International Journal of Innovative Computing, Information and Control
Volume **13**, Number **3**, June **2017**
ICIC International ©2017 ISSN 1349-4198
pp. 809–822

NEW STABILITY CRITERIA FOR A CLASS OF MARKOVIAN JUMPING GENETIC REGULATORY NETWORKS WITH TIME-VARYING DELAYS

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Received December 2016; revised April 2017

Abstract. *This paper addresses the problem of stability analysis for a class of genetic regulatory networks (GRNs) with Markovian jumping parameters and time-varying delays. By constructing a novel Lyapunov-Krasovskii functional (LKF) and using an appropriate enlargement scheme, new stability criteria are proposed in terms of linear matrix inequalities, which can guarantee the mean square stability of Markovian jumping GRNs. The novelty of this paper is that the less conservative stability criteria are obtained by utilizing Wirtinger-type integral inequality to estimate the weak infinitesimal operator of LKF. Furthermore, two illustrative examples are given to show the effectiveness of the theoretical results and the significant improvement on the existing results.*
Keywords: Genetic regulatory networks (GRNs), Markovian jumping parameters (MJPs), Stability, Linear matrix inequality (LMI), Time-varying delays

1. **Introduction.** In the past decade, the research of genetic regulatory networks (GRNs) has been attracting considerable attention due to important growth in a wide range of applications including genetic engineering and biological background areas, see, e.g., [1, 2, 3, 4, 5, 6, 7]. In nature, a GRN is a dynamic system to describe interactions among genes (mRNA) and its products (proteins). Generally speaking, mathematical models of GRNs have been broadly classified into two types: discrete-time models and continuous-time models. A continuous-time one can be described by a set of differential equations, in which the derivatives of the unknown functions can be introduced to characterize the continuous change of mRNA and protein concentrations [8, 9, 10, 11]. Consequently, the differential equation models have been used to understand the complex properties of GRNs.

As is well known, the existence of time delays often leads to unsatisfactory performances or even system instability [12, 13, 14, 15, 16, 17, 18]. In practical applications, time delays, caused by the finite translation speed in modeling gene regulation process, are primary factors impacting the dynamic behavior of whole gene network [19, 20, 21]. Also, the mathematical modeling of GRNs considering without delays may lead to wrong predictions of the concentrations of mRNAs and proteins. In recent years, a series of literature addressed the analysis and synthesized problems of delayed GRNs, which are referred to stability analysis and stabilization [22, 23], H_∞ filter design [24, 25, 26], state estimation [13, 20, 27] and so on.

In addition, the intrinsic noises and exogenous disturbances are always inevitable in most of practical gene regulation process. Note that the inherent noises may be generated randomly by individual molecules as well as the exogenous disturbances could affect gene systems under the assumption of the Brownian motion [8, 24, 27, 29]. As a result, it is the best viewed to modeling gene systems with a stochastic process. Note that the stability analysis plays an important role in designing or controlling delayed GRNs with Markovian jumping parameters (MJPs), which has encouraged the research in this field. To date, a number of effective techniques have been employed in literature to obtain less conservative stability criteria, such as free-weighting matrices techniques [8, 28], convex combination method [29], and delay (or delay-range) partition approaches [30, 31, 32]. More recently, it is worth mentioning that Wirtinger-type integral inequality is adopted in stability analysis of delayed GRNs to estimate accurately the derivative of LKF, which is helpful to obtain some less conservative stability criteria [10, 15, 21, 25].

In this paper, we investigate the stability problem for a class of stochastic GRNs with time-varying delays as well as seek further improvement covering the existing works. The main contributions of this study consist of the following aspects: (i) By choosing an appropriate LKF, new delay-dependent mean-square asymptotic stability criteria are given; (ii) Within this framework, the Wirtinger-type integral inequality and convex combination technique are introduced to estimate the double-integral items in the weak infinitesimal operator of the LKF, which are useful to establish the stability criteria; (iii) According to numerical comparisons, it is clear to observe that the stability criteria proposed in the paper are less conservative and more effective than the one in [29]. The rest of the paper is organized as follows: the problem is formulated and some preliminaries are given in Section 2; in Section 3, mean-square asymptotic stability criteria for the delayed stochastic GRNs are established; a numerical example is provided in Section 4; finally, we conclude this paper in Section 5.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ represents the set of all $m \times n$ real matrices, and I stands for the identity matrix with an appropriate dimension. Matrix $P > 0$ ($P \geq 0$) means P is positive definite (positive semi-definite). For a matrix A, A^T denotes its transpose. $\|\cdot\|$ denotes the Euclidean norm of vectors or matrices, $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the maximum and minimum eigenvalues of a real symmetric matrix respectively. Let $\mathrm{col}(\cdots)$ and $\mathrm{diag}(\cdots)$ be the block column matrix and diagonal matrix formed by the elements in brackets, respectively. $\mathcal{I}_N$ refers to the set $= \{1, 2, \ldots, N\}$ for any positive integer N.

2. **Problem Statement and Preliminaries.**

In this paper, we consider the following differential GRNs model composed of n mRNAs and n proteins with time-varying delays:

$$\dot{m}_i(t) = -a_i m_i(t) + \sum_{j=1}^{N} b_{ij} f_j(p_j(t - h(t))) + J_i, \tag{1a}$$

$$\dot{p}_i(t) = -c_i p_i(t) + d_i m_i(t - \tau(t)), \quad i = 1, 2, \ldots, n, \tag{1b}$$

where $m_i(t)$ and $p_i(t)$ are the concentrations of mRNA and protein of the ith gene at the time t, respectively. $a_i > 0$ and $c_i > 0$ denote the degradation rates of the mRNA and the protein, respectively. Let $d_i > 0$ denote the translation rate of the ith gene. The time-varying delays $h(t)$ and $\tau(t)$ are assumed to satisfy the following conditions

$$0 < \tau_1 \leq \tau(t) \leq \tau_2, \quad \mu_{11} \leq \dot{\tau}(t) \leq \mu_1, \quad 0 < h_1 \leq h(t) \leq h_2, \quad \mu_{22} \leq \dot{h}(t) \leq \mu_2, \tag{2}$$

b_{ij} is defined as follows

$$b_{ij} = \begin{cases} 0, & \text{if there is no connection between } j \text{ and } i, \\ \alpha_{ij}, & \text{if transcription factor } j \text{ activates gene } i, \\ -\alpha_{ij}, & \text{if transcription factor } j \text{ represses gene } i, \end{cases} \tag{3}$$

where α_{ij} represents the dimensionless transcriptional rate of transcription factor j to gene i, $J_i = \sum_{j=\mathcal{S}_i} \alpha_{ij}$ is the basal rates, in which $\mathcal{S}_i$ is the set of repressors of the i-th gene. Here, $f_j(x) = (x/\beta_j)^{\alpha_j}/(1 + (x/\beta_j)^{\alpha_j})$, $x \in \mathbb{R}$, which is a nonlinear feedback regulation function, where h_j is the Hill coefficient, and $\beta_j > 0$ is a constant.

Assumption 1. For $j = 1, 2, \ldots, n$, the nonlinear function $f_j(\nu)$ is continuous and bounded, and satisfies the following inequality:

$$f_j(0) = 0, \quad 0 \leq \frac{f_j(\nu_1) - f_j(\nu_2)}{\nu_1 - \nu_2} \leq h_j, \quad \nu_1 \neq \nu_2, \tag{4}$$

where h_j, $j = 1, 2, \ldots, n$, are the known constants.

