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DESIGN A DISTRIBUTED CONTROLLER FOR MICROGRIDS

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Abstract. *This paper addresses the problem of distributed energy management system (EMS) design considering network-induced delays. First of all, the distribution power network incorporating solar panels is modelled as a discrete-time linear state-space equation and its measurements/control information is obtained using a set of sensors and actuators. These electronic devices send the observation information to the EMS via unreliable communication links. In order to stabilize the system, we propose discrete-time distributed static output feedback control strategies based on the linear matrix inequality and semidefinite programming approaches. Using the Taylor series method, delays are integrated into the closed loop system so that the proposed controller runs with less energy resources and computation. Simulation results demonstrate that the proposed method is able to stabilize the system in a fairly short time.*
Keywords: Delays, Energy management system, Feedback controller, Microgrid

1. **Introduction.** There are many different feedback control techniques available for the power system stability in the literature. To begin with, the load frequency control scheme is modelled considering delays, and then the proportional integral (PI) controller is used for stabilizing the system. [1] focuses on the prediction based H_∞ control design strategy taking delay into account in the feedback control signals. Next, the centralized control strategy considering delay is suggested for a large-scale interconnected power system [2]. It shows that time delay in the feedback loop can destabilize the system. In other words, the communication impairments can mislead the design engineer and consumers even causing blackouts in the system. Recently, a unified distributed control strategy for the DC microgrid is proposed in [3]. It shows that the standard distributed PI voltage controllers are no longer able to regulate the average DC microgrid bus voltage, so the distributed voltage controllers are replaced by double integrator controllers. Interestingly, the three types of H_∞ delay free feedback controllers (centralized), namely static output feedback controllers, dynamic output feedback controllers, and observer-based output feedback controllers are investigated for linear discrete-time systems in [4]. The controller synthesis in discrete-time linear systems with uncertainty is proposed in [5]. Due to the simplicity and easy implementation point of view, the static output feedback controller is preferred in the complex and mission critical network such as smart grid. Usually, the partial system state information is only available so the static output feedback controller design remains an open question in the control and smart grid communities. Finally, computing machines have finite memory and temporal resolution [6], so the discrete-time controller is obviously preferred from the engineering aspects. Inspired by the above discussions and analysis, this paper focuses on the delay-dependent distributed static output feedback control strategy in the context of smart grids. After transforming the continuous-time

state space model into a discrete-time framework, the optimal output feedback gain is calculated using a convex optimization process. The efficacy of the developed approaches is verified through numerical simulations.

The remainder of this paper is organized as follows. A microgrid model is illustrated in Section 2. In Section 3, the delay-dependent control strategy is proposed. Section 4 presents simulation results. This paper ends with a conclusion in Section 5.

Notation: Bold face upper and lower case letters are used to represent matrices and vectors, respectively. Superscripts $\mathbf{x}'$ denotes the transpose of $\mathbf{x}$, $eig(\mathbf{X})$ denotes eigen values of $\mathbf{X}$, $|\mathbf{X}|_2$ denotes the 2-norm of $\mathbf{X}$, diag$(\mathbf{x})$ denotes diagonal matrix and $\mathbf{I}$ denotes the identity matrix.

2. **Distribution Power Network Incorporating Microgrids.** The considered solar cells are connected through the IEEE-4 bus distribution system shown in Figure 1 [7, 8]. The state space framework of this microgrid is written as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \tag{1}$$

where $\mathbf{x}(t) = \mathbf{v}_s - \mathbf{v}_{ref}$ is the point of common coupling (PCC) state voltage deviation, $\mathbf{v}_s = (v_1\ v_2\ v_3\ v_4)'$, v_i is the ith PCC voltage, $\mathbf{v}_{ref}$ is the PCC reference voltage, $\mathbf{u}(t) = \mathbf{v}_p - \mathbf{v}_{pref}$ is the distributed energy resource (DER) control input deviation, $v_p = (v_{p1}\ v_{p2}\ v_{p3}\ v_{p4})'$, v_{pi} is the ith DER input voltage and $\mathbf{v}_{pref}$ is the reference control effort. The matrices $\mathbf{A}$ and $\mathbf{B}$ are given in [7, 8, 9].

