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POLYNOMIAL STATIC OUTPUT FEEDBACK H_∞ CONTROL FOR CONTINUOUS-TIME LINEAR SYSTEMS VIA DESCRIPTOR APPROACH

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Abstract. *This paper deals with the problem of the robust static output feedback H_∞ control (SOFC) for continuous linear systems with polytopic uncertainties. The controller has been gotten by the use of descriptor redundancy. Under this approach a sufficient condition is provided for the existence of a solution to the problem. Thus, the advantage of this method is to obtain more free matrices in the design condition, also the polynomial approach helps to have a less conservative result. In the end, the performance of the method is shown by several examples.*

Keywords: Robust static output feedback H_∞ control, Linear matrix inequalities (LMIs), Polytopic uncertainty, Homogenous matrices

1. **Introduction.** The problem of the output feedback H_∞ controller design has been extensively studied in recent years for linear continuous systems with polytopic uncertainties using LMI-based convex conditions and has been programmed by means of simple interfaces (LMI control toolbox, YALMIP) and solved by efficient algorithms (LMI toolbox and SeDuMi) [3, 4, 5]. On the other hand, in control theory and practice output feedback control is very useful since it can be easily implemented with low cost [1]. As a specific case, the robust output feedback controller design for linear systems with polytopic uncertainties has been considered in many researches [9, 10, 11, 31]. The design problem of such controller can be represented as a bilinear matrix inequality (BMI) problem, which is non-convex [25, 26].

Moreover, when the problem is to synthesize a robust controller for uncertain systems, Lypunov theory and LMI can also be applied. However, even synthesis problems that are convex for the precisely known case, as in the synthesis of state feedback stabilizing gain, become non convex when the considered system is uncertain; consequently, the LMI conditions are only sufficient. The synthesis of a robust static output feedback gain for uncertain systems is an even more involved problem, being the main subject of research in several papers [9, 10, 11, 12, 13].

Concerning uncertain systems with parameters in a polytopic domain, the analysis conditions for precisely known systems can be adapted and applied to the vertices of the polytopic, imposing the same matrix for entire set. Such approach, however, have results on conditions that are only sufficient and usually conservative. To reduce the conservativeness, the matrix variable may also be parameter-dependent [6, 7]. Moreover, use homogenous polynomial parameter dependent (HPPD) variables with various degrees [2, 8, 17, 18, 27, 29], in order to get less conservative results.

Motivated by the fact that for the SOF controllers problems, the conditions guaranteeing the existence of H_∞ control law are given in terms of BMI, recently, the descriptor approach was an alternative solution to get good results [14]. Thus, the use of the descriptor formulation may help to avoid appearance of crossing terms between the system matrices and the designed ones.

This paper tackles the static output feedback H_∞ control problem for linear continuous-time system based on polynomial approach. The interest is to design a static output feedback control for polytopic system, which guaranteed a prescribed H_∞ performance level. The main idea is to rewrite the closed loop system by the use of descriptor redundancy where the target is to separate system matrices and the controller gain. Different from [24] that uses optimization solution "fminsearch" [30], an iterative LMI based procedure involving decision variables is proposed in this paper to improve the H_∞ SOF controllers, which gives opportunity to use more slack variables in order to reduce the conservatism. In the end, numerical examples are provided to make clear the efficiency of the proposed approach.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are presented in Section 2. In Section 3, we present two results. First, we obtain a new H_∞ performance analysis for the SOF control. Thereafter, based on this analysis, we propose a design method for H_∞ SOF control. Simulation studies are given to demonstrate the approach effectiveness in Section 4. Section 5 concludes this paper.

Notation: we use standard notations throughout this paper. The notation $P > 0$ (< 0) is used for positive (negative) definite matrices. $*$ stands for the symmetric term of the diagonal elements of square symmetric matrix. I_m denotes the identity matrix of dimension m. The superscript "T" and the notation sym(Y) represent respectively the transpose and $Y + Y^T$.

2. **System Description and Problem Statement.**

Consider the following state-space representation for a class of uncertain continuous-time

$$\begin{cases} \dot{x}(t) = A(\alpha)x(t) + E(\alpha)w(t) + B(\alpha)u(t) \\ z(t) = C_1(\alpha)x(t) + F(\alpha)w(t) + D(\alpha)u(t) \\ y(t) = C_2(\alpha)x(t) + H(\alpha)w(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, $y(t) \in \Re^p$ is the output measurement, $u(t) \in \Re^m$ is the control input vector and $w(t) \in \Re^f$ is the exogenous input in $L_2[0,\infty)$ and $z(t) \in \Re^q$ the controlled output variable. The matrices $A(\alpha)$, $B(\alpha)$, $E(\alpha)$, $C_1(\alpha)$, $D(\alpha)$, $F(\alpha)$, $C_2(\alpha)$ and $H(\alpha)$ are the system matrices of appropriate dimensions.