Let $m(t) = \mathrm{col}(m_1(t), m_2(t), \ldots, m_n(t))$ and $p(t) = \mathrm{col}(p_1(t), p_2(t), \ldots, p_n(t))$. Then the dynamics of GRNs (1) can be written as the following form

$$\begin{cases} \dot{m}(t) = -Am(t) + Bf(p(t - h(t))) + J, \\ \dot{p}(t) = -Cp(t) + Dm(t - \tau(t)), \end{cases} \tag{5}$$

where $f(p(t-h(t))) = \mathrm{col}(f_1(p_1(t-h(t))), f_2(p_2(t-h(t))), \ldots, f_n(p_n(t-h(t))))$, $A = \mathrm{diag}(a_1, a_2, \ldots, a_n)$, $C = \mathrm{diag}(c_1, c_2, \ldots, c_n)$, $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$, $B = [b_{ij}]$, and $J = \mathrm{col}(J_1, J_2, \ldots, J_n)$.

Denote

$$x(t) = m(t) - m^*, \quad y(t) = p(t) - p^*, \tag{6}$$

where (m^*, p^*) represents the equilibrium point of GRN (5), i.e., (m^*, p^*) is a solution of the following equation:

$$\begin{cases} -Am^* + Bf(p^*) + J = 0, \\ -Cp^* + Dm^* = 0. \end{cases} \tag{7}$$

Then, shifting the intended equilibrium point (m^*, p^*) of (5) to the origin, we obtain

$$\begin{cases} \dot{x}(t) = -Ax(t) + Bg(y(t - h(t))), \\ \dot{y}(t) = -Cy(t) + Dx(t - \tau(t)), \end{cases} \tag{8}$$

where $g(s) = f(s + p^*) - f(p^*)$ satisfying the following conditions:

$$g_i(0) = 0, \quad 0 \leq \frac{g_i(x)}{x} \leq h_i, \quad i = 1, 2, \ldots, n, \quad \forall 0 \neq x \in \mathbb{R}.$$

Next, taking the Markovian jumping parameters into (8), we have

$$\begin{cases} \dot{x}(t) = -A_\imath x(t) + B_\imath g(y(t - h(t))), \\ \dot{y}(t) = -C_\imath y(t) + D_\imath x(t - \tau(t)), \end{cases} \tag{9}$$

where

$$\begin{bmatrix} A_\imath & B_\imath & C_\imath & D_\imath \end{bmatrix} = \begin{bmatrix} A(r(t)) & B(r(t)) & C(r(t)) & D(r(t)) \end{bmatrix}.$$

Here, $r(t)$ $(t \geq 0)$ represents a Markov process on the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ which takes values in a finite state space $\mathcal{I}_N$ with generator $\Gamma = [\gamma_{\imath\jmath}]_{N\times N}$, and the mode transition probabilities are given by

$$\gamma_{\imath\jmath} := P\{r(t + \Delta t) = \jmath | r(t) = \imath\} = \begin{cases} \gamma_{\imath\jmath}\Delta t + o(\Delta t), & \imath \neq \jmath, \\ 1 + \gamma_{\imath\jmath}\Delta t + o(\Delta t), & \imath = \jmath, \end{cases}$$

where $\Delta t > 0$ satisfying $\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0$, $\gamma_{\imath\jmath} \geq 0$ ($\jmath \neq \imath$) is the transition rate from mode $\imath$ to mode $\jmath$, and $\gamma_{\imath\imath} = -\sum_{\jmath=1,\jmath\neq\imath}^{N} \gamma_{\imath\jmath}$.

Remark 2.1. *According to* [8, 24, 27]*, the system matrices of* (8) *may change randomly by Markov process, due to the inherent noises of individual molecules as well as the exogenous disturbances could affect gene system under the assumption of Brownian motion. Therefore, it is necessary to consider MJPs in a real GRN model.*

Definition 2.1. *Let* $(x(t,\phi,\imath_0), y(t,\varphi,\imath_0))$ *denote the solution of* (9)*, then the system* (9) *is said to be mean-square asymptotically stable if, for each* $\varepsilon > 0$*, there exists a* $\delta(\varepsilon) > 0$ *such that*

$$\mathbb{E}||x(t,\phi,\imath_0)||_2^2 < \varepsilon, \quad \mathbb{E}||y(t,\varphi,\imath_0)||_2^2 < \varepsilon$$

and

$$\lim_{t\to\infty} \mathbb{E}\left\{||x(t,\phi,\imath_0)||_2^2\right\} = 0, \quad \lim_{t\to\infty} \mathbb{E}\left\{||y(t,\varphi,\imath_0)||_2^2\right\} = 0,$$

when the initial condition $(\phi,\varphi,\imath_0)$ *satisfies*

$$\sup_{-\tau^*\leq s\leq 0} ||\phi(s)||_2^2 < \delta(\varepsilon), \quad \sup_{-\tau^*\leq s\leq 0} ||\varphi(s)||_2^2 < \delta(\varepsilon),$$

where $\tau^* = \max\{\tau_2, h_2\}$.

Definition 2.2. [33] *Let* $V(x_t,t,r(t)=\imath)$ *be the stochastic positive LKF. The weak infinitesimal operator is defined as*

$$\begin{aligned} &\mathcal{L}V(x_t,t,r(t)=\imath) \\ &= \lim_{\Delta t\to 0} \frac{1}{\Delta t}\left[\varepsilon\left\{V(x|_{t+\Delta t}), t+\Delta t, r(t+\Delta t)|x_t, r(t)=\imath\right\} - V(x(t),t,\imath)\right]. \\ &= V_t(x_t,t,\imath) + V_x(x_t,t,\imath)f(x_t,t,\imath) + \sum_{\jmath=1}^{N}\gamma_{\imath\jmath}V(x_t,t,\jmath), \end{aligned} \tag{10}$$

where

$$V_t(x_t,t,\imath) = \frac{\partial V(x_t,t,\imath)}{\partial t}, \quad V_x(x_t,t,\imath) = \left(\frac{\partial V(x_t,t,\imath)}{\partial x_1}, \ldots, \frac{\partial V(x_t,t,\imath)}{\partial x_n}\right).$$

The main purpose of this paper is to investigate stability problem for the delayed GRN (9) with Markovian jumping parameters. In order to establish the delay-dependent stability criteria, we introduce the following lemmas.

Lemma 2.1. [34] ***(Jensen's Inequality)*** *For given a positive definite matrix* $M^T = M \in \mathbb{R}^{n\times n}$*, a scalar* $\tau > 0$*, and a vector function* $\omega(\cdot) : [-\tau, 0] \to \mathbb{R}^n$ *such that the integrations concerned are well defined, then the following inequities hold:*

$$\int_{-\tau}^{0} \omega^T(s)M\omega(s)\mathrm{d}s \geq \frac{1}{\tau}\left(\int_{-\tau}^{0}\omega(s)\mathrm{d}s\right)^T M \left(\int_{-\tau}^{0}\omega(s)\mathrm{d}s\right),$$

$$\int_{-\tau}^{0}\int_{\theta}^{0} \omega^T(s)M\omega(s)\mathrm{d}s\mathrm{d}\theta \geq \frac{2}{\tau^2}\left(\int_{-\tau}^{0}\int_{\theta}^{0}\omega(s)\mathrm{d}s\mathrm{d}\theta\right)^T M \left(\int_{-\tau}^{0}\int_{\theta}^{0}\omega(s)\mathrm{d}s\mathrm{d}\theta\right).$$

Lemma 2.2. [35] ***(Wirtinger-Type Integral Inequality)*** *For given a positive definite matrix* $M^T = M \in \mathbb{R}^{n\times n}$ *of appropriate size, two scalars* a *and* b *with* $a < b$*, and a derivable vector function* $\omega(\cdot) : [a,b] \to \mathbb{R}^n$*, then the following inequality holds:*

$$\int_a^b \dot{\omega}^T(s)M\dot{\omega}(s)\mathrm{d}s \geq \frac{1}{b-a}\begin{bmatrix}\Omega_0\\ \Omega_1\end{bmatrix}^T \tilde{M}\begin{bmatrix}\Omega_0\\ \Omega_1\end{bmatrix}, \tag{11}$$

where $\Omega_0 = \omega(b) - \omega(a)$, $\Omega_1 = \omega(b) + \omega(a) - \frac{2}{b-a}\int_a^b \omega(s)\mathrm{d}s$*, and* $\tilde{M} = \mathrm{diag}(M, 3M)$.