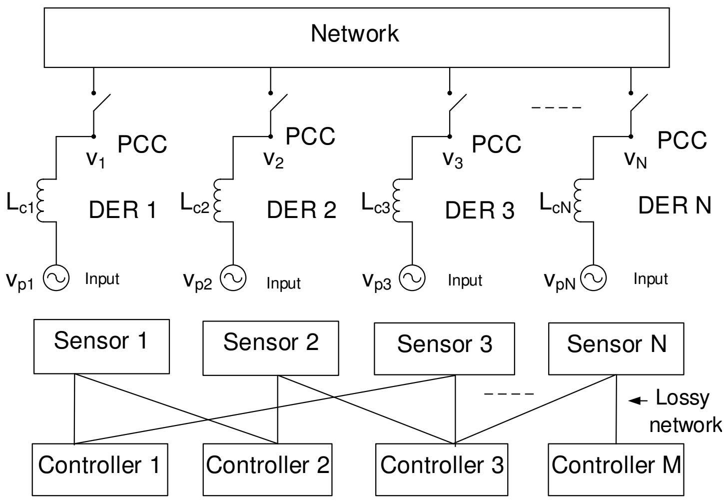

FIGURE 1. Micro-sources are connected to the distribution power network [7].

The microgrid state information is obtained by a set of smart sensors as follows:

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t), \tag{2}$$

where $\mathbf{y}(t)$ is the system measurements.

3. **Proposed Delay-Dependent Distributed Control Strategy.** The feedback control is employed for stabilizing the microgrid states, which is given by:

$$\mathbf{u}(t) = \mathbf{F}\mathbf{y}(t) = \mathbf{F}\mathbf{C}\mathbf{x}(t). \tag{3}$$

Here, $\mathbf{F}$ is the sparse feedback gain to be designed. If there is no connection between a sensor and a controller, the corresponding element of $\mathbf{F}$ is zero. For example, in Figure 1,

$\mathbf{F}$ belongs to the following structure:

$$\mathbb{F} = \left\{ \mathbf{F} \mid \mathbf{F} = \begin{bmatrix} F_{11} & F_{12} & 0 & 0 \\ 0 & F_{22} & F_{23} & 0 \\ F_{31} & 0 & F_{33} & 0 \\ 0 & 0 & F_{43} & F_{44} \end{bmatrix} \right\}. \tag{4}$$

Here, the feedback element F_{NM} is the connection between subsystem sensor N and controller M. Using $\mathbf{u}(t)$, (1) can be written as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{Ax}(t) + \mathbf{BFCx}(t) = (\mathbf{A} + \mathbf{BFC})\mathbf{x}(t) = \tilde{\mathbf{A}}\mathbf{x}(t), \tag{5}$$

where $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{BFC}$ is the closed loop system state matrix. Now, (5) is expressed as a discrete-time state-space model as follows:

$$\mathbf{x}(k+1) = \tilde{\mathbf{A}}_d\mathbf{x}(k), \tag{6}$$

where $\tilde{\mathbf{A}}_d = \mathbf{A}_d + \mathbf{B}_d\mathbf{FC}$, $\mathbf{A}_d = \mathbf{I} + \mathbf{A}\Delta t$, Δt is the discretization step size parameter, $\mathbf{B}_d = \mathbf{B}\Delta t$, $\mathbf{C} = \mathbf{C}_d$ and $\mathbf{F} = \mathbf{F}_d$ [10]. If there exists a gain matrix $\mathbf{F}$ given symmetric positive-definite matrix $\mathbf{P}$, then the following LMI holds:

$$\begin{aligned} &\tilde{\mathbf{A}}'_d\mathbf{P}\tilde{\mathbf{A}}_d - \mathbf{P} < \mathbf{0},\ \mathbf{P} > \mathbf{0} \\ &(\mathbf{A}_d + \mathbf{B}_d\mathbf{FC})'\mathbf{P}(\mathbf{A}_d + \mathbf{B}_d\mathbf{FC}) - \mathbf{P} < \mathbf{0},\ \mathbf{P} > \mathbf{0}. \end{aligned} \tag{7}$$