The time-invariant uncertain matrices belong to a polytopic domain given by:

$$M(\alpha) = \sum_{i=1}^{N} \alpha_i M_i, \quad \alpha \in \Lambda_N$$

$$\Lambda_N = \left\{ \alpha \in \Re^N : \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0, i = 1, \dots, N \right\}$$

Matrix $M(\alpha)$ represents any uncertain matrix given in (1), N is the number of vertices of the polytopic, $M_i = 1, \ldots, N$ are the vertices and Λ_N is the unit simplex.

The output-feedback controller under consideration is of the form

$$u(t) = Ky(t) \tag{2}$$

where $K \in \Re^{m,p}$ is the controller gain to be designed.

In [14], LMI based design for state feedback controller using the descriptor redundancy has been proposed to reduce computational cost. To take advantage of a descriptor redundancy formulation, (1) and (2) can be easily rewritten with the above defined notations respectively as:

$$\left\{ \begin{array}{l} \tilde{E}\dot{\xi}(t) = \tilde{A}(\alpha)\xi(t) + \tilde{B}(\alpha)w(t) \\ z(t) = \tilde{C}(\alpha)\xi(t) + \tilde{F}(\alpha)w(t) \end{array} \right. \tag{3}$$

where $\xi(t) = \begin{bmatrix} x(t)^T & y(t)^T & u(t)^T \end{bmatrix}^T$ and $\tilde{E} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\tilde{A}(\alpha) = \begin{bmatrix} A(\alpha) & 0 & B(\alpha) \\ C_2(\alpha) & -I & 0 \\ 0 & K & -I \end{bmatrix}$, $\tilde{C}(\alpha) = [C_1(\alpha) \quad 0 \quad D(\alpha)]$, $\tilde{F}(\alpha) = F(\alpha)$, $\tilde{B}(\alpha) = \begin{bmatrix} E(\alpha) \\ H(\alpha) \\ 0 \end{bmatrix}$.

Remark 2.1. *Rewriting the closed loop system (1) by the use of descriptor redundancy* [14] *allows to avoid appearance of crossing terms between the system matrices and the designed ones (i.e., K). It makes it easier for the LMI formulation of synthesis conditions.*

For the closed-loop system represented as in (3), we define H_∞ performance as follows.

- The closed-loop system (3) is asymptotically stable when $w(t) = 0$.
- The closed-loop system (3) has a prescribed level γ of H_∞ noise attenuation; i.e., the zero initial condition $x(0) = 0$ the condition
$$\|z(t)\|_2 < \gamma \|w(t)\|_2 \tag{4}$$
is satisfied for any nonzero $w(t) \in L_2[0, \infty)$ over the entire polytope Λ_N.

3. **Main Result.** In this section, we shall present a numerically efficient technique to find the SOFC gains in such a way to stabilize the system in closed loop, i.e., to ensure that (3) is asymptotically stable. We summarize the resulting LMI design conditions in the following theorem.

Theorem 3.1. *Suppose that the controller gain K is known, the system in (3) is asymptotically stable if for a prescribed scalar $\gamma > 0$ of H_∞ performance, if there exists matrice $X(\alpha)$ such that the following conditions are satisfied:*

$$\Psi = \begin{bmatrix} \aleph(\alpha) & \tilde{C}^T(\alpha) & X^T(\alpha)\tilde{B}(\alpha) \\ * & -I & \tilde{F}(\alpha) \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{5}$$

$$\tilde{E}^T X(\alpha) = X(\alpha)^T \tilde{E} \geq 0 \tag{6}$$

where $\aleph(\alpha) = \tilde{A}^T(\alpha)X(\alpha) + X^T(\alpha)\tilde{A}(\alpha)$.

Proof: Considering the following Lyapunov function candidate:

$$V = \xi^T(t)\tilde{E}^T X(\alpha)\xi(t) \tag{7}$$

where

$$\tilde{E}^T X(\alpha) = X(\alpha)^T \tilde{E} \geq 0$$

with $X(\alpha)$ having the following form:

$$X(\alpha) = \begin{bmatrix} X_{11}(\alpha) & X_{12}(\alpha) & X_{13}(\alpha) \\ X_{21}(\alpha) & X_{22}(\alpha) & X_{23}(\alpha) \\ X_{31}(\alpha) & X_{32}(\alpha) & X_{33}(\alpha) \end{bmatrix} \tag{8}$$

in order to satisfy (6), it is clear that $X_{11}(\alpha) = X_{11}(\alpha)^T > 0$; moreover, the matrices $X_{12}(\alpha)$ and $X_{13}(\alpha)$ should be equal to zero. We have the derivative of Lyapunov function (7) over the dynamic system (3) as follows:

$$\dot{V} = \dot{\xi}^T(t)\tilde{E}^T X(\alpha)\xi(t) + \xi^T(t)X^T(\alpha)\tilde{E}\dot{\xi}(t)$$