Lemma 2.3. [36] ***(Lower Bounds Theorem)*** *Let $f_1, f_2, \ldots, f_n : \mathbb{R}^m \to \mathbb{R}$ have positive finite values in an open subset $\mathbb{D} \subseteq \mathbb{R}^m$. Then the reciprocally convex combination of f_i over $\mathbb{D}$ satisfies*

$$\min_{\left\{\alpha_i:\alpha_i>0,\sum_i \alpha_i=1\right\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t) \tag{12}$$

subject to

$$g_{i,j} : \mathbb{R}^m \to \mathbb{R},\ g_{i,j}(t) = g_{j,i}(t),\ \begin{bmatrix} f_i(t) & g_{j,i}(t) \\ g_{j,i}(t) & f_i(t) \end{bmatrix} \geq 0. \tag{13}$$

3. **Main Results.** In this section, we discuss the stability problem of GRN (9) by defining a novel LKF and using the Wirtinger-type integral inequality. Before introducing the main results, the following notations are denoted:

$$e_i = [0_{n\times(i-1)n} \quad I \quad 0_{n\times(18-i)n}], \quad i = 1, 2, \ldots, 18,$$

$$\xi_1(t) = \mathbf{col}\left(x(t), x(t-\tau_1), \int_{t-\tau_1}^{t} x(s)\mathrm{d}s, \int_{t-\tau_2}^{t-\tau_1} x(s)\mathrm{d}s\right),$$

$$\xi_2(t) = \mathbf{col}\left(y(t), y(t-h_1), \int_{t-h_1}^{t} y(s)\mathrm{d}s, \int_{t-h_2}^{t-h_1} y(s)\mathrm{d}s\right),$$

$$L_1(\tau) = \mathbf{col}\left(e_1, e_3, e_{13}, (\tau-\tau_1)e_{14} + (\tau_2-\tau)e_{15}\right),$$

$$L_2(h) = \mathbf{col}\left(e_6, e_8, e_{16}, (h-h_1)e_{17} + (h_2-h)e_{18}\right),$$

$$L_{1\imath} = \mathbf{col}\left(-A_\imath e_1 + B_\imath e_{13}, e_5, e_1 - e_3, e_3 - e_4\right),\ \imath \in \mathcal{I}_N,$$

$$L_{2\imath} = \mathbf{col}\left(-C_\imath e_6 + D_\imath e_2 e_{10} e_6 - e_8 e_8 - e_9\right),\ \imath \in \mathcal{I}_N,$$

$$\begin{aligned}\xi(t) = \mathrm{col}\Bigg(& x(t), x(t-\tau(t)), x(t-\tau_1), x(t-\tau_2), \dot{x}(t-\tau_1), y(t), \\ & y(t-h(t)), y(t-h_1), y(t-h_2), \dot{y}(t-h_1), f(y(t)), \\ & f(y(t-h(t))), \int_{t-\tau_1}^{t} x(s)\mathrm{d}s, \frac{1}{\tau(t)-\tau_1}\int_{t-\tau(t)}^{t-\tau_1} x(t)\mathrm{d}s, \\ & \frac{1}{\tau_2-\tau(t)}\int_{t-\tau_2}^{t-\tau(t)} x(s)\mathrm{d}s, \int_{t-h_1}^{t} y(s)\mathrm{d}s, \frac{1}{h(t)-h_1}\int_{t-h(t)}^{t-h_1} y(s)\mathrm{d}s, \\ & \frac{1}{h_2-h(t)}\int_{t-h_2}^{t-h(t)} y(s)\mathrm{d}s \Bigg),\end{aligned} \tag{14}$$

$$R = \mathbf{col}\left(K_3, K4\right),\ T = \mathbf{col}\left(K_5, K6\right),$$

$$\widetilde{X_i} = \mathbf{diag}\left(X_i, 3X_i\right),\ \widetilde{U_i} = \mathbf{diag}\left(U_i, 3U_i\right),\ i = 1, 2, 3, 4,$$

$$K_1 = \mathbf{col}\left(e_1 - e_3, e_1 + e_3 - \frac{2}{\tau_1}e_{13}\right),\ K_2 = \mathbf{col}\left(e_3 - e_2, e_3 + e_2 - 2e_{14}\right),$$

$$K_3 = \mathbf{col}\left(e_6 - e_8, e_6 + e_8 - \frac{2}{h_1}e_{16}\right),\ K_4 = \mathbf{col}\left(e_2 - e_4, e_2 + e_4 - 2e_{15}\right),$$

$$K_5 = \mathbf{col}\left(e_8 - e_7, e_8 + e_7 - 2e_{17}\right),\ K_6 = \mathbf{col}\left(e_7 - e_9, e_7 + e_9 - 2e_{18}\right).$$

Based on the previous preparation, a new stability criterion for the stochastic GRN (9) is proposed as follows.

Theorem 3.1. *Consider GRN* (9) *with Assumption 1. For given scalars* $\tau_2 > \tau_1 > 0$, $h_2 > h_1 > 0$, μ_1, μ_2, μ_{11} *and* μ_{22} *satisfying* (2), *the trivial solution of GRN* (9) *is mean-square asymptotically stable if there exist positive-definite matrices* $P_{1\imath} > 0$, $P_{2\imath} > 0$ $(\imath = 1, 2)$, $Q_l > 0$ $(l = 1, \ldots, 8)$, $S > 0$, $X_b > 0$, $U_b > 0$, $G_b > 0$ $(b = 1, \ldots, 4)$, *diagonal matrices* $\triangle := \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n)$, *and matrices* M *and* N, *of appropriate sizes, such that the following inequalities hold for* $\tau \in \{\tau_1, \tau_2\}$ *and* $h \in \{h_1, h_2\}$:

$$J_1 := \begin{bmatrix} \widetilde{X_2} & M \\ M^T & \widetilde{X_2} \end{bmatrix} \geq 0, \ J_2 := \begin{bmatrix} \widetilde{X_4} & N \\ N^T & \widetilde{X_4} \end{bmatrix} \geq 0,$$

$$\Phi_\imath(\tau, h) = \Phi_{1\imath}(\tau, h) + \Phi_{2\imath} + \Phi_{3\imath} + \Phi_{4\imath}(\tau, h) + \Phi_{5\imath} - R^T J_1 R - T^T J_2 T < 0, \quad \imath \in \mathcal{I}_N, \tag{15}$$

where

$$\begin{aligned} \Phi_{1\imath}(\tau, h) = {} & L_1^T(\tau) P_{1\imath} L_{1\imath} + L_{1\imath}^T P_{1\imath} L_1(\tau) + L_2^T(h) P_{2\imath} L_{2\imath} + L_{2\imath}^T P_{2\imath} L_2(h) \\ & + L_1^T(\tau) \left(\sum_{\jmath=1}^{2} \gamma_{\imath\jmath} P_{1\jmath} \right) L_1(\tau) + L_2^T(h) \left(\sum_{\jmath=1}^{2} \gamma_{\imath\jmath} P_{2\jmath} \right) L_2(h), \end{aligned}$$

$$\Phi_{2\imath} = -2e_{11}^T \Delta C_\imath e_6 + 2e_{11}^T \Delta D_\imath e_2 + e_{11}^T S e_{11} - (1 - \mu_2) e_{12}^T S e_{12},$$