It can be seen that the Lypunov matrix $\mathbf{P}$ is unknown which makes (7) quite difficult to solve. Inspired by the two-step procedure in the continuous-time case [7], $\mathbf{P}$ can be firstly computed as follows:

$$(\beta\mathbf{A}_d)'\mathbf{P}(\beta\mathbf{A}_d) - \mathbf{P} < \mathbf{0}. \tag{8}$$

Here, $\beta = 1/[\gamma \max\{eig(\mathbf{A}_d)\}]$, $\gamma > 1$ is a free parameter and $\max\{eig(\mathbf{A}_d)\}$ is the maximum eigen values of $\mathbf{A}_d$. The quantity γ ensures eigenvalues of the scaled close loop system strictly less than one. Now according to the standard Schur's complement, (8) can be transformed into the following LMI form:

$$\begin{bmatrix} -\mathbf{P} & \beta\mathbf{A}'_d\mathbf{P} \\ \beta\mathbf{P}\mathbf{A}_d & -\mathbf{P} \end{bmatrix} < \mathbf{0}. \tag{9}$$

Consider there is a delay, d, between sensors and controllers. Now expand $\mathbf{x}(t-d)$ using the Taylor series as:

$$\mathbf{x}(t-d) \approx \mathbf{x}(t) - d\dot{\mathbf{x}}(t) \approx [\mathbf{I} - d(\mathbf{A} + \mathbf{BFC})]\mathbf{x}(t). \tag{10}$$

The control effort with delay can be written as follows:

$$\mathbf{u}(t) = \mathbf{Fy}(t-d) = \mathbf{FC}[\mathbf{I} - d(\mathbf{A} + \mathbf{BFC})]\mathbf{x}(t). \tag{11}$$

Now the state-space model (1) with delay can be written as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{Ax}(t) + \mathbf{BFC}[\mathbf{I} - d(\mathbf{A} + \mathbf{BFC})]\mathbf{x}(t) \\ &= (\mathbf{A} + \mathbf{BFC})(\mathbf{I} - d\mathbf{BFC})\mathbf{x}(t). \end{aligned} \tag{12}$$

For a real-time implementation, transform (12) into a discrete-time form:

$$\mathbf{x}(k+1) = \left(\tilde{\mathbf{A}}_d + \mathbf{G}\right)\mathbf{x}(k). \tag{13}$$

Here, $\tilde{\mathbf{A}}_d = \mathbf{A}_d + \mathbf{B}_d\mathbf{FC}$, $\mathbf{A}_d = \mathbf{I} + \mathbf{A}\Delta t$, $\mathbf{I}$ is the identity matrix, $\mathbf{B}_d = \mathbf{B}\Delta t$ and $\mathbf{G} = -d(\mathbf{A} + \mathbf{BFC})\mathbf{B}_d\mathbf{FC}$. Due to the complexity of $\mathbf{G}$, we treat it as uncertainty in the following analysis. It is easy to verify that

$$\mathbf{G}'\mathbf{G} \leq \alpha^2\mathbf{I}, \tag{14}$$

where α is given by:

$$\alpha = d\left(||\mathbf{A}||_2 + ||\mathbf{B}||_2\bar{f}||\mathbf{C}||_2\right)||\mathbf{B}_d||_2\bar{f}||\mathbf{C}||_2. \quad (15)$$

Here, $||\bullet||_2$ denotes the induced l_2 norm and $\bar{f}$ is given an upper bound of $||\mathbf{F}||_2$ for preventing $\mathbf{F}$ being excessively large. To enforce the upper bound $\bar{f}$ of $\mathbf{F}$, i.e., $||\mathbf{F}||_2 < \bar{f}$, the following LMI is needed:

$$||\mathbf{F}||_2 < \bar{f} \;\Rightarrow\; \begin{bmatrix} -\bar{f}^2\mathbf{I} & \mathbf{F}' \\ \mathbf{F} & -\mathbf{I} \end{bmatrix} < 0. \quad (16)$$