Let

$$\begin{aligned} J =& \dot{V} + z^T(t)z(t) - \gamma^2 w^T(t)w(t) \\ =& \left[\xi^T(t)\tilde{A}^T(\alpha) + w^T(t)\tilde{B}^T(\alpha)\right] X(\alpha)\xi(t) + \xi^T(t)X^T(\alpha)\left[\tilde{A}(\alpha)\xi(t) + \tilde{B}(\alpha)w(t)\right] \\ &+ \left[\xi^T(t)\tilde{C}^T(\alpha) + w^T(t)\tilde{F}^T(\alpha)\right]\left[\tilde{C}(\alpha)\xi(t) + \tilde{F}(\alpha)w(t)\right] - \gamma^2 w^T(t)w(t) \end{aligned}$$

by taking $\zeta(t) = \begin{bmatrix} \xi(t)^T & w(t)^T \end{bmatrix}^T \neq 0$ then J becomes as the following:

$$J = \zeta(t)^T \, \Phi \, \zeta(t)$$

with

$$\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 \\ \Phi_2^T & \Phi_3 \end{bmatrix}$$

where $\Phi_1 = sym\left(\tilde{A}^T(\alpha)X(\alpha)\right) + \tilde{C}^T(\alpha)\tilde{C}(\alpha)$, $\Phi_2 = X^T(\alpha)\tilde{B}(\alpha) + \tilde{C}^T(\alpha)\tilde{F}(\alpha)$, $\Phi_3 = \tilde{F}^T(\alpha)\tilde{F}(\alpha) - \gamma^2 I$, so $J < 0$ for any $\zeta(t) = \left[\xi(t)^T \quad w(t)^T\right]^T \neq 0$ if $\Phi < 0$. by applying Schur complement and some row transformations on Φ, LMI (5) is obtained.

If (5) is satisfied, we have $J < 0$ for any $\zeta(t) = \left[\xi(t)^T \quad w(t)^T\right]^T \neq 0$, which implies $V(\xi(\infty)) - V(\xi(0)) + \int_0^\infty z^T(t)z(t)dt - \gamma^2\int_0^\infty w^T(t)w(t)dt < 0$. With zero initial condition $\xi(0) = 0$ and $V(\xi(\infty)) > 0$, we obtain $\int_0^\infty z^T(t)z(t)dt < \gamma^2\int_0^\infty w^T(t)w(t)dt$ for any nonzero $w(t) \in L_2[0,\infty)$. This ends the proof. □

Remark 3.1. *The static output feedback H_∞ controller design was founded by many approaches* [21, 23, 24]. *However, it was necessary to impose some equality constraints on the systems matrices* [21, 23], *also by making conditions about their rank. Moreover, they require to give some descriptions about the output matrix $C_2(\alpha)$ if it belongs to the polytopic domain or not* [24]. *Thus, in our study the constraints given* [24] *have been considered and by avoiding the ones proposed in* [21, 23], *our approach presents suitable results for general case.*

Now, based on Theorem 3.1 we present our aim result on the static output feedback H_∞ controller design in the following theorem.

Theorem 3.2. *The system in (3) is asymptotically stable for a prescribed scalar $\gamma > 0$ of H_∞ performance, if there exist matrices $X_{11}(\alpha) > 0$, $X_{21}(\alpha)$, $X_{22}(\alpha)$, $X_{23}(\alpha)$, X_{33}, T_1, T_2 and Z satisfying the following matrix inequality:*

$$\begin{bmatrix} \Pi_{11} & * & * & * & * \\ \Pi_{21} & \Pi_{22} & * & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * & * \\ \Pi_{41} & H^T X_{22}(\alpha) & H^T X_{23}(\alpha) & -\gamma^2 I & * \\ C_1(\alpha) & 0 & D(\alpha) & F^T(\alpha) & -I \end{bmatrix} < 0 \tag{9}$$

where

$$\begin{aligned}
\Pi_{11} &= A^T(\alpha)X_{11}(\alpha) + X_{11}(\alpha)A(\alpha) + G^T(\alpha)X_{21}(\alpha) + X_{21}^T(\alpha)G(\alpha).\\
\Pi_{21} &= -X_{21}(\alpha) + Z^T T_1 + X_{22}^T(\alpha)G(\alpha).\\
\Pi_{22} &= -X_{22}(\alpha) - X_{22}^T(\alpha) + Z^T T_2 + T_2^T Z_2.\\
\Pi_{31} &= B^T(\alpha)X_{11}(\alpha) + X_{23}^T(\alpha)G(\alpha).\\
\Pi_{32} &= -X_{33}T_2 - X_{23}^T(\alpha) + Z.\\
\Pi_{33} &= -X_{33} - X_{33}^T.\\
\Pi_{41} &= E^T(\alpha)X_{11}(\alpha) + H^T(\alpha)X_{21}(\alpha).
\end{aligned}$$

with

$$G(\alpha) = \begin{cases} C_2C_2^TC_2, & \textit{if } C_2(\alpha) \textit{ is fixed, i.e., } C_2(\alpha) = C_2 \textit{ and } C_2 \textit{ is of full row rank;}\\ C_2, & \textit{if } C_2(\alpha) \textit{ is fixed, i.e., } C_2(\alpha) = C_2 \textit{ and } C_2 \textit{ is of non-full row rank;}\\ C_2(\alpha), & \textit{if } C_2(\alpha) \textit{ is non-fixed.}\end{cases}$$