$$\begin{aligned} \Phi_{3\imath} = {} & e_1^T Q_1 e_1 - e_3^T Q_1 e_3 + e_3^T Q_2 e_3 - (1 - \mu_1) e_2^T Q_2 e_2 + (1 - \mu_{11}) e_2^T Q_3 e_2 - e_4^T Q_3 e_4 \\ & + e_6^T Q_4 e_6 - e_8^T Q_4 e_8 + e_8^T Q_5 e_8 - (1 - \mu_2) e_7^T Q_5 e_7 + (1 - \mu_{22}) e_7^T Q_6 e_7 - e_9^T Q_6 e_9 \\ & + (-A_\imath e_1 + W_\imath e_{12})^T Q_7 (-A_\imath e_1 + W_\imath e_{12}) - e_5^T Q_7 e_5 \\ & + (-C_\imath e_6 + D_\imath e_2)^T Q_8 (-C_\imath e_6 + D_\imath e_2) - e_{10}^T Q_8 e_{10}, \end{aligned}$$

$$\begin{aligned} \Phi_{4\imath}(\tau, h) = {} & \tau_1^2 (-A_\imath e_1 + W_\imath e_{12})^T X_1 (-A_\imath e_1 + W_\imath e_{12}) - K_1^T \widetilde{X_1} K_1 + (\tau_2 - \tau_1)^2 e_5^T X_2 e_5 \\ & - \frac{1}{\tau_1} e_{13}^T G_1 e_{13} + h_1^2 (-C_\imath e_6 + D_\imath e_2)^T X_3 (-C_\imath e_6 + D_\imath e_2) - K_3^T \widetilde{X_3} K_3 \\ & + (h_2 - h_1)^2 e_{10}^T X_4 e_{10} + \tau_1 e_1^T G_1 e_1 + (\tau_2 - \tau_1) e_3^T G_2 e_3 + h_1 e_6^T G_3 e_6 \\ & - \frac{1}{h_1} e_{16}^T G_3 e_{16} + (h_2 - h_1) e_8^T G_4 e_8 - (\tau - \tau_1) e_{14}^T G_2 e_{14} - (\tau_2 - \tau) e_{15}^T G_2 e_{15} \\ & - (h - h_1) e_{17}^T G_4 e_{17} - (h_2 - h) e_{18}^T G_4 e_{18}, \end{aligned}$$

$$\begin{aligned} \Phi_{5\imath} = {} & \frac{\tau_1^4}{2} (-A_\imath e_1 + W_\imath e_{12})^T U_1 (-A_\imath e_1 + W_\imath e_{12}) - 2(\tau_1 e_1 - e_{13})^T U_1 (\tau_1 e_1 - e_{13}) \\ & + \frac{(\tau_2 - \tau_1)^2}{2} e_5^T U_2 e_5 - 2(e_3 - e_{14})^T U_2 (e_3 - e_{14}) + \frac{(h_2 - h_1)^2}{2} e_{10}^T U_4 e_{10} \\ & - 2(e_2 - e_{15})^T U_2 (e_2 - e_{15}) - 2(h_1 e_6 - e_{16})^T U_3 (h_1 e_6 - e_{16}) \\ & + \frac{h_1^4}{2} (-C_\imath e_6 + D_\imath e_2)^T U_3 (-C_\imath e_6 + D_\imath e_2) \\ & - 2(e_8 - e_{17})^T U_4 (e_8 - e_{17}) - 2(e_7 - e_{18})^T U_4 (e_7 - e_{18}). \end{aligned}$$

Proof: Firstly, we define the following LKF candidate for GRN (9):

$$V(\imath, t, x(t), y(t)) = V_1(\imath, t, x(t), y(t)) + \sum_{k=2}^{5} V_k(t, x(t), y(t)), \tag{16}$$

where

$$V_1(\imath, t, x(t), y(t)) = \xi_1^T(t) P_{1\imath} \xi_1(t) + \xi_2^T(t) P_{2\imath} \xi_2(t),$$

$$V_2(t,x(t),y(t)) = 2\sum_{j=1}^{n}\delta_j\int_0^{y_j(t)} f_j(s)\mathrm{d}s + \int_{t-h(t)}^{t} f^T(y(s))Sf(y(s))\mathrm{d}s,$$

$$\begin{aligned}V_3(t,x(t),y(t)) \;=\; & \int_{t-\tau_1}^{t} x^T(s)Q_1x(s)\mathrm{d}s + \int_{t-\tau(t)}^{t-\tau_1} x^T(s)Q_2x(s)\mathrm{d}s \\ & + \int_{t-\tau_2}^{t-\tau(t)} x^T(s)Q_3x(s)\mathrm{d}s + \int_{t-h_1}^{t} y^T(s)Q_4y(s)\mathrm{d}s \\ & + \int_{t-h(t)}^{t-h_1} y^T(s)Q_5y(s)\mathrm{d}s + \int_{t-h_2}^{t-h(t)} y^T(s)Q_6y(s)\mathrm{d}s \\ & + \int_{t-\tau_1}^{t} \dot{x}^T(s)Q_7\dot{x}(s)\mathrm{d}s + \int_{t-h_1}^{t} \dot{y}^T(s)Q_8\dot{y}(s)\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_4(t,x(t),y(t)) \;=\; & \tau_1\int_{-\tau_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)X_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + h_1\int_{-h_1}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)X_3\dot{y}(s)\mathrm{d}s\mathrm{d}\theta \\ & +(\tau_2-\tau_1)\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1} \dot{x}^T(s)X_2\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ & +(h_2-h_1)\int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} \dot{y}^T(s)X_4\dot{y}(s)\mathrm{d}s\mathrm{d}\theta \\ & +\int_{-\tau_1}^{0}\int_{t+\theta}^{t} x^T(s)G_1x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t-\tau_1} x^T(s)G_2x(s)\mathrm{d}s\mathrm{d}\theta \\ & +\int_{-h_1}^{0}\int_{t+\theta}^{t} y^T(s)G_3y(s)\mathrm{d}s\mathrm{d}\theta + \int_{-h_2}^{-h_1}\int_{t+\theta}^{t-h_1} y^T(s)G_4y(s)\mathrm{d}s\mathrm{d}\theta,\end{aligned}$$

$$\begin{aligned}V_5(t,x(t),y(t)) \;=\; & {\tau_1}^2\int_{-\tau_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta \\ & +\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{-\tau_1}\int_{t+\lambda}^{t-\tau_1} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta \\ & +{h_1}^2\int_{-h_1}^{0}\int_{\theta}^{0}\int_{t+\lambda}^{t} \dot{y}^T(s)U_3\dot{y}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta \\ & +\int_{-h_2}^{-h_1}\int_{\theta}^{-h_1}\int_{t+\lambda}^{t-h_1} \dot{y}^T(s)U_4\dot{y}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\theta.\end{aligned}$$

According to Definition 2.2, the weak infinitesimal operator $\mathcal{L}$ of the stochastic process $\{r(t)=\imath,\, t\geq 0\}$ is given by

$$\mathcal{L}V(\imath,t,x(t),y(t)) = \mathcal{L}V_1(\imath,t,x(t),y(t)) + \sum_{k=2}^{5}\mathcal{L}V_k(t,x(t),y(t)). \tag{17}$$

It is not difficult to obtain that

$$\xi_1(t) = L_1(\tau(t))\xi(t), \quad \xi_2(t) = L_2(h(t))\xi(t), \tag{18}$$

$$\dot{\xi}_1(t) = L_{1\imath}\xi(t), \quad \dot{\xi}_2(t) = L_{2\imath}\xi(t). \tag{19}$$

In view of (10), (18), and (19), we obtain

$$\mathcal{L}V_1(\imath,t,x(t),y(t)) = \xi^T(t)\Phi_{1\imath}(\tau(t),h(t))\xi(t). \tag{20}$$

From (2), we have

$$\begin{aligned}\mathcal{L}V_2(t,x(t),y(t)) \le 2f^T(y(t))\Delta\dot{y}(t) + f^T(y(t))Sf(y(t))\\ - (1-\mu_2)f^T(t-h(t))Sf(t-h(t)).\end{aligned} \tag{21}$$

