TC - d^{-1}e^{-\alpha_p h}S_p - \alpha_p P_p, \\ \xi(t) = {} & \left[\ x^T(t) \quad x^T(t-d) \quad \dot{x}^T(t-\tau) \quad w^T(t)\ \right]. \end{aligned}$$

In view of LMI (30), we get

$$\dot{V}(t) - \alpha_p V(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) < 0. \tag{a16}$$

Letting $\gamma^2 w^T(s)w(s) - y^T(s)y(s) = \Gamma(s)$, following the proof line of (a5), for $\forall t \in [0,T]$, we have

$$\begin{aligned} V(t) < {} & \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(0,t)} \exp\left\{\sum_{p=1}^{M} \alpha_p T_p(0,t)\right\} V(0) \\ &+ \int_0^t \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(s,t)} \exp\left\{\sum_{p=1}^{M} \alpha_p T_p(s,t)\right\} \Gamma(s)ds. \end{aligned} \tag{a17}$$

Note that $\Gamma(s) \leq \gamma^2 w^T(s)w(s)$,

$$\begin{aligned} V(t) < & \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(0,t)} \exp\left\{\sum_{p=1}^{M} \alpha_p T_p(0,t)\right\} V(0) \\ & + \gamma^2 \int_0^t \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(s,t)} \exp\left\{\sum_{p=1}^{M} \alpha_p T_p(s,t)\right\} w^T(s)w(s)ds. \end{aligned} \tag{a18}$$

Following the same proof line of Theorem 3.1, from (30)-(36) and (a18), we can conclude that

$$x^T(t)Rx(t) < c_2.$$

Then from Definition 2.2, the switched neutral system (1) is finite-time bounded with respect to $(c_1, c_2, T, R, d_1^2, \sigma(t))$.

Under zero initial condition, we have $V(0) = 0$; thus

$$\begin{aligned} 0 < & \int_0^t \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(s,t)} \exp\left\{\sum_{p=1}^{M} \alpha_p T_p(s,t)\right\} \Gamma(s)ds \\ < & \prod_{p=1}^{M} \mu_p^{N_{\sigma,p}(0,t)} \exp\left\{\sum_{p=1}^{M} \alpha_p T_p(0,t)\right\} \int_0^t \Gamma(s)ds, \end{aligned}$$

which implies that

$$\int_0^t \Gamma(s)ds > 0.$$

It is equivalent to

$$\int_0^T y^T(t)y(t)dt < \gamma^2 \int_0^T w^T(t)w(t)dt.$$

By Definition 2.2, we know that system (1) is finite-time bounded with H_∞ performance γ^2. The proof is completed. □