and the static output feedback H_∞ controller law is given by:

$$K = X_{33}^{-T}Z \tag{10}$$

Proof: Based on Theorem 3.1, the system in (3) is asymptotically stable with H_∞ performance γ, if conditions (5) and (6) are satisfied. Note that the system matrices in these conditions contain uncertainties. We assume that matrix $X_{11}(\alpha) > 0$ depends on the system uncertainties and to cast the condition (6) into LMI, we define matrices $X(\alpha)$ as:

$$X(\alpha) = \begin{bmatrix} X_{11}(\alpha) & 0 & 0\\ X_{21}(\alpha) & X_{22}(\alpha) & X_{23}(\alpha)\\ X_{33}T_1 & X_{33}T_2 & X_{33}\end{bmatrix} \tag{11}$$

which makes an obligation to put $X_{11}(\alpha)$ as follows

$$X_{11}(\alpha) > 0$$

now by assuming that (5) and (6) hold and by considering the structure given of $X(\alpha)$, we replace each element by its equivalent in Theorem 3.1 then we obtain the following transformation:

$$Z = X_{33}^T K \tag{12}$$

and from Π_{33} we conclude that X_{33} is invertible then (9) and (10) hold. This complete the proof. □

Remark 3.2. *By using the descriptor redundancy formulation we have obtained a sufficient condition for the design of the controller law. The target is to separate system matrices and the controller gain by means of free matrices derives from the descreptor approach, which helps to reduce the conservatism.*

Remark 3.3. *To solve the robust LMI condition of Theorem 3.2, the technique given in* [17] *in the aim to handle robust LMIs with parameters in the unit simplex can be applied. The parameter-dependent variables in Theorem 3.2 are supposed as homogenous polynomials of arbitrary degree g, although different degrees can be used producing results with distinct complexity and accuracy. Let $Q_g(\alpha)$ be any parameter dependent variable in (10), of arbitrary degree g, denoted by:*

$$Q_g(\alpha) = \sum_{k\in\mathbf{k}(g)} \alpha_1^{k_1}\dots\alpha_N^{k_N}Q_k, \qquad k = k_1k_2\dots k_N \tag{13}$$

where $\alpha_1^{k_1} \dots \alpha_N^{k_N}$, $\alpha \in \Lambda_N$, $k_i \in \mathbb{Z}^+$, $i = 1, \dots, N$ *are the monomials, and* $Q_k \in \mathbb{R}$, $\forall k \in \mathbf{k}(g)$ *are matrix-valued coefficients.* $\mathbf{k}(g)$ *is the* N*-tuples obtained as all possible combinations of non-negative integers* K_i, $i = 1, \dots, N$, *such that* $k_1 + k_2 + \dots + k_N = g$. *To illustrate this notation, consider a homogenous polynomial of degree* $g = 2$ *with* $N = 2$. *The set* $\mathbf{k}$ *is given by* $\mathbf{k} = 02, 11, 20$, *corresponding to the generic from* $Q_2(\alpha) = \alpha_2^2 Q_{02} + \alpha_1\alpha_2 Q_{11} + \alpha_1^2 Q_{20}$. *This choice for the decision variables provides less conservative results with the increase of* g *at the price of greater complexity and computational effort. Robust LMIs with parameters in the unit simplex can be fully characterized by means of homogenous polynomial solutions, without loss of generality* [18]. *That is, if a solution of degree* $\bar{g}$ *exits, a sequence of LMIs providing sufficient conditions for the existence of homogenous polynomials of increasing degree* $g > \bar{g}$ *can be used, with convergence assured for a large of enough* g [17]. *The LMI conditions, expressed only in terms of the vertices of the system, were obtained with the ROLMIP (Robust LMI Parser) toolbox available at http://www.dt.fee.unicamp.br//agulhari/rolmip/rolmip.htm. The toolbox is developed for Matlab and works jointly with YALMIP, returning the entire set of LMIs through simple commands that describe the structure of the matrices involved and the robust LMI conditions to be programmed.*

Remark 3.4. *We mention that in (9), there are products between the variables* T_i $(i = 1, 2)$ *and matrices* Z *and* X_{33}. *In special case where* T_i $(i = 1, 2)$ *equal zero in (11), Theorem 3.2 can be solved with good results. Thus, less conservative* H_∞ *bounds can be obtained by taking account of variables* T_i $(i = 1, 2)$ *as proposed in (9) based on the algorithm* [19] *shown in the following step.*

Thus, to solve the non linearities problem in (9) caused by the product between T_i $(i = 1, 2)$ and Z, we propose the following algorithm [19].