Substituting (9) into (21) yields

$$\mathcal{L}V_2(t,x(t),y(t)) \le \xi^T(t)\Phi_{2\imath}\xi(t). \tag{22}$$

Similarly, it follows that

$$\begin{aligned}&\mathcal{L}V_3(t,x(t),y(t))\\ &\le x^T(t)Q_1x(t) - x^T(t-\tau_1)Q_1x(t-\tau_1) + x^T(t-\tau_1)Q_2x(t-\tau_1)\\ &\quad - (1-\mu_1)x^T(t-\tau(t))Q_2x(t-\tau(t)) + (1-\mu_{11})x^T(t-\tau(t))Q_3x(t-\tau(t))\\ &\quad - x^T(t-\tau_2)Q_3x(t-\tau_2) + y^T(t)Q_4y(t) - y^T(t-h_1)Q_4y(t-h_1)\\ &\quad + y^T(t-h_1)Q_5y(t-h_1) - (1-\mu_2)y^T(t-h(t))Q_5y(t-h(t))\\ &\quad + (1-\mu_{22})y^T(t-h(t))Q_6y(t-h(t)) + \dot{x}^T(t)Q_7\dot{x}(t) - y^T(t-h_2)Q_6y(t-h_2)\\ &\quad - \dot{x}^T(t-\tau_1)Q_7\dot{x}(t-\tau_1) + \dot{y}^T(t)Q_8\dot{y}(t) - \dot{y}^T(t-h_1)Q_8\dot{y}(t-h_1)\\ &\le \xi^T(t)\Phi_{3\imath}\xi(t)\end{aligned} \tag{23}$$

and

$$\begin{aligned}&\mathcal{L}V_4(\imath,t,x(t),y(t))\\ &\le \tau_1^2\dot{x}^T(t)X_1\dot{x}(t) - \tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)X_1\dot{x}(s)\mathrm{d}s + (\tau_2-\tau_1)^2\dot{x}^T(t-\tau_1)X_2\dot{x}(t-\tau_1)\\ &\quad - (\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)X_2\dot{x}(s)\mathrm{d}s + h_1^2\dot{y}^T(t)X_3\dot{y}(t) - h_1\int_{t-h_1}^{t}\dot{y}^T(s)X_3\dot{y}(s)\mathrm{d}s\\ &\quad + (h_2-h_1)^2\dot{y}^T(t-h_1)X_4\dot{y}(t-h_1) - (h_2-h_1)\int_{t-h_2}^{t-h_1}\dot{y}^T(s)X_4\dot{y}(s)\mathrm{d}s\\ &\quad + \tau_1x^T(t)G_1x(t) - \int_{t-\tau_1}^{t}x^T(s)G_1x(s)\mathrm{d}s + (\tau_2-\tau_1)x^T(t-\tau_1)G_2x(t-\tau_1)\\ &\quad - \int_{t-\tau_2}^{t-\tau_1}x^T(s)G_2x(s)\mathrm{d}s + h_1y^T(t)G_3y(t) - \int_{t-h_1}^{t}y^T(s)G_3y(s)\mathrm{d}s\\ &\quad - \int_{t-h_2}^{t-h_1}y^T(s)G_4y(s)\mathrm{d}s + (h_2-h_1)y^T(t-h_1)G_4y(t-h_1).\end{aligned} \tag{24}$$

By using Lemma 2.1, we have

$$\int_{t-\tau_1}^{t}x^T(s)G_1x(s)\mathrm{d}s \ge \frac{1}{\tau_1}\xi^T(t)e_{13}^TG_1e_{13}\xi(t), \tag{25a}$$

$$\int_{t-h_1}^{t}y^T(s)G_3y(s)\mathrm{d}s \ge \frac{1}{h_1}\xi^T(t)e_{16}^TG_3e_{16}\xi(t), \tag{25b}$$

$$\int_{t-\tau(t)}^{t-\tau_1}x^T(s)G_2x(s)\mathrm{d}s \ge \frac{1}{\tau(t)-\tau_1}\xi^T(t)e_{14}^TG_2e_{14}\xi(t), \tag{26a}$$

$$\int_{t-\tau_2}^{t-\tau(t)}x^T(s)G_2x(s)\mathrm{d}s \ge \frac{1}{\tau_2-\tau(t)}\xi^T(t)e_{15}^TG_2e_{15}\xi(t), \tag{26b}$$

$$\int_{t-h(t)}^{t-h_1} y^T(s)G_4y(s)\mathrm{d}s \geq \frac{1}{h(t)-h_1}\xi^T(t)e_{17}^T G_4 e_{17}\xi(t), \tag{27a}$$

$$\int_{t-h_2}^{t-h(t)} y^T(s)G_4y(s)\mathrm{d}s \geq \frac{1}{h_2-h(t)}\xi^T(t)e_{18}^T G_4 e_{18}\xi(t). \tag{27b}$$

Also, based on Lemma 2.2, we obtain

$$-\tau_1\int_{t-\tau_1}^{t} \dot{x}^T(s)X_1\dot{x}(s)\mathrm{d}s \leq -\xi^T(t)K_1^T\widetilde{X_1}K_1\xi(t), \tag{28a}$$

$$-h_1\int_{t-h_1}^{t} \dot{y}^T(s)X_3\dot{y}(s)\mathrm{d}s \leq -\xi^T(t)K_3^T\widetilde{X_3}K_3\xi(t), \tag{28b}$$

$$\begin{aligned} &-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)X_2\dot{x}(s)\mathrm{d}s \\ \leq &-\frac{\tau_2-\tau_1}{\tau(t)-\tau_1}\xi^T(t)K_2^T\widetilde{X_2}K_2\xi(t) - \frac{\tau_2-\tau_1}{\tau_2-\tau(t)}\xi^T(t)K_4^T\widetilde{X_2}K_4\xi(t) \end{aligned} \tag{29a}$$

$$\begin{aligned} &-(h_2-h_1)\int_{t-h_2}^{t-h_1} \dot{y}^T(s)X_4\dot{y}(s)\mathrm{d}s \\ \leq &-\frac{h_2-h_1}{h(t)-h_1}\xi^T(t)K_5^T\widetilde{X_4}K_5\xi(t) - \frac{h_2-h_1}{h_2-h(t)}\xi^T(t)K_6^T\widetilde{X_4}K_6\xi(t). \end{aligned} \tag{29b}$$

Then, according to (25)-(29), and (9), we can rewrite (24) as

$$\begin{aligned} \mathcal{L}V_4(t,x(t),y(t)) \leq \xi^T(t)\Bigg(&\Phi_{4\imath}(\tau(t),h(t)) - \frac{\tau_2-\tau_1}{\tau(t)-\tau_1}K_2^T\widetilde{X_2}K_2 - \frac{\tau_2-\tau_1}{\tau_2-\tau(t)}K_4^T\widetilde{X_2}K_4 \\ &- \frac{h_2-h_1}{h(t)-h_1}K_5^T\widetilde{X_4}K_5 - \frac{h_2-h_1}{h_2-h(t)}K_6^T\widetilde{X_4}K_6\Bigg)\xi(t). \end{aligned} \tag{30}$$

By using Lemma 2.3, we have,

$$\mathcal{L}V_4(t,x(t),y(t)) \leq \xi^T(t)\left[\Phi_{4\imath}(\tau(t),h(t)) - R^TJ_1R - T^TJ_2T\right]\xi(t). \tag{31}$$

Next, based on the second inequality of Lemma 2.1, it is known that

$$\mathcal{L}V_5(t,x(t),y(t)) \leq \xi^T(t)\Phi_{5\imath}\xi(t). \tag{32}$$

Thus, combining (20), (22), (23), (31), (32) with (17), we have

$$\mathcal{L}V(\imath,t,x(t),y(t)) \leq \xi^T(t)\Phi(\tau(t),h(t))\xi(t) < 0. \tag{33}$$