In order to design $\mathbf{F}$, consider the following Lyapunov function:

$$V(k+1) = \mathbf{x}'(k+1)\mathbf{P}\mathbf{x}(k+1) = \mathbf{x}'(k)\left(\tilde{\mathbf{A}}_d + \mathbf{G}\right)'\mathbf{P}\left(\tilde{\mathbf{A}}_d + \mathbf{G}\right)\mathbf{x}(k). \quad (17)$$

The Lyapunov function increment $\Delta V(k+1) = V(k+1) - V(k)$ is:

$$\Delta V(k+1) = \mathbf{x}'(k)\left[\left(\tilde{\mathbf{A}}_d + \mathbf{G}\right)'\mathbf{P}\left(\tilde{\mathbf{A}}_d + \mathbf{G}\right) - \mathbf{P}\right]\mathbf{x}(k). \quad (18)$$

The system is stable if $\Delta V(k+1) < 0$. This is achievable if:

$$(\mathbf{A}_d + \mathbf{B}_d\mathbf{F}\mathbf{C} + \mathbf{G})'\mathbf{P}(\mathbf{A}_d + \mathbf{B}_d\mathbf{F}\mathbf{C} + \mathbf{G}) - \mathbf{P} < \mathbf{0},\ \mathbf{P} > \mathbf{0}. \quad (19)$$

Given $\mathbf{P}$ in (9), the inequality (19) can be transformed into the following form:

$$\begin{bmatrix} -\mathbf{P} & (\mathbf{A}_d + \mathbf{B}_d\mathbf{F}\mathbf{C})'\mathbf{P} \\ \mathbf{P}(\mathbf{A}_d + \mathbf{B}_d\mathbf{F}\mathbf{C}) & -\mathbf{P} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{G}'\mathbf{P} \\ \mathbf{P}\mathbf{G} & \mathbf{0} \end{bmatrix} < \mathbf{0}. \quad (20)$$

The second term in (20) can be written as follows:

$$\begin{bmatrix} \mathbf{0} & \mathbf{G}'\mathbf{P} \\ \mathbf{P}\mathbf{G} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{P} \end{bmatrix}\mathbf{G}[\mathbf{I}\ \mathbf{0}] + \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}\mathbf{G}'[\mathbf{0}\ \mathbf{P}]. \quad (21)$$

The following lemma is used to simplify the equality (21).

Lemma 3.1. *Let* $\mathbf{E}$*,* $\mathbf{M}(k)$*, and* $\mathbf{H}$ *be real matrices of appropriate dimensions with* $\mathbf{M}'(k)\mathbf{M}(k) \le \mathbf{I}$, $\forall k \ge 0$*, then for any scalar number* $\epsilon > 0$*, the following inequality holds:*

$$\mathbf{E}\mathbf{M}(k)\mathbf{H} + \mathbf{H}'\mathbf{M}'(k)\mathbf{E}' \le \epsilon^{-1}\mathbf{E}\mathbf{E}' + \epsilon\mathbf{H}'\mathbf{H}. \quad (22)$$

Using (22), the inequality (21) becomes:

$$\begin{bmatrix} \mathbf{0} & \mathbf{G}'\mathbf{P} \\ \mathbf{P}\mathbf{G} & \mathbf{0} \end{bmatrix} \le \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \epsilon^{-1}\mathbf{P}^2 \end{bmatrix} + \alpha^2\begin{bmatrix} \epsilon\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}. \quad (23)$$

Using (23), the inequality (20) becomes:

$$\begin{bmatrix} -\mathbf{P} + \alpha^2\epsilon\mathbf{I} & (\mathbf{A}_d + \mathbf{B}_d\mathbf{F}\mathbf{C})'\mathbf{P} & \mathbf{0} \\ \mathbf{P}(\mathbf{A}_d + \mathbf{B}_d\mathbf{F}\mathbf{C}) & -\mathbf{P} & \mathbf{P} \\ \mathbf{0} & \mathbf{P} & -\epsilon\mathbf{I} \end{bmatrix} \le \mathbf{0}. \quad (24)$$

In summary, the proposed sparse distributed feedback gain is designed by solving (9) for $\mathbf{P}$ and (16), (24) for gain $\mathbf{F}$.