Algorithm 1. [19]*: Initialize matrices* T_i *and scalar* γ_T *and* γ_Z
while $\left|\frac{\gamma_T - \gamma_Z}{\gamma_T}\right| > \epsilon$ ***and*** *(Maximum number of iterations not reached)* ***do***

Solve Theorem 3.2 with T_i, *minimizing* γ, $\gamma_T \longleftarrow \gamma$
find and store matrices X_{33} *and* Z
Solve Theorem 3.2 with X_{33} *and* Z *obtained in the previous step, minimizing* γ,
$\gamma_Z \longleftarrow \gamma$, *find matrices* T_i.

end while.

Remark 3.5. *This algorithm provides less conservative results and helps to have extra free weighting matrices* T_i $(i = 1, 2)$. *Thus, this method provides better results than ones in* [24].

4. **Numerical Examples.** This section is devoted to numerical evaluation of the conservatism of the proposed design methodology as compared with other method in literature. All the simulations given were implemented in Matlab, version 7.6.0 using the tool boxes Yalmip [3] and SeDuMi [5].

Based on Theorem 3.2 and the algorithm given in the previous section by initializing T_i $(i = 1, 2)$ for each example with appropriate dimension, $\gamma_T = 2$, $\gamma_Z = 1.5$, $\epsilon = 0.00000001$ and the maximum number of iterations in Algorithm 1 has been fixed in 40. Other initial conditions for T_i $(i = 1, 2)$ could provide different results. ϵ $\left(\left|\frac{\gamma_T - \gamma_Z}{\gamma_T}\right| > \epsilon\right)$ is chosen in the form to get the closed values between γ_T and γ_Z, which means that the obtained γ is stationary.

4.1. **Example 1.** Consider the continuous-time system [24] with the vertices number being $N = 2$ given as the following:

$$A_1 = \begin{bmatrix} -2.98 & -0.57 & 0 & -0.034 \\ -0.99 & -0.21 & 0.035 & -0.0011 \\ 0 & 0 & 0 & 1 \\ 0.39 & -5.5550 & 0 & -1.89 \end{bmatrix}, \; A_2 = \begin{bmatrix} -2.98 & 2.43 & 0 & -0.034 \\ -0.99 & -0.21 & 0.035 & -0.0011 \\ 0 & 0 & 0 & 1 \\ 0.39 & -5.5550 & 0 & -1.89 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0.032 \\ 0 \\ 0 \\ 1.6 \end{bmatrix}, \; E_1 = E_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$C_{11} = C_{12} = \begin{bmatrix} 1 & 0 & 0 & 2 \end{bmatrix}, \; D_1 = D_2 = 1, \; F_1 = F_2 = 0,$$

$$C_{21} = C_{22} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; H_1 = H_2 = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$$

and T_i $(i = 1, 2)$ has been initialized by $T_1 = [1 \quad 1 \quad 1 \quad 1]$ and $T_2 = [1 \quad 1]$.

Table 1 shows the H_∞ performance bounds obtained for different value of g. As can be seen, the proposed approach provides less conservative results than the ones in [24]. Note that, N_D increases by increasing the polynomial degree. Due to that, by increasing degrees the conservatism decreases even if the problem becomes more complicated.

TABLE 1. Comparison results of H_∞ performance

	Theorem 3.2 $g = 2$	Theorem 3.2 $g = 1$	[24]
$\gamma_{\min}$	0.6163	0.6163	0.6581
N_D	76	52	27

with N_D being the decision variable number

Now we show the output controlled $z(t)$ and the ratio $r(t) = \sqrt{\frac{\int_0^t z^T(i)z(i)di}{\int_0^t w^T(i)w(i)di}}$ in Figure 1 and Figure 2 respectively with $w(t) = \frac{\sin(t)}{t^{1.5}+1}$ and initial condition of the state $x(t)$ are null, under the obtained control law value $K = [-5.4885 \quad -3.3617]$.

Figure 2 shows that the ratio tends toward a constant value 0.5847, which is less than prescribed value $\gamma_{\min} = 0.6163$.

Figure 3 depicts the evolution of γ_Z and γ_T in Algorithm 1 proposed, where the improvements obtained through the iterative procedure in terms of smaller bounds are apparent and tend toward constant values.

4.2. **Example 2.** In this example, make a comparison between Theorem 3.2 and the approaches given in [23, 24], where the system parameters are in the following form with the vertices number being $N = 2$:

$$A_1 = \begin{bmatrix} -0.9896 & 17.41 & 96.15 \\ 0.2648 & -0.8512 & -11.39 \\ 0 & 0 & -30 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.702 & 50.72 & 263.5 \\ 0.2201 & -1.418 & -31.99 \\ 0 & 0 & -30 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -97.78 \\ 0 \\ 30 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -85.09 \\ 0 \\ 30 \end{bmatrix}, \quad E_1 = E_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix},$$

$$C_{11} = C_{12} = 1, \; D_1 = 0, \; D_2 = 0, \; C_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; C_{22} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \; H_1 = H_2 = 0$$