Taking the mathematical expectation on both sides of (33), it implies

$$\mathbb{E}\left\{\mathcal{L}V(\imath,t,x(t),y(t))\right\} \leq \mathbb{E}\left\{\xi^T(t)\Phi(\tau(t),h(t))\xi(t)\right\}. \tag{34}$$

Since $\Phi(\tau(t),h(t))$ depends affinely on τ and h, we obtain from (15) that $\Phi(\tau(t),h(t)) < 0$, $\forall\tau\in[\tau_1,\tau_2]$, $h\in[h_1,h_2]$. So,

$$\mathbb{E}\left\{\mathcal{L}V(\imath,t,x(t),y(t))\right\} \leq -\varpi\left(\mathbb{E}\left\{||x(t,\phi,\imath_0)||_2^2 + ||y(t,\varphi,\imath_0)||_2^2\right\}\right) < 0, \tag{35}$$

where $\varpi = \min_{\tau\in[\tau_1,\tau_2],h\in[h_1,h_2]}\lambda_{\min}(-\Phi(\tau,h))$, which means

$$\mathbb{E}\left\{\mathcal{L}V(\imath,t,x(t),y(t))\right\} \leq -\varpi\mathbb{E}||x(t,\phi,\imath_0)||_2^2,\ \mathbb{E}\left\{\mathcal{L}V(\imath,t,x(t),y(t))\right\} \leq -\varpi\mathbb{E}||y(t,\varphi,\imath_0)||_2^2. \tag{36}$$

Then, integrate both sides of (36) from 0 to

$$\begin{aligned}&\mathbb{E}\{V(\imath,t,x(t),y(t))-V(\imath(0),0,x(0),y(0))\}\le-\varpi\int_0^t\mathbb{E}||x(s,\phi,\imath_0)||_2^2\mathrm{d}s,\\&\mathbb{E}\{V(\imath,t,x(t),y(t))-V(\imath(0),0,x(0),y(0))\}\le-\varpi\int_0^t\mathbb{E}||y(s,\varphi,\imath_0)||_2^2\mathrm{d}s.\end{aligned}\tag{37}$$

According to the LKF (16), one can know that

$$\mathbb{E}V(\imath,t,x(t),y(t))>0.$$

Then, we have

$$\begin{aligned}&\int_0^t\mathbb{E}||x(s,\phi,\imath_0)||_2^2\mathrm{d}s<\varpi^{-1}V(\imath(0),0,x(0),y(0)),\\&\int_0^t\mathbb{E}||y(s,\varphi,\imath_0)||_2^2\mathrm{d}s<\varpi^{-1}V(\imath(0),0,x(0),y(0)).\end{aligned}\tag{38}$$

Letting $z(t)=\begin{bmatrix}x^T(t) & y^T(t)\end{bmatrix}$, it follows from (38) that the solution $z(t)$ of system (9) is uniformly bounded, which implies $z(t)$ is uniformly continuous. Applying the Barbalat's Lemma to (38) yields

$$\lim_{t\to\infty}\mathbb{E}||x(t,\phi,\imath_0)||_2^2=0,\quad \lim_{t\to\infty}\mathbb{E}||y(t,\varphi,\imath_0)||_2^2=0.$$

On the other hand, it is not difficult to obtain that

$$\mathbb{E}\{V(\imath,t,x(t),y(t))\}<\mathbb{E}\{V(\imath,0,x(0),y(0))\}.\tag{39}$$

Note that

$$\begin{aligned}\mathbb{E}\{V(\imath(0),0,x(0),y(0))\}&=V_1(\imath,0,x(0),y(0))+\sum_{k=2}^{5}V_k(0,x(0),y(0))\\&\le\lambda_{11}||\phi(s)||_2^2+\lambda_{22}||\varphi(s)||_2^2,\end{aligned}\tag{40}$$

$$\begin{aligned}&\mathbb{E}\left\{V(\imath,t,x(t),y(t))\right\}\ge\lambda_{\min}(P_{1\imath})\mathbb{E}\left\{||x(t,\phi,\imath_0)||_2^2\right\}\ge\lambda_{\min}(P_{1,2})\mathbb{E}\left\{||x(t,\phi,\imath_0)||_2^2\right\},\\&\mathbb{E}\left\{V(\imath,t,x(t),y(t))\right\}\ge\lambda_{\min}(P_{1,2})\mathbb{E}\left\{||y(t,\varphi,\imath_0)||_2^2\right\},\end{aligned}\tag{41}$$

where $\lambda_{\min}(P_{1,2})$ is the minimum eigenvalue of $\mathrm{diag}(P_{1\imath},P_{2\imath})$, and

$$\begin{aligned}\lambda_{11}=&\,\tau_2[\lambda_{\max}(P_{1\imath})+\lambda_{\max}(Q_1)+\lambda_{\max}(Q_2)+\lambda_{\max}(Q_3)+\lambda_{\max}(Q_7)+\lambda_{\max}(X_1)\\&+\lambda_{\max}(X_2)+\lambda_{\max}(G_1)+\lambda_{\max}(G_2)+\lambda_{\max}(U_1)+\lambda_{\max}(U_2)],\\\lambda_{22}=&\,h_2[\lambda_{\max}(P_{2\imath})+\lambda_{\max}(Q_4)+\lambda_{\max}(Q_5)+\lambda_{\max}(Q_6)+\lambda_{\max}(Q_8)+\lambda_{\max}(X_3)\\&+\lambda_{\max}(X_4)+\lambda_{\max}(G_3)+\lambda_{\max}(G_4)+\lambda_{\max}(U_3)+\lambda_{\max}(U_4)].\end{aligned}\tag{42}$$

Therefore, from (39)-(41), we have

$$\begin{aligned}\mathbb{E}||x(t,\phi,\imath_0)||_2^2&\le\frac{\lambda_{11}||\phi(t)||_2^2+\lambda_{22}||\varphi(t)||_2^2}{\lambda_{\min}(P_{1,2})},\\\mathbb{E}||y(t,\varphi,\imath_0)||_2^2&\le\frac{\lambda_{11}||\phi(t)||_2^2+\lambda_{22}||\varphi(t)||_2^2}{\lambda_{\min}(P_{1,2})}.\end{aligned}\tag{43}$$

Furthermore, for any $\varepsilon>0$, choose

$$\delta(\varepsilon)=\min\left\{\frac{\varepsilon\lambda_{\min}(P_{1,2})}{2\lambda_{11}},\frac{\varepsilon\lambda_{\min}(P_{1,2})}{2\lambda_{22}}\right\}.$$

Then $\mathbb{E}||x(t,\phi,\imath_0)||_2^2<\varepsilon$ and $\mathbb{E}||y(t,\varphi,\imath_0)||_2^2<\varepsilon$ when $\sup_{-\tau^*\le s\le0}||\phi(s)||_2^2<\delta(\varepsilon)$ and $\sup_{-\tau^*\le s\le0}||\varphi(s)||_2^2<\delta(\varepsilon)$. Based on Definition 2.1, the Markovian jumping GRN (9) is mean-square asymptotically stable. The proof is completed. □

Remark 3.1. *Note that the methodology of this work is different from the ones in* [29, 30, 31, 32]. *By constructing a novel LKF and utilizing Wirtinger-type integral inequality technique, a new stability criterion is proposed to ensure the mean square stability of the stochastic Markovian jumping GRNs with time-varying delays.*

Next, when $\mu_{11} = \mu_{22} = 0$, the following corollary is obtained easily from Theorem 3.1.