4. **Numerical Results and Discussion.** The system parameters are specified in Table 1. In this table, the process and measurement noise covariance matrices are denoted by $\mathbf{Q}_n$ and $\mathbf{R}_w$. The considered measurement noise covariance is a diagonal matrix [11, 12]. The sampling period for discretization is 0.1 ms.

It is assumed that the delay is uniformly distributed between 0.2 ms to 1 ms, and there are process and measurement noises in the system. It is assumed that the sensing measurement delay is fixed in the range of 2-4 samples period [13]. To begin with, considering the measurement delay is 2 samples, the simulation result is presented in Figure 2. It can be observed that the proposed algorithm can stabilize the system within a short time. This stabilization is due to the fact that the proposed controller properly calculates the

TABLE 1. Parameter values for delay-dependent controller design.

Parameters	Values	Parameters	Values
$\mathbf{Q}_n$	$0.001 * \mathbf{I}_4$	$\mathbf{R}_w$	$0.01 * \mathbf{I}_4$
γ	2	Delay	Uniform distribution

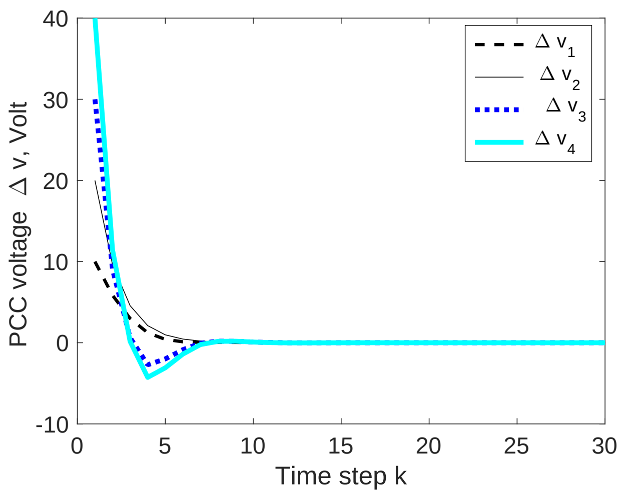

FIGURE 2. System states response with 2 delay samples

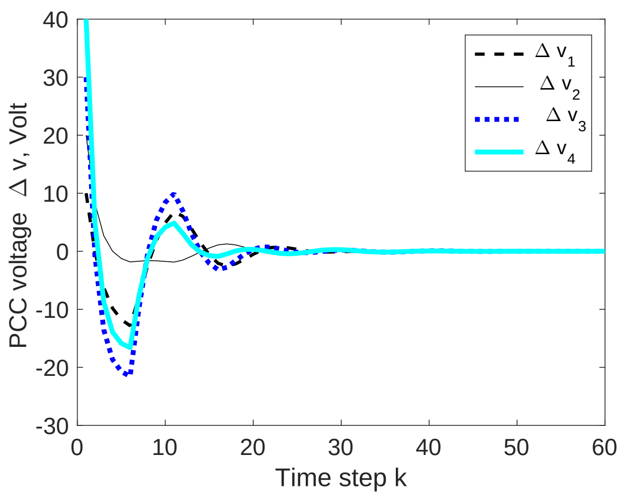

FIGURE 3. System states response with 4 delay samples

controller gain such that the system states will be stable in a fairly short time even if certain amount of delay exist. On the other hand, when the sample delay is increased to 4, then the simulation result is presented in Figure 3. It is expected that it suffers more damping and requires more time for stabilizing the system compared with the 2 samples delay in Figure 2. In other words, the system performance is deteriorated if the network induced more delays.

5. **Conclusions.** This study presents the discrete-time distributed output feedback control strategy under the condition of network induced delays. Based on the linear matrix inequality and semidefinite programming approaches, the proposed algorithm and control synthesis of the power network is derived. The proposed control framework could properly determine the output feedback gain such that the system states will be stable in a fairly short time. Simulations are carried out to verify the effectiveness and feasibility of the proposed methods. Future study includes design of the distributed controller considering both communication delays and packet losses.

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