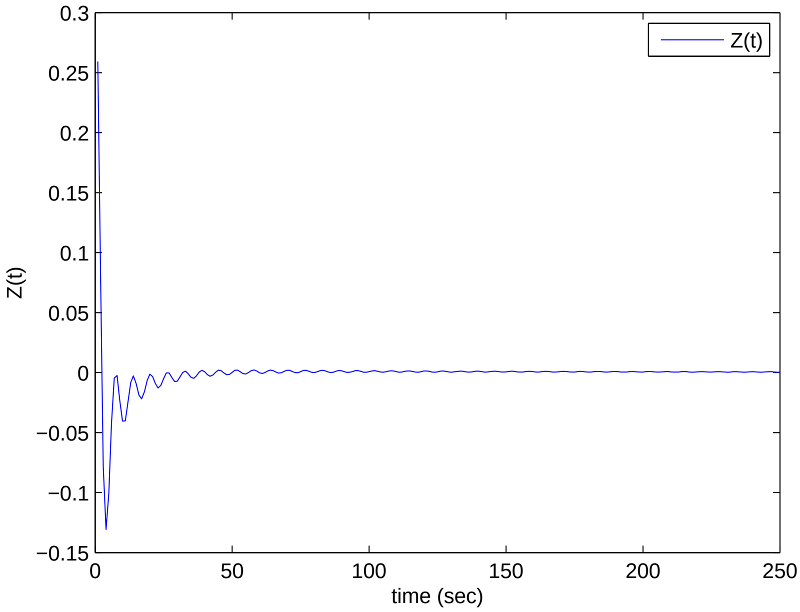

FIGURE 1. Response of the controlled output $z(t)$

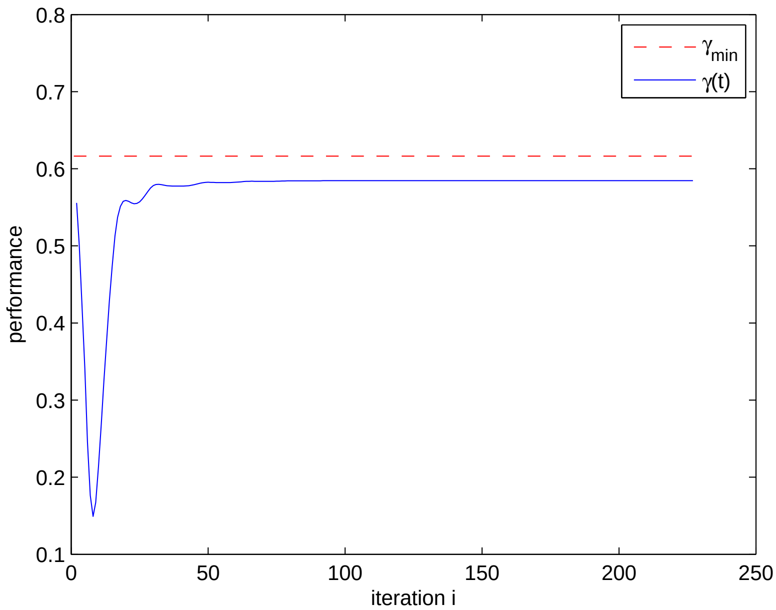

FIGURE 2. Comparison between the ratio $r(t)$ and the prescribed value $\gamma_{\min}$

and T_i $(i = 1, 2)$ has been initialized by $T_1 = [0 \quad 0 \quad 0]$ and $T_2 = [0 \quad -1]$. The results obtained via Theorem 3.2 are presented in Table 2 for different degrees g. It can be mentioned that through this example the effectiveness of the proposed approach is clear, specially by increasing the polynomial degree g. Compared to [23, 24], the H_∞ performance values obtained for different degrees are always the best.

In Figure 4 we present the computational result of transfer function (TF1 and TF2) for the two vertices derived from the system (1) compared with the prescribed value $\gamma_{\min}$.