Corollary 3.1. *Consider GRN* (9) *with Assumption 1. For given scalars* $\tau_2 > \tau_1 > 0$, $h_2 > h_1 > 0$, μ_1 *and* μ_2 *satisfying* (2) *with* $\mu_{11} = \mu_{22} = 0$, *the trivial solution of GRN* (9) *is mean-square asymptotically stable if there exist positive-definite matrices* $P_{1\imath} > 0$, $P_{2\imath} > 0$ $(\imath = 1, 2)$, $Q_l > 0$ $(l = 1, 2, 4, 5, 7, 8)$, $S > 0$, $X_b > 0$, $U_b > 0$, $G_b > 0$ $(b = 1, \ldots, 4)$, *diagonal matrices* $\triangle := \mathrm{diag}(\delta_1, \delta_2, \ldots, \delta_n)$, *and matrices* M *and* N *of appropriate sizes, such that the following inequalities hold with* $\tau \in \{\tau_1, \tau_2\}$ *and* $h \in \{h_1, h_2\}$:

$$J_1 \geq 0,\ \ J_2 \geq 0,\ \ \Phi_{1\imath}(\tau, h) + \Phi_{2\imath} + \tilde{\Phi}_{3\imath} + \Phi_{4\imath}(\tau, h) + \Phi_{5\imath} - R^T J_1 R - T^T J_2 T < 0,\ \ \imath \in \mathcal{I}_N,$$

where

$$\begin{aligned}\tilde{\Phi}_{3\imath} &= e_1^T Q_1 e_1 - e_3^T Q_1 e_3 + e_3^T Q_2 e_3 - (1-\mu_1) e_2^T Q_2 e_2 + e_6^T Q_4 e_6 - e_8^T Q_4 e_8 + e_8^T Q_5 e_8 \\ &\quad - (1-\mu_2) e_7^T Q_5 e_7 + (-A_\imath e_1 + W_\imath e_{12})^T Q_7 (-A_\imath e_1 + W_\imath e_{12}) - e_5^T Q_7 e_5 \\ &\quad + (-C_\imath e_6 + D_\imath e_2)^T Q_8 (-C_\imath e_6 + D_\imath e_2) - e_{10}^T Q_8 e_{10}.\end{aligned}$$

Remark 3.2. *In contrast to* [29, Theorem 3.1], *Corollary 3.1 is a less conservative stability criterion for stochastic GRNs, which will be illustrated by one example in the next section.*

4. **A Numerical Example.**

In this section, an example is provided to demonstrate the effectiveness of the proposed results in this paper.

Example 4.1. *Consider the GRN model described by* (9) *with the following parameters:*

$$A_1 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix},\quad B_1 = \begin{bmatrix} 1 & -2 \\ 0.8 & 0 \end{bmatrix},\quad C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix},\quad B_2 = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},\quad C_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\quad D_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is assumed that $f(x) = x^2/(1+x^2)$, *and then,* $K = \begin{bmatrix} 0.65 & 0 \\ 0 & 0.65 \end{bmatrix}$. *Also, the transition probabilities are selected as* $\Gamma = \begin{bmatrix} -0.4 & 0.4 \\ 0.6 & -0.6 \end{bmatrix}$.

When $\tau_1 = 0.2$, $\mu_1 = 1$, $h_1 = 0.1$, $\mu_2 = 0.1$, *and* $h_2 = 0.3$. *Firstly, by using the YALMIP toolbox of MATLAB to solve LMI* (15), *the maximal allowable upper bounds* τ_2 *can be obtained by Corollary 3.1 and* [29, Theorem 3.1], *which are shown in Table 1. It can be seen from Table 1 that Corollary 3.1 is the less conservative than* [29, Theorem 3.1]. *Also, when the initial conditions are chosen as* $\phi_0 \equiv [0.62 \quad 0.52]^T$, $\psi_0 \equiv [0.81 \quad 0.43]^T$, *the simulation results of the trajectories are given in Figure 1. In addition, when* $\tau_1 = 0.2$, $\mu_1 = 1$, $\mu_{11} = \mu_{22} = 0.01$, $h_1 = 0.1$, $\mu_2 = 0.1$, *and* $h_2 = 0.3$, *we can obtain the maximal allowable upper bound as* $\tau_2 = 3.29$ *by Theorem 3.1.*

Remark 4.1. *It follows from Example 4.1 that, the less-conservative calculation results compared with* [29, Theorem 3.1] *can be obtained by solving the LMIs in Corollary 3.1. Thus, our results are obviously applicable.*

TABLE 1. Maximal allowable values of τ_2 for Example 4.1

Methods	$\max \tau_2$
[29, Theorem 3.1]	*0.46*
Corollary 3.1	*3.03*

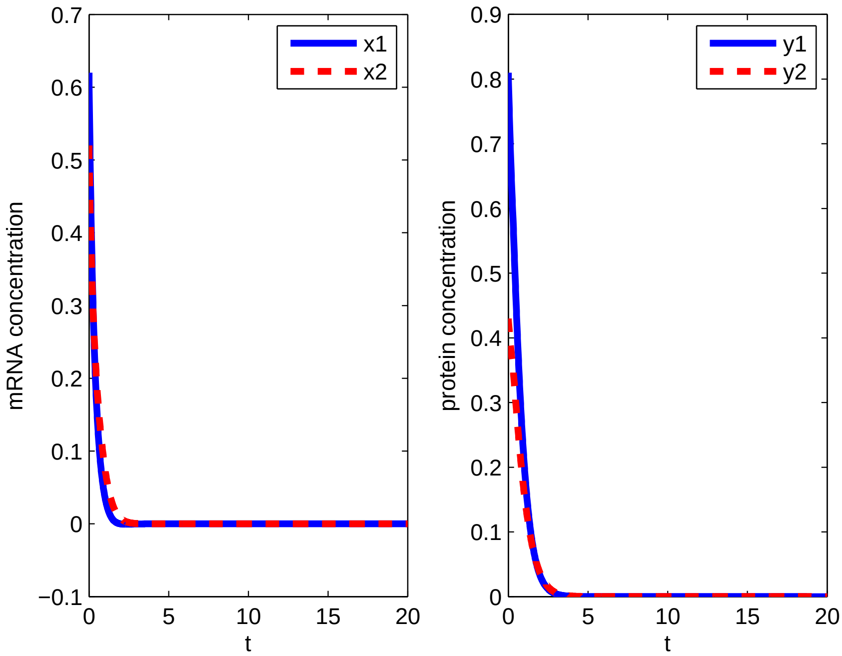

FIGURE 1. Trajectories of mRNA and protein concentrations for Example 4.1

5. **Conclusion.** In this study, we have investigated the stability problem for the Markovian jumping GRNs with time-varying delays. A novel delay-dependent LKF has been introduced to establish stability analysis, and Wirtinger-type integral inequality and convex technique have been employed to estimate the weak infinitesimal operator of LKF; then, a less-conservative stability criterion has been derived. Finally, the numerical examples have been provided to illustrate the advantage of the proposed method.

Acknowledgment. This work was partially supported by the National Natural Science Foundation of China (11371006), the Natural Science Foundation of Heilongjiang Province (F201326, A201416), and the Fund of Heilongjiang Education Committee (12541603). The authors thank the anonymous associate editor and referees for their helpful comments and suggestions.

REFERENCES

[1] H. De Jong, Modeling and simulation of genetic regulatory systems: A literature review, *Journal of Computational Biology*, vol.9, no.1, pp.67-103, 2002.

[2] S. Hardy and P. N. Robillard, Modeling and simulation of molecular biology systems using Petri nets: Modeling goals of various approaches, *Journal of Bioinformatics and Computational Biology*, vol.2, no.4, pp.619-637, 2004.

[3] L. Chen and K. Aihara, Stability of genetic regulatory networks with time delay, *IEEE Trans. Circuits and Systems I: Fundamental Theory and Applications*, vol.49, no.5, pp.602-608, 2002.

[4] J. D. Cao and F. Ren, Exponential stability of discrete-time genetic regulatory networks with delays, *IEEE Trans. Neural Networks*, vol.19, no.3, pp.520-523, 2008.

[5] C. Xu, M. Liao and Q. Zhang, On the mean square exponential stability for a stochastic fuzzy cellular neural network with distributed delays and time-varying delays, *International Journal of Innovative Computing, Information and Control*, vol.11, no.1, pp.247-256, 2015.