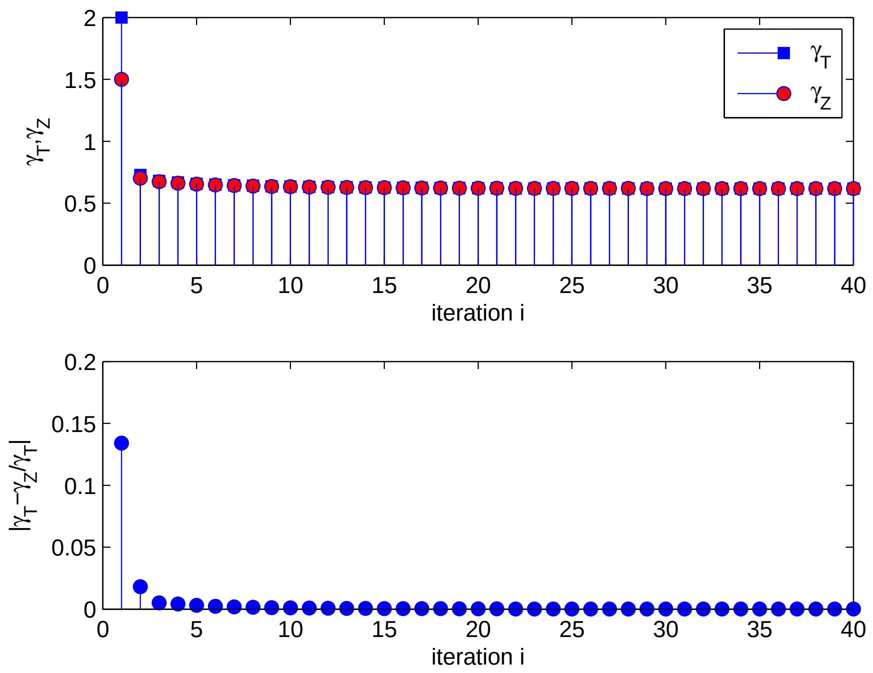

FIGURE 3. Behavior of γ_Z, γ_T and $\left|\frac{\gamma_T-\gamma_Z}{\gamma_T}\right|$ along the iteration of algorithm for Example 1 and $g=1$

TABLE 2. Comparison results of H_∞ performance

	Theorem 3.2 $g=2$	Theorem 3.2 $g=1$	[24]	[23]
$\gamma_{\min}$	2.8007	2.8026	6.6836	7.0362

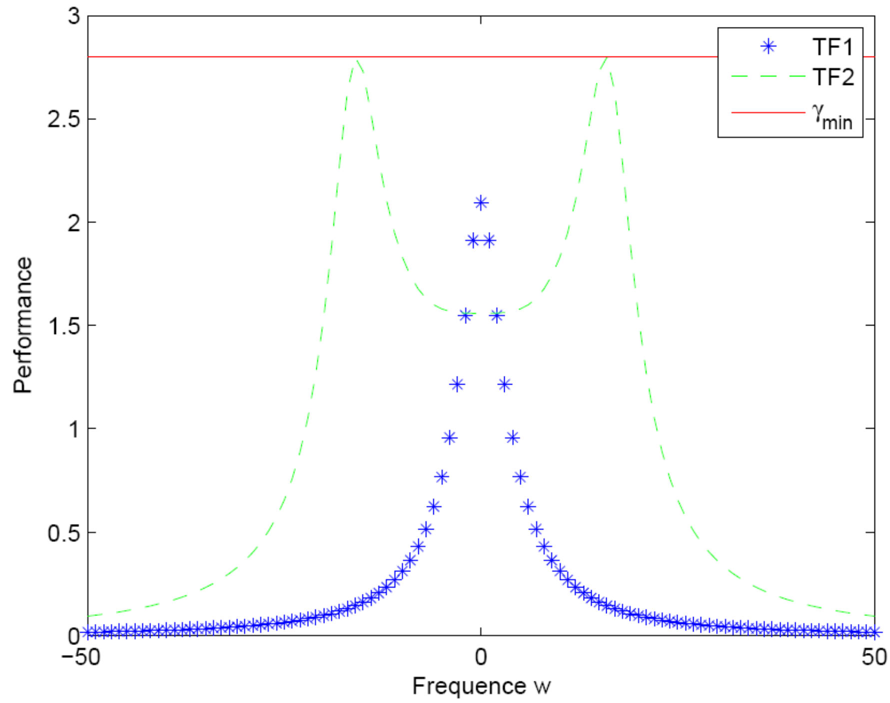

FIGURE 4. The computational result of transfer function

4.3. **Example 3.** In this example we present a real application of the H_∞ SOF control to show the interest of the proposed method in reality. Consider the continuous-time system given in [15] which represents 2 mass-spring model borrowed from [16] as the following:

$$\left[\begin{array}{c|c|c} A(\alpha) & B(\alpha) & E(\alpha) \\ \hline C_1(\alpha) & D(\alpha) & F(\alpha) \\ \hline C_2(\alpha) & H(\alpha) & 0 \end{array}\right] = \left[\begin{array}{cccc|c|c} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ -\frac{q_1(\alpha)+q_2}{m_1} & \frac{q_2}{m_1} & -\frac{C_0(\alpha)}{m_1} & 0 & 1 & m_1 \\ \frac{q_2}{m_2} & -\frac{q_2}{m_2} & 0 & -\frac{C_0(\alpha)}{m_2} & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

with $m_1 = 2$, $m_2 = 1$, $q_2 = 0.5$, $1 \leq q_1 \leq 4$ and $1 \leq c_0 \leq 4$, yielding a polytope of $N = 4$ vertices and T_i $(i = 1, 2)$ has been initialized by $T_1 = [1 \quad 1 \quad 1 \quad 1]$ and $T_2 = [1 \quad 1]$.

The approaches from [13, 15] and Theorem 3.2 in this paper are applicable for designing the robust static output feedback H_∞ controller (2) for this example, the minimum values of the H_∞ performance γ are given in Table 3. The computation results show that the proposed method for different value of g in this paper provides a better alternative design for this example.

TABLE 3. Comparison results of H_∞ performance with literature

	Theorem 3.2 $g = 3$	Theorem 3.2 $g = 2$	Theorem 3.2 $g = 1$	[15]	[13]
$\gamma_{\min}$	7.5662	7.5664	7.5698	7.63	17.58

We present the state $x(t)$ and the output controlled $z(t)$ in Figure 5 and Figure 6 respectively with $w(t) = \frac{\sin(t)}{t^{1.5}+1}$ and initial condition of the state $x(t)$ are null with the controller gain (2)

$$K = [-2.6277 \quad 1.9311] .$$

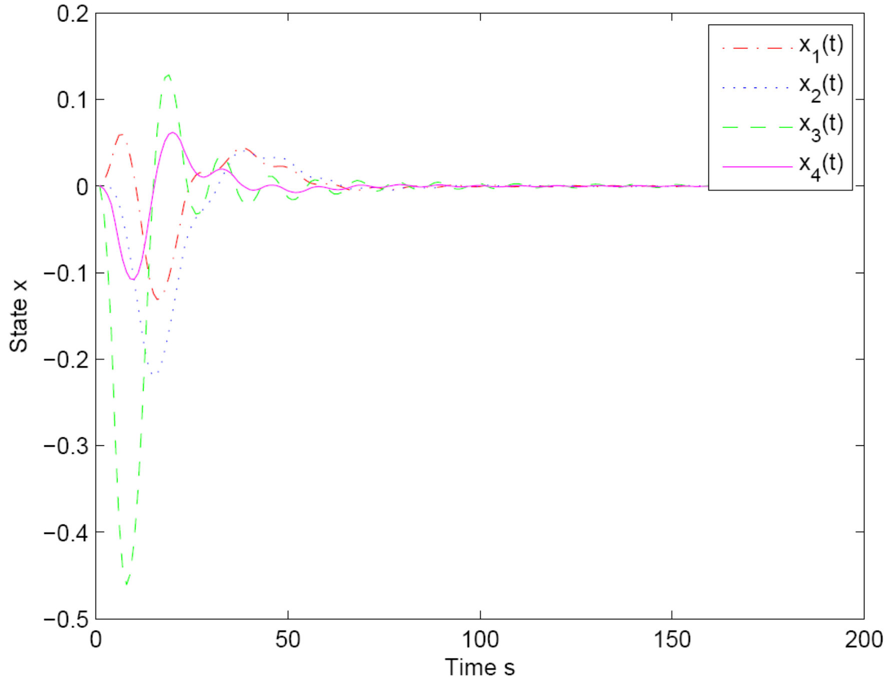

FIGURE 5. The state behavior of the system

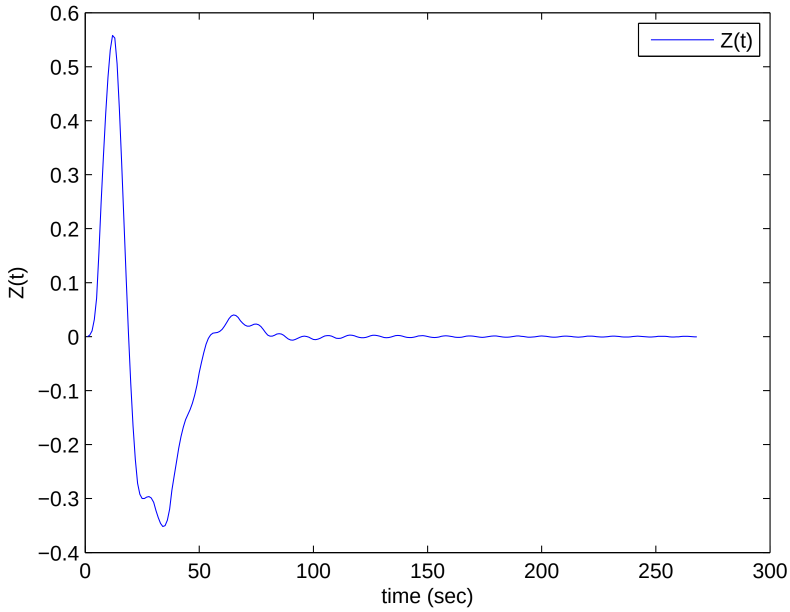

FIGURE 6. The output controlled $z(t)$ for Example 3 and $g = 1$

Remark 4.1. *Before concluding, from Table 1, Table 2 and Table 3 we want to clarify the complexity side of our approach and to put some comments on computational efficiency. Through this work, we were able to add some free matrices via descriptor redundancy approach in order to reduce the conservatism, but it increases the complexity* N_D *and the dimension of our LMIs. This constraint may be considered as the cost to be paid to obtain the best results.*

5. **Conclusion.** In this paper, the problem of static output-feedback H_∞ controller for continuous-time linear systems has been studied. Sufficient condition for controller design has been derived via solving a set of linear matrix inequalities based on an iterative algorithm and the polynomial approach. In particular, it has been proved that the new proposed conditions are more relaxed than the existing ones which reduce the conservatism. Finally, numerical examples have been considered to illustrate the effectiveness of the proposed approach.

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