[6] L. X. Zhang, Y. Z. Zhu, P. Shi and Y. X. Zhao, Resilient asynchronous H_∞ filtering for Markovian jump neural networks with unideal measurements and multiplicative noises, *IEEE Trans. Cybernetics*, vol.45, no.12, pp.2840-2852, 2015.

[7] X. Zhang, L. G. Wu and J. H. Zou, Globally asymptotic stability analysis for genetic regulatory networks with mixed delays: An M-matrix-based approach, *IEEE/ACM Trans. Computational Biology and Bioinformatics*, vol.13, no.1, pp.135-147, 2016.

[8] W. B. Zhang, J. A. Fang and Y. Tang, Stochastic stability of Markovian jumping genetic regulatory networks with mixed time delays, *Applied Mathematics and Computation*, vol.217, no.17, pp.7210-7225, 2011.

[9] W. B. Zhang, Y. Tang, X. T. Wu and J. A. Fang, Stochastic stability of switched genetic regulatory networks with time-varying delays, *IEEE Trans. Nanobioscience*, vol.13, no.3, pp.336-342, 2014.

[10] X. F. Fan, X. Zhang, L. G. Wu and M. Shi, Finite-time stability analysis of reaction-diffusion genetic regulatory networks with time-varying delays, *IEEE/ACM Trans. Computational Biology and Bioinformatics*, DOI: 10.1109/TCBB.2016.2552519, 2016.

[11] T. T. Liu, X. Zhang and X. Y. Gao, Stability analysis for continuous-time and discrete-time genetic regulatory networks with delays, *Applied Mathematics and Computation*, vol.274, pp.628-643, 2016.

[12] Z. G. Wu, P. Shi, H. Y. Su and J. Chu, Sampled-data exponential synchronization of complex dynamical networks with time-varying coupling delay, *IEEE Trans. Neural Networks and Learning Systems*, vol.24, no.8, pp.1177-1187, 2013.

[13] X. Zhang, L. G. Wu, Y. Y. Han and Y. T. Wang, State estimation for delayed genetic regulatory networks with reaction-diusion terms, *IEEE Trans. Neural Networks and Learning Systems*, DOI:10.1109/TNNLS.2016.2618899, 2016.

[14] P. Shi, Y. Zhang, M. Chadli and R. Agarwal, Mixed H_∞ and passive filtering for discrete fuzzy neural networks with stochastic jumps and time delays, *IEEE Trans. Neural Networks and Learning Systems*, vol.27, no.4, pp.903-909, 2016.

[15] Y. T. Wang, X. H. Zhang and X. Zhang, Neutral-delay-range-dependent absolute stability criteria for neutral-type Lur'e systems with time-varying delays, *Journal of the Franklin Institute*, vol.353, no.18, pp.5025-5039, 2016.

[16] Y. T. Wang, Y. Xue and X. Zhang, Less conservative robust absolute stability criteria for uncertain neutral-type Lur'e systems with time-varying delays, *Journal of the Franklin Institute*, vol.353, no.4, pp.816-833, 2016.

[17] H. Chen, P. Shi, C. Lim and P. Hu, Exponential stability for neutral stochastic Markov systems with time-varying delay and its applications, *IEEE Trans. Cybernetics*, vol.46, no.6, pp.1350-1362, 2016.

[18] R. Yang, Z. Zhang and P. Shi, Exponential stability on stochastic neural networks with discrete interval and distributed delays, *IEEE Trans. Neural Networks*, vol.21, no.1, pp.169-175, 2010.

[19] W. Q. Wang, S. K. Nguang, S. M. Zhong and F. Liu, Robust stability analysis of stochastic delayed genetic regulatory networks with polytopic uncertainties and linear fractional parametric uncertainties, *Communications in Nonlinear Science and Numerical Simulation*, vol.19, no.5, pp.1569-1581, 2014.

[20] T. H. Lee, S. Lakshmanan, J. H. Park and P. Balasubramaniam, State estimation for genetic regulatory networks with mode-dependent leakage delays, time-varying delays, and Markovian jumping parameters, *IEEE Trans. Nanobioscience*, vol.12, no.4, pp.363-375, 2013.

[21] X. Zhang, L. G. Wu and S. C. Cui, An improved integral to stability analysis of genetic regulatory networks with interval time-varying delays, *IEEE/ACM Trans. Computational Biology and Bioinformatics*, vol.12, no.2, pp.398-409, 2015.

[22] X. Zhang, Y. Y. Han, L. G. Wu and J. H. Zou, M-matrix-based globally asymptotic stability criteria for genetic regulatory networks with time-varying discrete and unbounded distributed delays, *Neurocomputing*, vol.174, pp.1060-1069, 2016.

[23] X. Zhang, A. H. Yu and G. D. Zhang, M-matrix-based delay-rangedependent global asymptotical stability criterion for genetic regulatory networks with time-varying delays, *Neurocomputing*, vol.113, pp.8-15, 2013.

[24] B. Shen, Z. D. Wang, J. L. Liang and X. H. Liu, Sampled-data H_∞ filtering for stochastic genetic regulatory networks, *International Journal of Robust and Nonlinear Control*, vol.21, no.15, pp.1759-1777, 2011.

[25] Y. T. Wang, X. Zhang and Z. R. Hu, Delay-dependent robust H_∞ filtering of uncertain stochastic genetic regulatory networks with mixed time-varying delays, *Neurocomputing*, vol.166, pp.346-356, 2015.

[26] Y. T. Wang, X. M. Zhou and X. Zhang, H_∞ filtering for discretetime genetic regulatory networks with random delay described by a Markovian chain, *Abstract and Applied Analysis*, vol.2014, Article ID 257971, 2014.

[27] B. Lv, J. L. Liang and J. D. Cao, Robust distributed state estimation for genetic regulatory networks with Markovian jumping parameters, *Communications in Nonlinear Science and Numerical Simulation*, vol.16, no.10, pp.4060-4078, 2011.

[28] W. B. Zhang, J. A. Fang and Y. Tang, Robust stability for genetic regulatory networks with linear fractional uncertainties, *Communications in Nonlinear Science and Numerical Simulation*, vol.17, no.4, pp.1753-1765, 2012.

[29] G. He, J. A. Fang and X. Wu, Robust stability of Markovian jumping genetic regulatory networks with mode-dependent delays, *Mathematical Problems in Engineering*, vol.2012, Article ID 504378, 2012.

[30] Y. T. Wang, A. H. Yu and X. Zhang, Robust stability of stochastic genetic regulatory networks with time-varying delays: A delay fractioning approach, *Neural Computing and Applications*, vol.23, no.5, pp.1217-1227, 2013.

[31] S. Lakshmanan, F. A. Rihan, R. Rakkiyappan and J. H. Park, Stability analysis of the differential genetic regulatory networks model with time-varying delays and Markovian jumping parameters, *Nonlinear Analysis: Hybrid Systems*, vol.14, pp.1-15, 2014.

[32] T. T. Yu, J. Wang and X. Zhang, A less conservative stability criterion for delayed stochastic genetic regulatory networks, *Mathematical Problems in Engineering*, vol.2014, Article ID 768483, 2014.

[33] X. R. Mao, Stability of stochastic differential equations with Markovian switching, *Stochastic Processes and Their Applications*, vol.79, no.1, pp.45-67, 1999.

[34] K. Gu, An integral inequality in the stability problem of time-delay systems, *Proc. of the 39th IEEE Conference on Decision and Control*, pp.2805-2810, 2000.

[35] A. Seuret and F. Gouaisbaut, Wirtinger-based integral inequality: Application to time-delay systems, *Automatica*, vol.49, no.9, pp.2860-2866, 2013.

[36] P. G. Park, J. W. Ko and C. Jeong, Reciprocally convex approach to stability of systems with time-varying delays, *Automatica*, vol.47, no.1, pp.235-238, 2011.